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OPTIMIZATION OF AN AUTOMATED MANUFACTURING SYSTEM SIMULATION MODEL USING SIMULATED ANNEALING

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ABSTRACT

The purpose of this study is to investigate the feasibility of using a simulated annealing algorithm in conjunction with a simulation model to find the optimal parameter levels at which to operate the system being simulated. In particular, we discuss an effort to use simulated annealing to find a combination of input parameter values for an automated manufacturing system which optimizes a nonconvex, nonconcave objective function of the input parameters.

This paper contains a brief description of an automated manufacturing system used to assemble three products. The problem objective is to maximize profit as a function of the levels of three parameters – batch size of arriving products, distribution of products in the batches, and machine output buffer size. Simulated annealing is then used to search for the optimal combination of input parameter levels. By experimenting with the simulated annealing parameters, the algorithm parameters are chosen such that the annealing program will consistently find the global optimum after evaluating approximately 30% of the input variable combinations.

1. INTRODUCTION

A typical optimization problem is to maximize a real-valued function $F(\mathbf{x})$ where feasible points $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ are restricted to some constraint set $X \subset \mathcal{R}^n$. If this problem is analytically intractable, as is often the case in manufacturing or other practical environments, then the analyst may choose to simulate the system. The optimization of this simulated system is the focus of this paper.

The aim of simulation optimization is to determine the inputs, sometimes called *parameters*, which optimize the outputs, or *performance measures*, of the simulation experiment. The performance measures may be stochastic functions of the parameters because of the random nature of the experimental model's inputs. For the purposes of optimization, the simulation is simply a 'black box' which computes a (stochastic) function value for a given combination of the parameter values.

Meketon (1987) surveys approaches and recent results in simulation optimization. These approaches include standard non-linear programming techniques, response surface methodologies and stochastic approximation techniques. A drawback of such approaches is that they terminate upon finding a local optimum. In another survey, Glynn (1986) presents a framework of optimization problems and some of their corresponding solution approaches. The approaches discussed by Meketon are, according to Glynn, more appropriate for a class of problems characterized by a finite number of continuous parameters. However, these approaches are apparently not effective when the parameters are discrete, i.e., take on only a countable number of values.

In this paper, we consider the simulation optimization problem of maximizing the total expected profit of a small, hypothetical automated manufacturing system. There are three discrete parameters used to describe the operation of this system. The expected profit is, by construction, a complicated function which only implicitly depends on the parameters and has multiple local maxima. Hence, this problem is characterized by a finite number of discrete parameters and is not amenable to optimization via the aforementioned approaches. We propose, instead, to solve this problem using *simulated annealing*.

Simulated annealing is an iterative stochastic search method, analogous to the physical annealing process whereby material is gradually cooled so that a minimal energy state is achieved. This method, rather than have a sequence of feasible solutions *always* follow a descending objective function value path, allows this sequence to descend along a path *most* of the time. In this way, the path traversed may leave the area around a local minimum to locate either a better local minimum or, perhaps, the global minimum. Of course, this approach may just as easily be applied to maximization problems. Metropolis *et al.* (1953) first introduced the simulated annealing concept while Kirkpatrick *et al.* (1983) successfully applied the approach to deterministic optimization problems. Bulgak *et al.* (1988) have recently demonstrated one application of simulated annealing to simulation optimization.

The goals of this paper are to demonstrate that simulated annealing is a viable and efficient approach to solving problems characterized by a finite number of discrete parameters and to suggest ways to overcome some obstacles inherent to simulated annealing. In the next section, the model of the automated manufacturing system is briefly described. The simulated annealing procedure and its associated concerns are discussed in the third section. The experimental results are contained in the fourth section and our concluding comments are in the last section.

2. THE MODEL

The automated manufacturing system modeled in this study consists of four machines used to assemble three similar products and a carrousel used for transportation between the machines. Each machine has both an input and output buffer. Product batches enter into the system's initial inventory area at fixed and regular time periods. A more detailed description of the system is supplied by Manz (1989).

The three parameters, i.e., the decision variables of the simulation, describing the variable features of the system's operation are 1) the size of the arrival batches, 2) the distribution of products within the arrival batches, and 3) the size of the output buffers at each machine. Each parameter may take on multiple values. There are six different batch size values, five different batch distributions of products and four different local output buffer configurations.

The performance measure is total expected profit, averaged over a time interval excluding the initial transient period. Profit is defined as the total revenue minus the total cost. Revenue is accrued from the sale of each of the products while costs are incurred from the purchase of raw material, holding partially completed products in inventory, and assembly costs. This profit function has multiple local maxima and is therefore representative of many objective functions in practical situations. Such functions tend to be troublesome for traditional mathematical programming algorithms.

3. THE ALGORITHM

Simulated annealing is a stochastic optimization method which performs well on combinatorial problems and only uses specific function values, i.e., derivatives or other special function information are not required. Hence, simulated annealing ought certainly to be a candidate solution approach for simulation optimization.

The algorithm begins by randomly choosing a point c in the parameter space, evaluating that point (i.e., running a simulation experiment) and assigning to V_c the corresponding function evaluation. Subsequently an adjacent feasible point a satisfying $\| c - a \| = 1$ is randomly selected and then evaluated, and the corresponding function evaluation is assigned to V_a. The point a becomes the current point c with probability $\frac{1}{1+e^{(\frac{V_c - V_a}{T})}}$, where T is a parameter describing the current temperature of the annealing process. The temperature T is then decreased and this iteration is repeated.

The algorithm's behavior is strongly dependent on the existing temperature. At high temperatures, the probability of accepting the adjacent point a is large, thus allowing acceptance of downhill moves on the way to larger uphill moves. As the temperature decreases, however, the probability of accepting downhill moves decreases while the probability of accepting uphill moves increases.

A sequence of non increasing temperatures

$\{T(j)\}_{j=1}^{\infty}$, where $T(j)$ is the temperature during the jth iteration, define an *annealing schedule*. Geman *et al.* (1984) demonstrate the convergence of the simulated annealing approach when $T(j)$ satisfies $T(j) \geq \frac{C}{\log(1+j)}$ for some constant C which is independent of j. Hence, to ensure that convergence to a global optimum occurs, the temperature should be decreased gradually. However, the applicability of this result to stochastic functions, i.e., simulation evaluations, is unknown and, further, the convergence may be too slow to be useful in practice. Therefore, we chose a more rapid, heuristic annealing schedule which Ackley (1987), among others, has used.

Our annealing schedule is determined by a set of four parameters, $\{T_i, T_f, r, k\}$. The algorithm is initialized at a starting temperature T_i. The current temperature T_{now} is maintained for k iterations and then decreased at a rate r so that $T_{new} = r \cdot T_{now}$. The algorithm stops after the current temperature passes below a threshold value T_f.

An important aspect to the efficient use of this simulated annealing algorithm is the selection of an annealing schedule so that the solution obtained is a global, or at least a local, optimum when no *a priori* knowledge of the objective function 'terrain' exists. If the temperature decreases too quickly, then the solution obtained may not even be a local optimum. On the other hand, if the temperature decreases too slowly, an excessive number of expensive function evaluations may have been performed. A brief discussion of the algorithm's performance for various values of these annealing schedule parameters may now be appropriate.

A high initial temperature means there is a high probability of accepting downhill moves at the start of a search while a lower initial temperature results in a lower probability. Similarly, using a relatively high final temperature implies there is a relatively high probability of accepting downhill moves at the end of a search while using a low final temperature results in a relatively low probability of accepting downhill moves and thereby increases the tendency to locate a local, if not global, optimum at the end of a search.

Temperature decay rates, r, closer to 1 cause the temperature to decrease slowly and thus allow the algorithm to search a relatively broad area and to accept a relatively large number of downhill moves. Hence, using such decay rates is, perhaps, preferable in steep, hilly terrain where the global optimum may be several peaks away from the current point. Conversely, use of decay rates closer to 0 may be appropriate for less hilly terrain where fewer steep downhill moves would be necessary to explore the space well.

The number of iterations to be performed at each temperature, k, also affects the performance of the algorithm. As k is increased, more points are evaluated and larger areas of the terrain may be searched. This enhances the algorithm's chances of locating a global optimum.

The choice of suitable parameters requires exploration of the parameter space and experimentation with combinations of simulated annealing parameters. Since this experimentation may require a large number of function evaluations, simulated annealing appears to be most attractive at present for those applications where a number of similarly landscaped variations on a particular model will be run. Only one startup effort would be needed to choose the initial parameters. Optimization of variations of the system would then proceed with the chosen parameters.

4. RESULTS

Each of the 120 combinations of automated manufacturing system parameters was simulated using the model developed in the SIMAN simulation language and the point estimate representing the expected total profit from each simulation run was placed in a datafile. This datafile was constructed to confirm that the objective function contained multiple local maxima, i.e., was neither convex nor concave. In fact, the expected total profit was nonconvex, nonconcave across batch size and across the range of distributions. The global optimum of 127.2 occurs at a batch size of 45, using Distribution 2 and Output Buffer Configuration 1.

Simulated annealing parameters were chosen by evaluating combinations of the initial and final temperatures, temperature decay rates, and number of iterations performed at each temperature. Computer runs were made at three rates of temperature decrease: 0.8, 0.6, and 0.4. The initial temperature was set at 100 for all of the runs and the final temperature was set to 10 for half of the runs and 5 for the other half. The number of points, k, evaluated at each temperature was varied from 3 to 8.

Five separate runs were made at each combination of simulated annealing parameters using five different starting points. The results of one set of simulated annealing searches are shown in Figure 1. (Less than five distinct points may appear at each iteration level, k, since some of the runs achieved the same maximum point.)

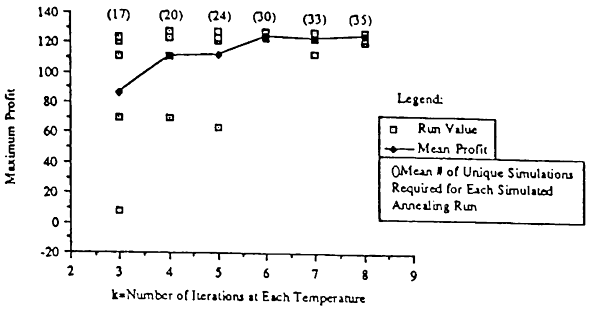

Figure 1a: Maximum Profit vs. Number of Iterations, Initial Temp=100, Final Temp=10, r=0.8

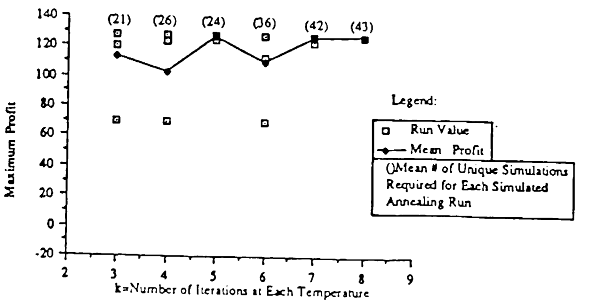

Figure 1b: Maximum Profit vs. Number of Iterations, Initial Temp=100, Final Temp=5, r=0.8

From Geman *et al.* (1984), recall that optimal annealing schedules decreased the temperature very slowly. Hence, if we hypothesize that an optimal annealing schedule applied to simulation optimization were to behave in a similar manner, then better solutions should be obtained with parameter values yielding slower sequences of temperature decrease. Similarly, a sequence which is longer in length ought to result in better solutions. Therefore, a lower final temperature ($T_f = 5$), a slower rate of temperature decrease ($r = 0.8$) and a larger number of iterations performed at each temperature ($k = 8$) should supply annealing schedules yielding better solutions. Our results do, in fact, support these conclusions. Furthermore, the annealing schedule arising from these preferences, $\{T_i = 100, T_f = 5, r = 0.8, k = 8\}$, was the most consistent at locating the global optimum. On average, 43 distinct points (each requiring a separate simulation run), or approximately one third of the system's parameter space, were evaluated using this annealing schedule.

5. SUMMARY

A simulated annealing algorithm was used in conjunction with a simulation model to locate the combination of input parameter values which optimize a nonconvex, nonconcave objective function of the input parameters of an automated manufacturing system. A SIMAN simulation model was developed to evaluate system profit as a function of the levels of three parameters — batch size of arriving products, distribution of products in the batches, and machine output buffer size. Simulated annealing parameters were chosen so that the simulated annealing program would consistently find the global optimum after evaluating approximately 36% of the 120 input variable combinations. This demonstrates the viability of this technique for simulation optimization when the problem is to determine which combination of input parameter values optimizes a system's performance.

Determining an appropriate annealing schedule which will produce a near-optimal solution when little or no information of the objective function terrain is known is difficult. In general, the investigator cannot afford to reduce the temperature too slowly and reducing the temperature too quickly may not be effective. Hence, some experimentation may be required to select suitable simulated annealing parameters. The simulated annealing algorithm would therefore appear to be most attractive for applications where a number of similarly landscaped model variations will be investigated. Once parameter values have been determined for one model, the resulting annealing schedule could be used for the other models.

At present, there are few, if any, alternate approaches to simulation optimization under the conditions considered in this paper. We are in the process of developing and testing an alternate method of optimization using an integer simplex search approach. We intend to compare this method with simulated annealing and to determine which technique would be most efficient given marginal knowledge of the output space terrain.

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