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An algorithm for testing serial dependence of simulation output data

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ABSTRACT

This paper presents an algorithm to determine where the correlation of data sequence has died out. This algorithm can be used to collect essentially uncorrelated observations from a sequence of correlated observations. The final purpose is to use such observations to derive statistical inferential procedures for the parameters concerned. Examples of M/M/1 queues are presented.

INTRODUTION

Simulation modeling is a quite useful technique to determine the behavior of the system under a variety of conditions, to identify means through which system performance may be improved or simply to better understand the behavior of the system. Hence an important objective in simulation output analysis is to estimate means of the system characteristics. These estimates are the primary sources from which the behavior of the system can be characterized. Examples of such estimates include the mean time that arrivals spend in the system or the waiting line, the mean length of the waiting line, and the mean idle time for the service channel or channels.

The data sequence in simulation output often comprises realizations of correlated random variables. This constitutes a major stumbling block for applying classical statistical inferential procedures to the output data of a simulation because these procedures are known to be unreliable if the assumed independence of sample observations is violated. Ho and Schmidt (1987) have shown that the predictability of classical procedures is affected in the presence of data correlation, especially for highly correlated sample observations.

Simulation output analysis has been an active field of research. Consequently various methods have been proposed to resolve the problem of data correlation. These include replication, batch means, parametric modeling, spectral analysis, regeneration cycles, and standardized time series. While each method has its own method of analysis, these procedures can be classified into four categories as in Law (1983): (1) those that seek independent observations. (2) those that seek to estimate correlation in output variables. (3) those that exploit the special probabilistic structure of the underlying process. (4) those that are based upon standardized time series. The procedure to be presented in this paper falls into the first category. Hence in the sections below the attempt to seek uncorrelated observations is the mode to reduce the potential disastrous impact when the assumption of independence is violated.

While the method of replication could provide independent observations, it is truly wasteful that one simulation run only yields a single observation. The method of batch means isn't quite expensive. This method divides a sample sequence into batches of equal size; then computes the sample mean of each batch. Motivation for batching observations is brought about by the anticipation that the absolute magnitude of correlation among batch means decreases asymptotically; thus successive batch means may be treated as uncorrelated if the sample size is large enough. However, Schmeiser and Kang (1981) have shown that the sequence formed by batch means still gives the same type of time series process as that of the original data sequence. Therefore in a batch means sequence the consecutive observations have the possiblity of being subjected to strong autocorrelation. This proposition is supported by empirical results reported by Law and Kelton (1979), where the correlation between batch means is the most serious error in the case of M/M/1 queues.

The problem of correlated batch means gives rise to procedures which seek to find a suitable batch size so that successive batch means can be treated as uncorrelated. Law (1983) discussed some approaches. One is to fix the number of batches and then increases the batch size until the estimated correlation of adjacent batch means is less than a small, user-defined number (Gross and Haris (1974)). The drawback of this approach is that the correlation estimator is generally biased and for small sample sizes is highly variable, thus less likely to produce a reasonable estimate of batch size. Another approach is developed by Fishman (1978). Fishman applied the von Neumann ratio test to determine whether observations in a batch means sequence are independent. The conclusion rendered in Fishman's empirical study is that his method might not perform well if the sample sequence is too positively autocorrelated.

Schmidt and Ho (1987) have proposed the

method of sequential systematic sampling to solve the problem of data cororelation. Similar to replication and batch means, sequential systematic sampling also employs uncorrelated observations to assist construction of inferential procedures. Nevertheless its sampling procedure could be viewed as the converse of batch means. While batch means groups a sequence of consecutive observations (one batch) together, sequential systematic sampling collects observations at intervals of some length (say k observations). If the correlation of sample sequence dies out at lag k, then observations drawn at intervals of k can be considered as essentially uncorrelated. Using a common value k as the batch size and the sampling interval, Ho and Schmit (1987) conducted a simulation study of comparing batch means and sequential systematic sampling. The comparison is based upon the predictability of confidence interval procedures applied to sample observations generated from autoregressive, simple moving average, and M/M/1 queuing models. The results that follow indicated that sequential systematic sampling is more satisfactory than batch means in terms of the predictability of the inferential procedure applied.

Apparently the method of sequential systematic sampling will require a procedure to determine a suitable value of k. The purpose of this paper is to present a procedure developed to identify where the correlation has died out, therefore being able to collect uncorrelated or nearly uncorrelated observations from a sequence of correlated observations. To assure effectiveness of the procedure developed, comparison with a parallel procedure already in use will be pursued in this paper.

Section 2 presents the problem of determing where the correlation has died out. The algorithm suggested by Fishman for determining a suitable batch size is reviewed. Fishman's algorithm will be used in this work as a benchmark for comparison. Section 3 presents an algorithm for determining where the lag correlation has died out. Section 4 presents empirical results for the M/M/1 queuing system applications.

THE PROBLEM

Consider a strictly stationary sequence $\{x_i\}$ with mean μ, variance σ^2 and autocovariance function $\{R_k = \text{Cov}(x_i, x_{i+k}), k=0,1,2,\ldots\}$. The lag correlation ρ_k is then given by $\rho_k = R_k/\sigma^2$. Our objective is to construct an interval estimate for μ. Based upon a sample record $x_1, x_2, \ldots, x_n$, we use the sample mean

$$\bar{x}_n = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad (2.1)$$

as an unbiased estimate for μ. To asses how close $\bar{x}_n$ is to μ, we need an estimate of

$$\text{Var}(\bar{x}_n) = [\,\sigma^2 + 2\sum_{s=1}^{n-1} (1-s/n)\, R_s\,] \;/\; n \qquad (2.2)$$

Hence an 100 (1-α)% confidence interval for is given by

$$\bar{x}_n \pm t_{n-1,1-\alpha/2}\sqrt{\hat{\text{Var}}(\bar{x}_n)} \qquad (2.3)$$

where $\hat{\text{Var}}(\bar{x}_n)$ is an unbiased estimate of $\text{Var}(\bar{x}_n)$ in (2.2).

The origin of our problem is to obtain an unbiased estimate of $\text{Var}(\bar{x}_n)$. This can be achieved if uncorrelated observations can be acquired from $\{x_i\}$. The method of batch means breaks the sample sequence into m batches of size k each, where n=mk. Let y_i be the sample mean of the ith batch. That is

$$y_i = 1/k \sum_{j=(i-1)k+1}^{ik} x_j \qquad (2.4)$$

$$i = 1, 2, \ldots, m.$$

Now each y_i is an unbiased estimate of μ. That is

$$E(y_i) = \mu$$

$$\text{and } \bar{x}_n = 1/n \sum_{j=1}^{m} x_j = 1/m \sum_{i=1}^{m} y_i$$

Now suppose we estimate $\text{Var}(\bar{x}_n)$ by (s_y^2/m) where

$$s_y^2 = \frac{1}{m-1}\sum_{i=1}^{m} (y_i - \bar{x}_n)^2 \qquad (2.5)$$

That is, we treat each y_i (batch mean) as though it were an individual observation.

Application of the method of batch means requires definition of batch size. A sufficiently large batch size might reduce bias in estimating $\text{Var}(\bar{x}_n)$. Several empirical studies including Law and Kelton (1979) have shown that for less congested queuing systems performance of batch means appears to be satisfactory if batch size is large enough. Fishman (1978) also suggested a procedure for determining batch size. The procedure is stated as follows:

1. Define α
2. Select n such that $n=2^N$ where N is an integer
3. r=0
4. r=r+1
5. $k=2^{r-1}$ (batch size)
6. $m=n/2^{r-1}$ (number of batches)
7. $y_1 = \frac{1}{k}\sum_{j=1}^{k} x_j$

$$y_2 = \frac{1}{k}\sum_{j=k+1}^{2k} x_j$$

$$y_i = \frac{1}{k} \sum_{j=(i-1)k+1}^{ik} x_j$$

$\vdots$

8. $$y_m = \frac{1}{k} \sum_{j=(m-1)k=1}^{mk} x_j$$

$$\bar{x}_n = \frac{1}{n} \sum_{j=1}^{n} x_j$$

9. $$C_m = 1 - \frac{\sum_{i=1}^{m-1} (y_i - y_{i+1})^2}{2 \sum_{i=1}^{m} (y_i - \bar{x}_n)^2}$$

10. If $\left| \frac{C_m}{\sqrt{(m-2)/(m^2-1)}} \right| < Z_{1-\alpha/2}$ the batch observations $y_1, y_2, \ldots, y_m$ may be considered independent random variables. Go to step 11. If

$$\left| \frac{C_m}{\sqrt{(m-2)/(m^2-1)}} \right| \geq Z_{1-\alpha/2}$$

go to step 4.

11. $$s_m = \sqrt{\frac{1}{m-1} \sum_{i=1}^{m} (y_i - \bar{x}_n)^2}$$

12. $$L, U = \bar{x}_n \pm \frac{s_m}{\sqrt{m}} t_{1-\alpha/2} (m-1)$$

13. Stop

Fishman's method relies upon the von Neumann ratio test to determine whether observations of a batch means sequence are independent. This method has been tested in the case of M/M/1 queues. Fishman concluded that his method performs well with large sample sizes if the system is not heavily congested.

The method of sequential systematic sampling (Schmidt and Ho (1987)) also attempts to acquire uncorrelated observations through regrouping observations in $\{x_i\}$. The method of regrouping may be considered as the converse of batch means. In applying the method of sequential systematic sampling, the sample observations $x_1, x_2, \ldots, x_n$ are sampled at intervals of length k, That is, the first systematically drawn sample consists of observartions $x_1, x_{k+1}, \ldots, x_{(m-1)k+1}$, the second consists of $x_2, x_{k+2}, \ldots, x_{(m-1)k+2}$, and so forth where n=mk. If the lag correlation ρ_k of $\{x_i\}$ dies out at lag k, then each of the k systematically drawn subsequences will comprise m essentially uncorrelated observations. Thus a procedure for determining where the correlation has died out becomes particularly important for the method of sequential systematic sampling if this method is to be implemented on correlated data.

Testing whether ρ_k dies out at lag k often involves the distribution theory of sample lag covariance and lag correlation. However, for finite n, the distribution theory of sample lag covariance and lag correlation is complicated. In fact, most of the past research on finite sampling distribution has been based upon the assumption of circular correlation to facilitate deriving the distribution of the following:

$$r_s = \sum_{t=1}^{n} x_t x_{t+s} \Big/ \sum_{t=1}^{n} x_t^2 \tag{2.6}$$

The formulation of r_s is given in general when $\mu=0$ is given in Hannan (1970).

Most of the exact results which have been obtained refer only to the distribution of r_1 for a simple model of $\{x_i\}$. References can be found in R.L. Anderson (1974), Koopman (1942), Dixon (1944), and Madow (1945). Various approximations have been derived for the distribution of higher autocorrelation (T. W. Anderson (1948), Hannan (1955), Leipnik (1947), and Hannan (1970)).

The approach using circular corelation not only resorts to a fictitious assumption but also entails considerable computation effort. These two drawbacks inevitably constrain circular correlation from formulating applicable distribution theory of correlation. On the other hand, the asymptotic theory may be used to derive limiting sampling distribution of autocorrelation. Priestley (1981) suggested that the asymptotic normality of the sample autocorelation at lag r may be approximated by

$$\hat{\rho}_r \sim N(\rho_r, \mathrm{Var}(\hat{\rho}_r)) \tag{2.7}$$

Suppose one wants to test the null hypothesis

$$H_0: \rho_k = 0$$

Under the null hypothesis

$$\mathrm{Var}(\hat{\rho}_k) = 1/n \{ 1+2 \sum_{\mu=1}^{q} \rho_\mu \}, \quad k > q \tag{2.8}$$

Two drawbacks are associated with this approach. First, the asymptotic normality may require a sufficiently large sample size which may be infeasible in terms of the final cost incurred. Secondly, the conventional correlation estimate is not unbiased, whose absolute magnitude of bias is often enlarged as the time lag increases. This phenomenon is illustrated in Figure 1, in which law and Kelton (1982b) estimated the correlation of the waiting time observations in an M/M/1 queuing system.

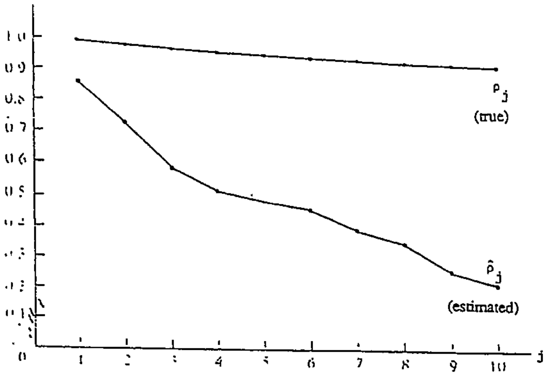

Figure 1. True and Estimated Correlations of the Waiting Time Process for the M/M/1 Queue with 0.9 Traffic Density.

Source: Law and Kelton (1982b).

PROPOSED METHOD

Testing whether coreelation dies out at some lag is a classical problem in time series analysis. The conventional approach to this problem is through construction of hypothesis testing procedures to determine where the correlation has died out. This generally will impose an arduous task in the light of distribution theory and estimation of sample autocorreltion. This section describes a procedure to determine where the correlation has died out. We approach the problem in two phases. In the first phase we propose a correlation estimate. Then we determine where the correlation has died out according to the criterion of Gross and Harris (1974). Hence a value k can be determined if the correlation at lag k is less than a small number. In phase 2 the von Neumann ratio test is used to test whether observations drawn at intervals of k are uncorrelated.

Suppose that the observations $x_1, x_2, \ldots, x_n$ can be represented by an autoregressive moving-average model given by

$$x_t - \phi_1 x_{t-1} - \phi_2 x_{t-2} - \cdots - \phi_p x_{t-p} = a_t - \theta_1 a_{t-1} - \cdots - \theta_q a_{t-q} \quad (3.1)$$

$$a_t \sim N(0, \sigma_a^2)$$

$$E(a_i a_j) = \begin{cases} \sigma_a^2 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

$$Cov(a_t x_s) = 0 \qquad \text{if } t > s$$

This is the familiar model referred to as ARMA(p,q) model in Box and Jenkins (1976). The thrust of their idea is to model the sample data according to the form of equation (3.1). The modeling procedure comprises identification, estimation, and diagnostic checking. If by these steps an AR(p) model is built and the estimates $(\phi_1, \phi_2, \ldots, \phi_p)$ are obtained, then the autocorrelation function ρ_k given by this model is

$$\rho_k = \phi_1 \rho_{k-1} + \phi_2 \rho_{k-2} + \ldots + \phi_p \rho_{k-p}, \quad k > 0 \qquad (3.2)$$

Employing the approach adopted by Gross and Harris (1974), we may consider that the lag correlation dies out at lag k if $\rho_k < c$ where c is a small, user-defined number.

For a stationary process $\{x_i\}$, if the value k is sufficiently large then by applying the method of sequential systematic sampling there are k systematic samples and each comprises m essentially uncorrelated observations. This can be tested empirically when the von Neumann ratio test is applied to such systematic samples. In the test if a value k is not accepted, increasing k will add to the likelihood of acceptance. The steps of determining k are described by the following algorithm:

1. Define α.
2. Define n.
3. n=mk, where m and k are to be determined.
4. For a sequence of observations $x_1, x_2, \ldots, x_n$. determine the order of p and q of an ARMA (p,q) model (We employed the corner method (Beguin, Gourieoux, and Monfort (1979)) to identify p and q).
5. Compute the maximum likeligood estimates ($\phi_1, \phi_2, \ldots, \phi_p, \theta_1, \theta_2, \ldots, \theta_q, \sigma_a^2$) according to Box and the Jenkins (1976)
6. Compute model residual correlation estimates r_j, $j=1,2,\ldots,t$
7. Compute $Q = n(n+2) \sum_{j=1}^{m} (n-j)^{-1} r_j^2$
8. If
   $$Q < \chi^2_{t-p-q}$$
   the sample observations $x_1, x_2, \ldots, x_n$ may considered being fitted by the ARMA (p,q) model in 4, then go to 9; otherwise, go to 4.
9. Compute the autocorrelation function of the model. For the case that q=0
   $$\rho_k = \phi_1 \rho_{k-1} + \phi_2 \rho_{k-2} + \cdots + \phi_p \rho_{k-p}, \quad k > 0$$
10. Determine k if $\rho_k < c$ where c is a small, user-defined number.
11. m=[n/k], wher [n/k] is the largest integer of n/k, if m<5 go to 18.
12. Generate a random number f, where $0 < f \leq 1$. Set i=[mf], where $1 \leq i \leq m$.
13. Compute $\bar{x}_n = \sum_{l=1}^{m} x_{(l-1)k+i} / m$
14. $$C_m = 1 - \sum_{l=1}^{m-1} (x_{(l-1)k+i} - x_{lk+i})^2 / 2 \sum_{i=1}^{m} (x_{(l-1)k+i} - \bar{x}_n)^2$$
15. If $\left| C_m / \sqrt{(m-2)/(m^2-1)} \right| < Z_{1-\alpha/2}$

the observations $x_{(1-1)k+i}$, $1=1,2,\ldots,m$ may be considered as independent random variables, go to 18; otherwise, go to 16.

16. Let k=k+10, m=[n/k].
    If m<5 go to 18. If m>5 go to 12.
17. k=[n/m]
18. Stop.

With the value k determined in the algorithm, an interval estimate for can be computed by applying the method of sequential systematic sampling. If the value k is chosen where the lag correlation is deemed to have died out, it is expected that the predictability of such interval estimate should be less affected by the presence of correlated data.

M/M/1 EXAMPLE

Comsider a stationary single-channel queuing system where service time is exponentially distributed with parameter λ and interarrival time is exponentially distributed with parameter ν. The rate $\tau(\lambda/\nu)$ gives proportion of the time the server is busy and is denoted as traffic density. If λ and ν are known, mean system time (mean time in the waiting line plus mean time in the service channel) μ is given as follows:

$$\mu = L/\lambda$$

where $L=(\nu-\lambda)/\nu$

Define ρ_n as the autocorrelation function of the waiting time observations in an M/M/1 queuing system with lag n. the formula for ρ_n is given in Daley (1968) by

$$\rho_n = \frac{(1-\tau)\ (1+\tau)}{2\pi\tau\ (2-\tau)} \int_0^a \frac{t\ t\ (a-t)}{(1-t)}\, dt$$

$$\text{where } a = \frac{4\tau}{(1+\tau)}, \quad 0<\tau<1$$

We first compared the correlation estimate in our algorithm and the conventional correlation estimate with respect to the induced bias in the case of M/M/1 queues. The comparison included the M/M/1 queuing model of τ=0.5,0.8, and 0.9, and for each model 1000 observations of the waiting time were generated. The results of the comparison are presented in Figure 2,3, and 4. As the results in Figure 2 indicated, for a less congested queuing system (τ=0.5), the bias of the conventional correlation estimate does not pose a serious problem. As for the results presented in Figure 3 and 4, where heavily cogested queuing systems (τ=0.8 and 0.9) were addressed, the conventional correlation estimate is subject to significant bias. This is especially true as the lag increases. By comparison our correlation estimate has led to substantial bias reduction.

We next comared sequential systematic sampling and batch means, in which the suggested algorithm and Fishman's procedure were applied to acquire the sampling interval

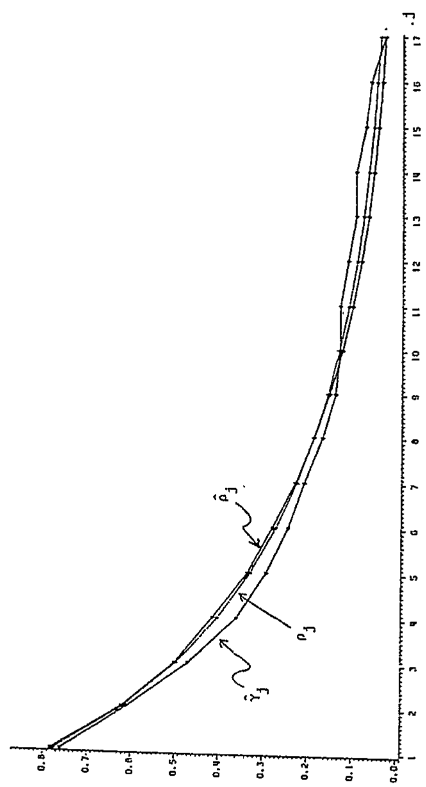

Figure 2.

Results of the Suggested Algorithm and the Conventional Procedure in Estimating Correlations of the M/M/1 Queue with τ = 0.5, ρ_j = True Correlation at Lag j, $\hat{\rho}_j$ = Estimated Correlation by the Suggested Algorithm, $\hat{\gamma}_j$ = Estimated Correlation by the Conventional Procedure.

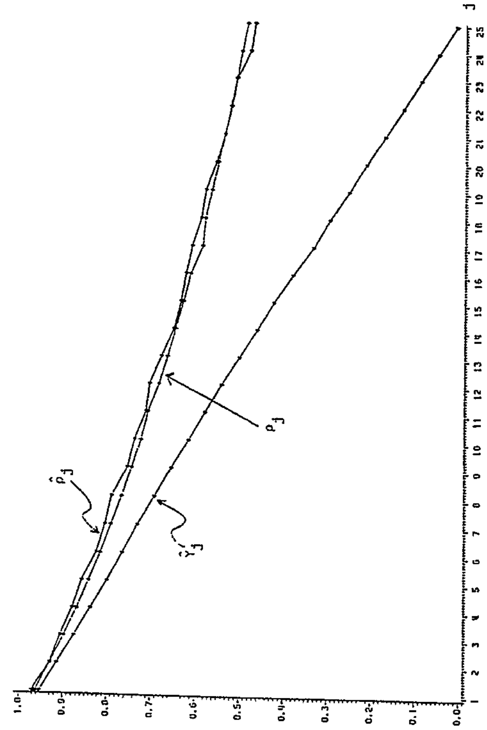

Figure 3.

Results of the Suggested Algorithm and the Conventional Procedure in Estimating Correlations of the M/M/1 Queue with τ = 0.8, ρ_j = True Correlation at Lag j, $\hat{\rho}_j$ = Estimated Correlation by the Suggested Algorithm, $\hat{\gamma}_j$ = Estimated Correlation by the Conventional Procedure.

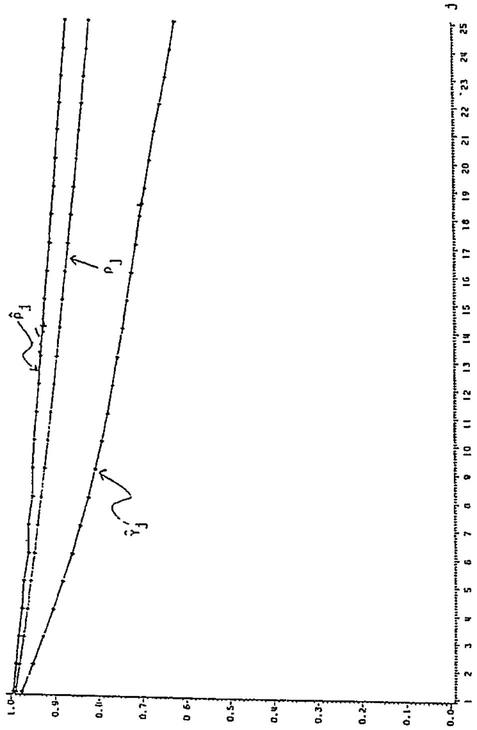

Figure 4. Results of the Suggested Algorithm and the Conventional Procedure in Estimating Correlation of the M/M/1 Queue with $\tau = 0.9$, ρ_j ≡ True Correlation at Lag j, $\hat{\rho}_j$ ≡ Estimated Correlation by the Suggested Algorithm, $\hat{\gamma}_j$ ≡ Estimated Correlation by the Conventional Procedure.

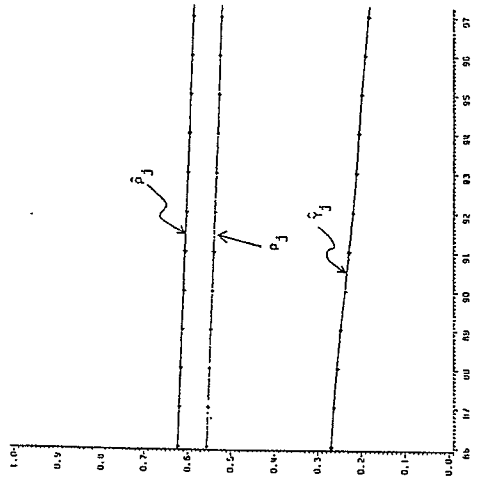

Figure 4. Continued

and batch size respectively. The comparison was based upon the coverage of confidence interval constructed. Again, the model of M/M/1 queues was the case under study. The two methods were applied to construct confidence intervals for the mean system time μ. Three sets of sampling experiments were conducted for the traffic density τ =0.5,0.8, and 0.9. For each set, 60 replications were used. As suggested by Fishman (1978) to avoid the problem of initial transient, the first 459 observations in every output process were deleted. The sample size used in each type of experiment was 2048, 4096, and 16384 respectively. In each replication Fishman's procedure was used to determine the batch size and employed the proposed algorithm for the sampling interval. Batch means and sequential systematic sampling were then applied to compute confidence interval. Batch means and sequential systematic sampling were then applied to compute confience intervals for μ . The coverage of confidence interval constructed is based upon the 60 replicates. The results were summarized in Table 1,2,and 3.

Table 1

Coverage of 80% Confidence Interval for the Method of Batch Means in M/M/1 Queueing Models.

Replication = 60

ρ \ n	2048	4096	8192	16384
0.5	0.8333	0.8136	0.9333	0.9333
0.8	0.6949	0.7627	0.7333	0.8667
0.9	0.2174	0.3966	0.6167	0.6610

Table 2

Proportion of Runs that Failed to Determine a Batch Size for the Method of Batch Means in M/M/1 Queueing Models.

ρ \ n	2048	4096	8192	16384
0.5	0.03	0.0	0.0	0.0
0.8	0.15	0.10	0.08	0.03
0.9	0.35	0.40	0.27	0.17

Table 3

Coverage of 80% Confidence Interval for the Method of Sequential Systematic Sampling in M/M/1 Queueing Models.

Replication = 60

ρ \ n	2048	4096	8192	16384
0.5	0.8000	0.7996	0.8305	0.8305
0.8	0.7797	0.7458	0.8136	0.8136
0.9	0.5652	0.6500	0.7667	0.8049

Table 1 presented the coverage of confidence interval when batch means is applied with Fishman's procedure to determine the batch size. As the results in table suggested, the resulting coverage for batch means is not satisfactory for $\tau=0.8$ and 0.9. In addition, according to Fishman's observations, the von Neumann ratio test may fail to determine the batch size for heavily congested queuing systems. Table 2 presented the proportion of replicates where such failure occurred. The results in Table 2 indicated that Fishman's procedure has difficulty in determining a suitable batch size for heavily congested queuing system data such as the system time observations with $\tau=0.9$. Based upon the reaults presented in Table 2, the performance of Fishman's procedure dose not seem to be satisfactory. Special precaution should be taken when Fishman's procedure is to be implemented on highly correlated data.

Table 3 presented the results in applying sequential systematic sampling where our suggested alorithm is used to determine the sampling interval. The user-defined value c in testing the autocorrelationis taken as 0.001 universally in all experiments. The results presented sugested that the algorithm proposed in this paper has led to an improvement on the coverage of the confidence interval applied.

CONCLUSION

The obtaining of uncorrelated observations has vital importance for the procedures of simulation output analysis that seek such observations. The algorithm presented in this paper is suited to the method of sequential systematic sampling for acquiring uncorrelated observations. The approach to this problem in the paper began with a correlation estimate which provides a reasonable approximation. This correlation estimate, together with the von Neumann ratio test, contributed to produce sequences of essentially uncorrelated observations from a sequence of correlated observations. With such observations at hand, sequential systematic sampling is able to negate the influence of data correlation and therefore leads to improvement of the coverage of confidence interval applied.

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