Proceedings of the 1987 Winter Simulation Conference
A. Thesen, H. Grant, W. David Kelton (eds.)

A NEW INITIAL BIAS DELETION RULE

Peter W. Glynn
Department of Industrial Engineering
University of Wisconsin
Madision, WI 53706

Donald L. Iglehart
Department of Operations Research
Stanford University
Stanford, CA 94305

ABSTRACT

A new initial bias deletion rule is described. Theoretical motivation for the rule is given, and some directions on further research are provided.

1. INTRODUCTION

Consider a real-valued stochastic process $Y = \{Y(t) : t \geq 0\}$ which is ergodic, in the sense that there exists a finite (deterministic) constant μ such that

$$\overline{Y}(t) \equiv \frac{1}{t}\int_0^t Y(s)ds \Rightarrow \mu \tag{1.1}$$

as $t \to \infty$. The steady-state simulation problem is concerned with the estimation of μ, and production of associated confidence intervals.

The limit relation (1.1) immediately suggests using the estimator $\overline{Y}(t)$, namely the sample mean of Y up to time t. A difficulty with this approach is that

$$E\overline{Y}(t) \neq \mu \tag{1.2}$$

in general. The bias in $\overline{Y}(t)$ is a consequence of using an atypical initial condition to start the simulation; atypical refers here to an initial condition which is not typical of the steady-state of the process. For example, queueing simulations of open queueing networks are typically initiated with all stations idle. In this setting, one generally sees the customer population build up to a "steady-state".

The bias difficulty (1.2) is often called the initial bias problem. This initial bias problem has historically attracted considerable attention within the simulation community.

One way to deal with this initial bias difficulty is to delete those initial observations most severely "contaminated" by the initial condition; such methods are called initial bias deletion rules. In Section 2, we briefly describe a new initial bias deletion rule, which we believe has some desirable theoretical properties. Section 3 offers some conclusions about the rule. An expanded version of the work described here will be available in a forthcoming paper (Glynn and Iglehart (1988)).

2. A NEW INITIAL BIAS DELETION RULE

Suppose that $Y = \{Y(t) : t \geq 0\}$ can be represented as $Y(t) = f(X(t))$, where $X = \{X(t) : t \geq 0\}$ is a continuous-time Markov chain on state space $S = \{0, 1, 2, \ldots\}$. If X is positive recurrent and irreducible, then

$$\mu = \sum_{i \in S} \pi_i f(i)$$

where the π_i's are the stationary probabilities of X. Suppose V is a r.v. independent of X such that

$$P\{V = i\} = \pi_i,$$

and let

$$T = \inf\{t > 0 : X(t) = V\}.$$

We claim that $\{X(T+t) : t \geq 0\}$ is a stationary continuous-time Markov chain, so that the post-T process is in "steady-state". To see this, observe that for suitably measurable functions $k(\cdot)$,

$$\begin{aligned} Ek(X(T+t) : t \geq 0) &= \sum_{i \in S} E\{k(X(T+t) : t \geq 0) | V = i\} \cdot \pi_i \\ &= \sum_{i \in S} \pi_i \, E\{k(X(T_i + t) : t \geq 0)\} \end{aligned}$$

where $T_i = \inf\{t > 0 : X(t) = i\}$. By the strong Markov property

$$E\{k(X(T_i + t) : t \geq 0)\} = E\{k(X(t) : t \geq 0) | X(0) = i\}$$

so

$$Ek(X(T+t):t\geq 0)=\sum_{i\epsilon S}\pi_i E\{k(X(t):t\geq 0)|X(0)=i\}$$

But the right-hand side is the expected value of the functional $k(\cdot)$ integrated against the probability distribution of a stationary continuous-time Markov chain.

Thus, if one could generate such a T, the initial bias deletion problem would be completely solved by deleting all observations collected prior to time T. Of course, since T involves using the stationary probabilities explicitly, construction of such a random time T in a practical context is infeasible.

As a consequence, it makes sense not to use this rule explicitly, but instead to attempt to "mimic" it. Rather than using the π_i's to generate T, we use the empirical steady-state distribution's $\hat{\pi}_i$'s. Specifically, let

$$\hat{\pi}_i(t)=\frac{1}{t}\int_0^t I(X(s)=i)ds.$$

Let $V(t)$ be the r.v. with the conditional distribution

$$P\{V(t)=i|X\}=\hat{\pi}_i(t)$$

and let

$$T(t)=\inf\{s\geq 0:X(s)=V(t)\}.$$

The idea is then to delete, from the data set $\{Y(s):0\leq s\leq t\}$, all observations collected prior to $T(t)$. This leaves the simulator with the (approximately) "steady-state" time series $\{Y(s):T(t)\leq s\leq t\}$.

3. CONCLUDING REMARKS

We expect that the deletion rule $T(t)$ described in Section 2 mimics the rule given by T, in the sense that

$$P\{(X(T(t)+u):u\geq 0)\epsilon\cdot\}\Rightarrow \sum_{i\epsilon S}\pi_i P\{(X(u):u\geq 0)\epsilon\cdot\}$$

as $t\to\infty$. However, this does not necessarily imply that the point estimate

$$\tilde{Y}(t)=\frac{1}{t-T(t)}\int_{T(t)}^t Y(s)ds$$

obtained after deletion of the initial segment $[0,T(t)]$ has low bias. The problem is that $\tilde{Y}(t)$ is a ratio of two random variables. Such ratio estimators are notoriously subject to bias difficulties.

This suggests that further theoretical and empirical study is necessary, in order to determine the efficacy of the deletion estimator $\tilde{Y}(t)$. This work will be reported in Glynn and Iglehart (1988).

ACKNOWLEDGEMENTS

This work was supported by U.S. Army Research Contract DAAG29-84-K-0030 and National Science Foundation Grants DCR 8509668 and ECS-840-4809.

REFERENCES

Glynn, P. W. and Iglehart, D. W. (1988). Forthcoming technical report, Department of Operations Research, Stanford University, Stanford, CA 94305.

AUTHOR'S BIOGRAPHY

PETER W. GLYNN is an associate professor in the Department of Industrial Engineering at the University of Wisconsin-Madison. He received a B.Sc. in mathematics from Carleton University in 1978 and a Ph.D. in operations research from Stanford University in 1982. His current research interests include Monte Carlo simulation, computational probability, and queueing theory. He is a member of ORSA, TIMS, the IMS, and the Statistical Society of Canada.

Peter W. Glynn
Department of Industrial Engineering
University of Wisconsin-Madison
1513 University Avenue
Madison, WI 53706
608/263-6790

DONALD L. IGLEHART is Professor and Chairman of Operations Research at Stanford University. Prior to 1967 he was an Assistant and Associate Professor in the Department of Operations Research at Cornell University. He was educated at Cornell University (Bachelor of Engineering Physics), Stanford University (M.S. and Ph.D. in Statistics), and Oxford University (Postdoctoral Research Fellowship). He has published papers in the areas of inventory theory, queueing theory, weak convergence of probability measures, and simulation output methodology. His current research interests center on developing new probabilistic and statistical methods for analyzing the output of computer simulations.

Donald L. Iglehart
Department of Operations Research
Stanford University
Stanford, CA 94305
415/497-4032