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FUTURE DIRECTIONS FOR THE FREQUENCY DOMAIN APPROACH

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1. Introduction

The frequency domain method for simulation experiments was introduced by Schruben and Cogliano (1981) as a means of sensitivity analysis and factor screening. The approach is to induce periodicities by oscillating parameter values sinusoidally throughout the run. Analysis of the frequency content of the resulting output indicates sensitivity tothe various inputs. Frequencies are chosen so that distinct combinations of factors have distinct frequencies at which potential peaks in the spectrum may be observed. The lack of such peaks is evidence that the corresponding factors are not influential. We note that the frequency domain approach differs from other spectral methods in that the periodicities are induced artificially by the experimenter. Most spectral approaches are concerned with measuring periodicities inherent in the system. In this discussion we will present some extensions and other uses of the frequency domain approach to simulation experiments.

2. Factor Screening

The frequency domain approach as it exists presently provides a qualitative analysis: the result of an experiment is a list of those terms which are present in the model and which are not. This may be determined in an ad hoc manner by visually inspecting the output power spectrum for peaks, or more quantitatively by performing two runs and a corresponding F test on the spectral ratio (see Schruben and Cogliano, 1987). This could be the first stage of a mixed approach. The factor screening is performed first using Schruben and Cogliano's approach, with the hope that most interaction terms will not be significant. The second stage consists of a conventional factorial design, with confounding on those factors determined to be insignificant by the factor screening.

Alternatively, the second stage could be another frequency domain experiment rather than a conventional one. Factor screening aids this by allowing for a wider bandwidth (assuming that some factors are eliminated from consideration) and thus shorter runs. Indeed, most frequency domain experiments with all possible factors included in the model are not able to achieve equal spacing of term indicator frequencies (see Jacobson, Buss, and Schruben; 1986), while the elimination of even a few frequencies usually allows the experimenter to choose frequencies such that equal spacing is possible.

3. Estimation

Frequency domain experiments may be used to estimate or fit a model to the simulation. Two approaches appear promising. One is essentially non-parametric and static in that the steady state response of the simulation is sought with no assumptions being made on the particular form of the transfer function(s). This approach is based on the relationships between the real and imaginary parts of the Fourier transform of the impulse response function called the Kramers-Kronig relations (Jackson; 1976). The steady state response is the Fourier transform evaluated at zero, and the Kramers-Kronig relations express this as the principle value integral of the imaginary part of the Fourier transform. If this integral is approximated by a sum, then a frequency domain experiment may be designed to estimate the imaginary part of the Fourier transform at spaced frequencies. The inputs typically must be oscillated at more than one frequency, so the problem of frequency selection is more complicated than in the factor screening experiments. It turns out that for a quadratic model one possible choice of frequencies consists of odd multiples of ratios of odd prime numbers, with the numerators and denominators being distinct for different factors. Determining optimal frequencies in this setting is an interesting unsolved problem.

The other approach to estimation assumes particular parametric forms of the impulse response functions. The Fourier transforms or transfer function (power spectrum) are thus functions of the given parameters. If frequencies are chosen as in the Kramers-Kronig approach above, then the parameters can then be estimated by a standard technique, such as least squares. For example, rational spectral densities could be assumed for the transfer function, and the frequency domain experiment results in estimates of the coefficients. One special case of this assumes exponential impulse response functions, with the parameters being the heights at zero and the rates of decay. See Schruben, Heath, and Buss (1987) for further details.

4. Optimization

The frequency domain approach may be used to estimate the optimum of a hypothesized response surface. The oscillating parameters explore a region of the surface. Jacobson (1987) is using this in conjunction with a

Newton type approach. The spectral estimates can be used to obtain directions and step sizes. The frequency selection problem for this is identical to that of the factor screening experiments. Naturally, an initial factor screening may be used to facilitate the optimization runs.

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