MARGINALIY SPECIFIC ALTERNATIVES TO NORMAL ARMA PROCESSES

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ABSTRACT

In many practical cases in time series analysis, marginal distributions in stationary situations are not Gaussian. It is therefore necessary to be able to generate and analyze non-Gaussian time series. Several non-Gaussian time series models are discussed in this paper. The marginal distributions are Laplace or I-Laplace distributions, and the correlation structure of the processes mimics that of the standard additive, linear, constant coefficient ARMA(p,q) models.

1. INTRODUCTION.

Random coefficient time series models and ordinary linear time series models with non-Gaussian marginal distributions have been developed for a variety of interesting situations in time series analysis to offer viable alternatives to the standard Gaussian assumptions. A problem with the ordinary linear models with non-Gaussian error structure is that the marginal distributions change with the correlation structure, and in particular tend to become Gaussian as the correlation in the processes gets large (Mallows, 1967).

There are, however, important exceptions to this result, and these exceptions lead to important alternatives. Gastwirth and Wolff (1965) developed a linear first order autoregressive process with a Laplace marginal distribution (called LAR(1)). Although it is a linear model, the marginal distributions are Laplace for all values of the correlation. This is because the distribution of the errors in the model is not absolutely continuous, which is a condition for the Mallows' results to hold. Independently Gaver and Lewis (1980) developed a linear first order autoregressive process with a Gamma marginal distribution, called GAR(1). Again the distribution of the errors in the model are not absolutely continuous. Subsequently both of these processes have been shown to be special cases of more general linear additive discrete-random-coefficient autoregressive models (Dewald and Lewis, 1985 and Lawrance and Lewis, 1981 and 1985). Other time series models using continuous-random-coefficients and having a specified marginal distribution are, for Gamma distributions, due to Lewis (1981), Hugus (1982) and Lewis, McKenzie and Hugus (1987), and, with Beta distributions, due to McKenzie (1985).

The Gamma models in particular provide a broad class of time series models for use in fields such as Operations Analysis, Hydrology and Meteorology. Nicholls and Quinn (1982) discuss general random coefficient autoregressive processes without reference to a particular marginal distribution; their results have applications in assessing statistical and asymptotic properties of all the models discussed here.

2. THE LAPLACE LAR(1) MODEL.

The Laplace LAR(1) model, and its generalization to higher order autoregressive and moving average correlation structures, was put forward as a model where two-sided symmetric random variables had larger kurtosis and longer tails than could be expected from Gaussian time series. A particular example is that of position errors in a large navigation system which were found by Hsu (1979) to have Laplace distributions. Again the N-S or E-W components of wind velocity data are often symmetric and long-tailed, especially in the tropics. For a summary of these applications and a summary of methods of generating the LAR(1) and its generalizations, see Dewald and Lewis (1986). In particular we note that generalizations to NLAR(1) and NLAR(2) models which are the analogs of the NEAR(1) and NEAR(2) processes of Lawrence and Lewis (1981 and 1985) are available and are discussed in this paper. All of these models are discrete-random-coefficients linear models.

The random coefficient linear process approach is not the only way to generate Laplace variables with a specified correlation structure. The literature contains numerous articles on the generation of random sequences. One approach put forward in several papers (Guja and Kavanagh, 1968; Hadad and Vallsalo, 1970; Li and Hammond, 1975, and Sondhi, 1983) involves passing white noise through a linear filter followed by a zero-memory nonlinear transformation. This is a general procedure that produces exactly the required marginal distribution and a good approximation to the autocorrelation structure. However, the scheme lacks the simplicity of the methods proposed for Laplace processes, which are just a random linear combination of Laplacian random variables. Moreover, the filtering approach produces, for example, in the first order autoregressive case, only one process. It is important to note in non-Gaussian time series there are infinitely many processes with the given marginal and autocorrelation structure. This dictates perhaps the use of higher order moments and an approach to this using a higher order residual analysis has been proposed by Lawrence and Lewis (1987).

3. THE I-LAPLACE MODELS.

Another application of the Laplace models arises in those cases where positive-valued time series are differentiated to remove trends. The resulting marginal random variables are two-sided and, in particular, differenced Gamma variables result in the I-Laplace family of distributions which we now consider.

The I-Laplace family of distributions is a natural generalization of the Laplace distribution, this being the I-Laplace
distribution when \( l=1 \). The \( l \)-Laplace distribution is infinitely
divisible, additive, symmetric and has extremely thick tails for
small values of the parameter \( l \). This case would be appro-
priate for modelling components of wind velocity data. These
data often exhibit very sporadic, long-tailed behavior. On the
other hand, as \( l \) approaches infinity, the \( l \)-Laplace distribu-
tion approaches a Gaussian distribution. Thus if the data is
Gaussian, no loss of generality arises from using the \( l \)-Laplace
distribution.

A square-root-Beta-Laplace transform is introduced which
allows one to transform one member of the \( l \)-Laplace fam-
ily into another in a simple way. As a result one can obtain an
\( l \)-Laplace process with first-order autoregressive structure,
the continuous-random-parameter just being the square root of
an appropriate Beta variable. These \( l \)-Laplace processes are
Markovian processes with the geometrically decaying au-
tocorrelation function which is typical of the Gaussian, first
order autoregressive (AR(1)) process. The basic structure is
a random coefficient autoregression, and we generalise the
structure of this model to encompass moving average (MA)
and mixed autoregressive moving average (ARMA) processes.
Again, both the structure of the process and their autocorre-
lation functions mirror those of the Gaussian ARMA processes.

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