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FREQUENCY DOMAIN METAMODELLING OF A FEEDBACK QUEUE

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ABSTRACT

Schruben and Cogliano [1987] introduced Frequency Domain Experiments as a tool for metamodel identification. To design a frequency domain experiment, the experimenter must choose appropriate values for certain experimental variables such as oscillation frequency, window size, and range of oscillation. In this paper, we demonstrate that these experimental variables affect the outcome of frequency domain experiments and the magnitude of these effects are model dependent.

1. INTRODUCTION

One of the purposes of building a simulation model is to determine which (input) factors significantly affect the model (system) response. A commonly used approach to determine how the input factors affect the model's response is to develop a response surface model. This usually takes the form of a K-order polynomial

$$E(Y) = \beta_0 + \sum_{j=1}^{t} \beta_j z_j, \quad (1)$$

where E(Y) is the expected value of the response Y; z_j is a term in the K-order polynomial, i.e., a particular product of non-negative integer powers of the input factors X_i, e.g., $X_1^2 X_2^3$, where the sum of the exponents is not greater than K, β_j is the co-efficient for the term z_j; and t is the number of potential terms in the prospective model. In subsequent discussion the K-order polynomial on the right hand side of (1) is referred to as a "metamodel" [Kleijnen, 1982].

Schruben and Cogliano [1987] describe how frequency domain experiments can be used for metamodel identification of linear systems. Contrary to conventional simulation runs, input factors of interest are oscillated sinusoidally during a frequency domain experiment. The spectrum of the output series of the simulation run is then estimated and examined for peaks at corresponding indicator frequencies. In this paper, we highlight some important issues connected with frequency domain experiments and use the example of a feedback queue for this purpose. Our choice of feedback queue as an example is motivated by the fact that this is a common component of many useful systems, e.g., models of computer systems [Som and Sargent, 1987], and open queues with no feedback are special cases of feedback queues with zero feedback.

2. SIMPLE FEEDBACK QUEUE

2.1 DESCRIPTION OF THE SYSTEM

The feedback queue used for the experiments described in this paper is shown in Figure 1. Customers arrive according to a Poisson Process with mean interarrival time A, join a FIFO (first in-first out) queue before a server which has an exponentially distributed random service time with mean S. After being served, a customer leaves the system with probability 1 - p or rejoins the queue with probability p. We consider mean service time S as the input factor for the system and the amount of time a customer spends in the system as its response.

2.2 OUTPUT AND INPUT SERIES

Output produced by a simulation model for the above system will typically consist of a series of values $\{Y_i\}$, where Y_i represents the time spent by the ith customer inside the system. In subsequent discussions, we assume that $\{Y_i\}$ has been ordered such that Y_i represents time spent inside the system by the ith customer entering the system. Let the service time for the ith customer be sampled from an exponential distribution having mean S_i. The series $\{S_i\}$ constitutes the input series for the input parameter 'service rate'.

2.3 BASIC METHOLDOLOGY

All the experiments described in this paper use the input series $\{S_i\}$, where

$$S_i = 0.5(U_s+L_s) + 0.5(U_s-L_s)\mathrm{Cos}(2\pi\omega_s i) \quad (2)$$

and U_s and L_s indicate lower and upper limits of the range of interest for input parameter S and ω_s indicates the oscillation frequency for parameter S. Each experiment consists of a pair of runs called a signal run and a control run. In a *signal run*, one uses the input parameter series $\{S_i\}$ with non-zero ω_s. In a *control run*, one uses $\omega_s = 0$, i.e., the input series does ***not*** oscillate during a control run. Spectra of the output series from the signal and control runs ($\hat{f}_s(\omega)$ and $\hat{f}_c(\omega)$, respectively) are then estimated and the spectral ratio $R(\omega) = \hat{f}_s(\omega)/\hat{f}_c(\omega)$ is computed. If a term in (1) is significant, then $R(\omega)$ should have a peak at the corresponding indicator frequencies [Schruben and Cogliano, 1987].

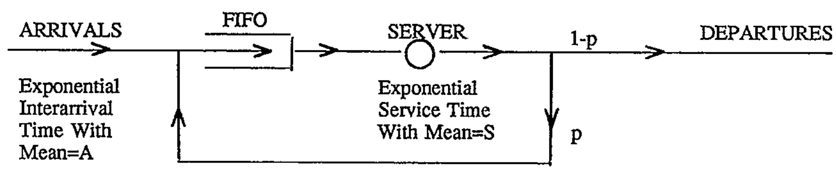

Figure 1

3. BASIC ISSUES IN FREQUENCY DOMAIN EXPERIMENTS

In designing a frequency domain experiment, the experimenter must specify appropriate values for the following experimental variables:

(i) Oscillation frequency, ω_s (eqn (2)),

(ii) Run length, i.e., length of the output series $\{Y_i\}$,

(iii) Window size 'm' used for calculating the spectrum of the output series (Schruben and Cogliano, 1987) and,

(iv) Range of values over which the parameter of interest oscilates.

Concerning (iv), we shall always assume that the input parameter is oscillated over the entire range of interest (as in eqn (2)) and the only way to change the range of values is to change its scale.

However, the nature of effect of the above variables on the outcome of frequency domain experiments is model dependent, e.g., oscillation frequency has significant effects for queues with high feedback but has little effect for queues with zero feedback (Section 4.1). Experiments conducted by Som, Sargent, and Schruben [1987] suggest that gain and noise characteristics of a system are two important factors that determine the nature of these effects and these characteristics should therefore be considered in conducting frequency domain experiments.

4. RESULTS OF SELECTED EXPERIMENTS

In this section, we present selected results from experiments with the feedback model described in Section 2 to demonstrate how the choice of the experimental variables affect the outcome of frequency domain experiments and that such effects are model dependent. A more complete discussion can be found in Som, Sargent, and Schruben, [1987].

4.1 EFFECT OF OSCILLATION FREQUENCY, ω_s

Figures 2 and 3 show plots of spectral ratios from experiments with oscillation frequency $\omega_s = 0.03$ and 0.48, respectively. In both cases, feedback is zero and the utilization factor $\rho = 0.7$. Figures 4 and 5 show plots of spectral ratios from experiments with $\omega_s = 0.35$ and 0.02, respectively. In both cases, feedback is 0.5 and the utilization factor $\rho \approx 0.8$. It can be easily seen that the spectral ratio is only slightly affected by the oscillation frequency in the former case but highly sensitive to the oscillation frequency for the later. Good results using low oscillation frequencies were also obtained for a closed loop computer system with high feedback [Som and Sargent, 1987].

4.2 EFFECT OF WINDOW SIZE

Figure 6 shows the plot of spectral ratio from an experiment with window size m = 200. All the other parameters are the same as in Figure 5 where m = 500. It can be observed that the change in the window size changed the spectral ratio at the indicator frequency 0.02 by about 70%.

4.3 EFFECT OF RANGE OF OSCILLATION

In general, a larger range of oscillation increases the spectral ratio at indicator frequencies and therefore gives sharper peaks. See Figures 2 and 7. Sometimes it can be done simply by changing the scale of input parameters, e.g., [5 seconds, 9 seconds] can be changed to [5000 milliseconds, 9000 milliseconds]. However, this technique does not work in queueing models where the noise levels are not independent of the input parameter level, e.g., with exponential distributions. See Figures 8 and 9.

5. CONCLUSIONS

Frequency domain experiments for metamodel identification promises to be an attractive alternative to current statistical methods because of the potential economy in experimentation [Schruben and Cogliano, 1987]. In order to produce the best results, the values of certain experimental variables must be chosen appropriately. For further discussion of these issues, see Som, Sargent, and Schruben [1987].

ACKNOWLEDGEMENTS

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spectrum	frequency
1.2123673	0.010
1.0794838	0.020
2.8696952	0.030
1.1126662	0.040
0.9632992	0.050
1.0616138	0.060
1.0650166	0.070
1.0085900	0.080
1.0218165	0.090
1.0099515	0.100
1.0186748	0.110
1.0144163	0.120
0.9884325	0.130
0.9965899	0.140
1.0966743	0.150
0.9739794	0.160
1.0109804	0.170
1.0651289	0.180
0.9801770	0.190
0.9415931	0.200
1.0448445	0.210
0.9560113	0.220
1.0463848	0.230
1.0151903	0.240
0.9713697	0.250
1.0163750	0.260
0.9829306	0.270
1.0272669	0.280
1.0079044	0.290
0.9376604	0.300
1.0337802	0.310
1.0400486	0.320
1.0556581	0.330
1.0081321	0.340
1.0900594	0.350
0.9718680	0.360
0.9353690	0.370
1.0693579	0.380
1.0711767	0.390
1.0443684	0.400
1.0396641	0.410
1.0143419	0.420
0.9862017	0.430
1.0164045	0.440
0.9616236	0.450
0.9612299	0.460
1.0541170	0.470
1.0187704	0.480
1.0220258	0.490
1.0456415	0.500

Feedback = 0, Mean Interarrival Time = 10 seconds, U_S = 9 seconds, L_S = 5 seconds, Utilization Factor $\rho \simeq 0.7$, Oscillation Frequency $\omega_S = 0.03$, Run length = 10,000, Tukey window size m = 200.

Figure 2: Plot of Spectral Ratio

spectrum	frequency
1.1021284	0.010
1.0215001	0.020
0.9764605	0.030
1.0734205	0.040
0.9651292	0.050
0.9521943	0.060
1.0377067	0.070
1.0552769	0.080
1.0012732	0.090
1.0267387	0.100
1.0095231	0.110
0.9927662	0.120
1.0256307	0.130
1.0139087	0.140
0.9992148	0.150
1.0134982	0.160
1.0039041	0.170
1.0336242	0.180
1.0297936	0.190
0.9806583	0.200
1.0049437	0.210
1.0454493	0.220
1.0102787	0.230
1.0374706	0.240
1.0354100	0.250
1.0056360	0.260
1.0891177	0.270
0.9768159	0.280
0.9867075	0.290
1.0070363	0.300
0.9997944	0.310
1.0284511	0.320
1.0574306	0.330
1.0304063	0.340
1.0479770	0.350
0.9944476	0.360
1.0121940	0.370
0.9848116	0.380
0.9995908	0.390
1.0141356	0.400
1.0212245	0.410
0.9952219	0.420
1.0397430	0.430
1.0164707	0.440
0.9452328	0.450
1.0233858	0.460
1.0238516	0.470
2.5737339	0.480
1.0640529	0.490
0.9850188	0.500

Feedback = 0, Mean Interarrival Time = 10 seconds, U_S = 9 seconds, L_S = 5 seconds, Utilization Factor $\rho \simeq 0.7$, Oscillation Frequency $\omega_S = 0.48$, Run length = 10,000, Tukey window size m = 200.

Figure 3: Plot of Spectral Ratio

spectrum	frequency
0.9975354	0.010
0.9643792	0.020
0.9209147	0.030
0.9649077	0.040
1.1131885	0.050
1.4741884	0.060
0.8658195	0.070
1.2745874	0.080
1.0883064	0.090
2.0334169	0.100
1.1522163	0.110
1.0687742	0.120
1.2670323	0.130
0.9520595	0.140
1.1276106	0.150
2.0280208	0.160
1.1557164	0.170
0.9086761	0.180
1.8932313	0.190
1.0582287	0.200
1.4783716	0.210
1.1094695	0.220
0.9956373	0.230
1.7111514	0.240
0.9013431	0.250
1.4234062	0.260
0.9098080	0.270
0.7139254	0.280
1.3526768	0.290
1.4395740	0.300
1.0109608	0.310
1.3079800	0.320
0.9582656	0.330
2.0187721	0.340
2.3950349	0.350
0.9174062	0.360
1.4178009	0.370
1.2936727	0.380
1.0354937	0.390
0.8086487	0.400
1.0753516	0.410
1.3427274	0.420
1.3130761	0.430
1.5385562	0.440
1.6236378	0.450
1.5488333	0.460
1.4180140	0.470
1.2176849	0.480
1.5310380	0.490
1.2144036	0.500

Feedback = 0.5, Mean Interarrival Time = 10 seconds, U_S = 5 seconds, L_S = 3 seconds, Utilization Factor $\rho \simeq 0.8$, Oscillation Frequency $\omega_S = 0.35$, Run length = 10,000, Tukey window size m = 500.

Figure 4: Plot of Spectral Ratio

spectrum	frequency
1.0925902	0.010
4.4874815	0.020
0.8395014	0.030
0.7815949	0.040
0.9004095	0.050
1.3563241	0.060
0.8935434	0.070
1.0863739	0.080
1.0716217	0.090
1.3383490	0.100
0.8720342	0.110
1.0381988	0.120
1.6930106	0.130
0.9178164	0.140
0.8046908	0.150
1.7284179	0.160
1.1733761	0.170
0.7666769	0.180
1.4874621	0.190
0.8413074	0.200
0.6844329	0.210
1.0112085	0.220
1.4626703	0.230
1.2578805	0.240
1.1829104	0.250
0.8952033	0.260
0.9707916	0.270
1.0419455	0.280
1.1047088	0.290
0.9468498	0.300
2.3258749	0.310
2.0958378	0.320
0.8298598	0.330
1.0015315	0.340
1.3220098	0.350
1.0815898	0.360
0.8748799	0.370
1.2701418	0.380
0.8273726	0.390
0.9314695	0.400
0.8283423	0.410
0.9120237	0.420
1.3036454	0.430
1.4424169	0.440
0.9859916	0.450
1.0529899	0.460
1.0916628	0.470
1.1253146	0.480
1.1196810	0.490
0.9663118	0.500

Feedback = 0.5, Mean Interarrival Time = 10 seconds, U_S = 5 seconds, L_S = 3 seconds, Utilization Factor $\rho \simeq 0.8$, Oscillation Frequency $\omega_S = 0.02$, Run length = 10,000, Tukey window size m = 500.

Figure 5: Plot of Spectral Ratio

spectrum	frequency
1.0314753	0.010
2.6546477	0.020
0.9738541	0.030
0.8630079	0.040
1.0478995	0.050
1.3116524	0.060
1.2764345	0.070
1.0769450	0.080
1.0373231	0.090
1.0440345	0.100
0.9882609	0.110
1.0111649	0.120
1.2970770	0.130
0.8662892	0.140
0.9726123	0.150
1.1504044	0.160
1.0240165	0.170
0.8751830	0.180
1.4409942	0.190
0.8679324	0.200
0.7334748	0.210
1.0083905	0.220
1.2353062	0.230
1.3287408	0.240
0.9870055	0.250
0.8917650	0.260
1.0166258	0.270
1.0579578	0.280
1.1253998	0.290
1.1917192	0.300
2.1271126	0.310
1.7739325	0.320
0.9356344	0.330
1.2114899	0.340
1.1319414	0.350
1.0130969	0.360
0.7979243	0.370
1.3221869	0.380
1.1874239	0.390
1.0158450	0.400
0.7941567	0.410
0.9738593	0.420
1.2583039	0.430
1.6565628	0.440
1.0475912	0.450
1.0248890	0.460
1.1212527	0.470
1.1274256	0.480
0.9878027	0.490
1.0005908	0.500

Feedback = 0.5, Mean Interarrival Time = 10 seconds, U_S = 5 seconds, L_S = 3 seconds, Utilization Factor $\rho \simeq 0.8$, Oscillation Frequency $\omega_S = 0.02$, Run length = 10,000, Tukey window size m = 200.

Figure 6: Plot of Spectral Ratio

spectrum	frequency
1.4241628	0.010
1.2401643	0.020
5.3552499	0.030
1.2782540	0.040
0.9999449	0.050
1.1363474	0.060
1.1444245	0.070
0.9892575	0.080
1.0547826	0.090
1.0607653	0.100
1.0555855	0.110
1.0229379	0.120
1.0607510	0.130
1.0129706	0.140
1.1705255	0.150
0.9943228	0.160
1.0264996	0.170
1.0824646	0.180
0.9693612	0.190
0.9536423	0.200
1.0950629	0.210
0.9613319	0.220
1.0496376	0.230
1.0154086	0.240
0.9986299	0.250
1.0655073	0.260
0.9863676	0.270
1.0601407	0.280
1.0289308	0.290
0.9205846	0.300
1.0425423	0.310
1.0565730	0.320
1.0485739	0.330
1.0218049	0.340
1.1559026	0.350
0.9689513	0.360
0.9512961	0.370
1.1071754	0.380
1.1093182	0.390
1.0364777	0.400
1.0678661	0.410
1.0239838	0.420
1.0518987	0.430
1.0657701	0.440
0.9660842	0.450
0.9397534	0.460
1.0589773	0.470
1.0158942	0.480
1.0265521	0.490
1.1091002	0.500

Feedback = 0, Mean Interarrival Time = 10 seconds, U_S = 10 seconds, L_S = 4 seconds, Utilization Factor $\rho \simeq 0.7$, Oscillation Frequency $\omega_S = 0.03$, Run length = 10,000, Tukey window size m = 200.

Figure 7: Plot of Spectral Ratio

spectrum	frequency
0.9990185	0.010
0.9976004	0.020
0.9882952	0.030
0.9981466	0.040
0.9981747	0.050
0.9990180	0.060
0.9977707	0.070
0.9993960	0.080
0.9995614	0.090
1.0005422	0.100
0.9992870	0.110
0.9982642	0.120
1.0007464	0.130
0.9983210	0.140
1.0012601	0.150
0.9998141	0.160
0.9984295	0.170
0.9997202	0.180
1.0000261	0.190
0.9975908	0.200
1.0001732	0.210
0.9990934	0.220
1.0008662	0.230
0.9994673	0.240
0.9992407	0.250
0.9988491	0.260
0.9958725	0.270
0.9990632	0.280
0.9988318	0.290
0.9995371	0.300
0.9996231	0.310
1.0004309	0.320
0.9965571	0.330
0.9985898	0.340
0.9981288	0.350
0.9984768	0.360
0.9984677	0.370
0.9983988	0.380
1.0020587	0.390
0.9993161	0.400
1.0007392	0.410
0.9997333	0.420
0.9989606	0.430
1.0012053	0.440
0.9973030	0.450
0.9986265	0.460
0.9994128	0.470
0.9987373	0.480
0.9962030	0.490
0.9963159	0.500

Feedback = 0, Mean Interarrival Time = 10 seconds, U_S = 9.9 seconds, L_S = 9.7 seconds, Utilization Factor $\rho \simeq 0.98$, Oscillation Frequency $\omega_S = 0.03$, Run length = 10,000, Tukey window size m = 200.

Figure 8: Plot of Spectral Ratio

spectrum	frequency
0.9863522	0.010
0.9985960	0.020
0.9957928	0.030
0.9964765	0.040
0.9985860	0.050
0.9989136	0.060
0.9991084	0.070
0.9990675	0.080
0.9992974	0.090
0.9996970	0.100
0.9994195	0.110
0.9988708	0.120
0.9991743	0.130
0.9994510	0.140
0.9996908	0.150
0.9993098	0.160
0.9989372	0.170
0.9991645	0.180
0.9988413	0.190
0.9988637	0.200
0.9994830	0.210
0.9993655	0.220
0.9994329	0.230
0.9997141	0.240
0.9995225	0.250
0.9988823	0.260
0.9985681	0.270
0.9989633	0.280
0.9991066	0.290
0.9992631	0.300
0.9993773	0.310
0.9985501	0.320
0.9977156	0.330
0.9979068	0.340
0.9981822	0.350
0.9982698	0.360
0.9983534	0.370
0.9989480	0.380
1.0001610	0.390
1.0001151	0.400
0.9993216	0.410
0.9987927	0.420
0.9985771	0.430
0.9985500	0.440
0.9982586	0.450
0.9982991	0.460
0.9986338	0.470
0.9986477	0.480
0.9982461	0.490
0.9979112	0.500

Feedback = 0, Mean Interarrival Time = 100 (one-tenth seconds), U_S = 99 (one-tenth seconds), L_S = 97 (one-tenth seconds), Utilization Factor $\rho \simeq 0.98$, Oscillation Frequency $\omega_S = 0.03$, Run length = 10,000, Tukey window size m = 200.

Figure 9: Plot of Spectral Ratio

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