Proceedings of the 1986 Winter Simulation Conference
J. Wilson, J. Henriksen, S. Roberts (eds.)

THE USE OF STATISTICAL EXPERIMENTAL DESIGN TECHNIQUES FOR THE SYSTEMATIC IMPROVEMENT OF AN AUTOMATED, HEURISTIC TARGETING ALGORITHM

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ABSTRACT

Force application, or rapid targeting or retargeting of weapons, is necessary to keep at risk high value military movable targets (MT) with a minimum number of weapons. Because of the targets' mobility, the system should be automated and able to deal with rapidly changing scenarios. If a multiple-warhead weapon is used, the targets must be arranged into sets of targets, each set being targeted by one weapon. The number of target sets formed must be kept to a minimum. This paper presents a heuristic, automated minimization algorithm created for that purpose. Statistical experimental design and optimization techniques were used to improve the efficiency of the algorithm in a wide variety of test scenarios by systematically selecting the best combination of algorithmic variations. Using a randomized block factorial design and numerous test scenarios with varying target distributions, competing versions of the algorithm were compared and the best combination of rules produced. The results indicate that experimental design techniques can be applied to heuristic rules to improve them in a systematic and unbiased way.

1. INTRODUCTION

Automated force application, or rapid retargeting of weapons, is necessary to continuously keep movable targets at risk with a minimum number of weapons. The target set minimization algorithm presented in this paper is part of a larger force application process. Once the targets are arranged into sets, further processing is needed, such as efficiently using available warheads, avoiding warhead detonation interference, and assigning an actual available weapon to each target set created.

The minimization algorithm utilizes a postulated multiple warhead missile with three warheads or re-entry vehicles (RV) available per missile. Three RVs per missile was arbitrarily chosen for algorithm development and testing, but the final algorithm can be modified to handle more or fewer than three RVs. At optimum efficiency, each missile will target three MTs, one with each available RV. The minimization algorithm, therefore, groups the targets in sets of three to minimize the number of missiles required for targeting. Whenever there are fewer than three targets left to be grouped, the targets are arranged in sets of two or, if that is not possible, singly. Three targets in a set is called a triplet; two targets in a set is called a doublet; and a single target in a set is called a singleton.

As an example, if a target scenario contains 10 MTs to be kept at risk, minimum use of weapons requires that the 10 targets be grouped into three MT triplets and one singleton. Four missiles would be used for targeting, and two RVs would be available for other use.

To this point, it appears that the force application process is not difficult: simply group the movable targets into as many triplets as possible, form doublets and singletons if needed, and assign a missile to each group. A closer look, however, shows that the arrangement of MTs into triplets is a complex process. Three targets can not always be grouped into one triplet MT set with total confidence that all three MTs can be targeted with one missile. Triplet creation with total targeting confidence is constrained because of the mobility of the targets.

The triplet minimization algorithm is, therefore, heuristic in nature, consisting of competing sets of seemingly sensible rules under specific constraints. In practice it is to be applied to targets that have a random distribution and an unknown density function. The final algorithm is used to minimize the number of weapons required, given any target scenario. For these reasons, a force application simulation (FASIM) was developed. FASIM models the force application process using three major steps. First, it simulates a movable target scenario and applies a triplet minimization algorithm to the set of targets. Second, it completes any doublet or singleton sets with lower priority targets so that RVs are used efficiently. This step also sequences the target sets so that RV detonation interference is avoided. Finally, it assigns a missile to each target set created using a modified Hungarian optimization-allocation algorithm. Since the experimental design techniques discussed in this paper were applied only to various methods for grouping targets into a minimum number of triplets, the latter two steps will not be discussed.

Statistical experimental design and optimization techniques were used to improve the efficiency of the minimization algorithm. Near optimization was accomplished by systematically selecting the best combination of algorithmic variations, keeping in mind certain geometric considerations and constraints inherent in the targeting process. The researchers do not claim to have produced the optimum minimization algorithm for all possible targeting cases, but it has been demonstrated that the final, improved algorithm does approach optimization.

2. TARGETS AND MISSILE COVERAGE REGIONS

Before the triplet minimization algorithm and its constraints are discussed in detail, movable targets and the missile coverage regions are explained briefly.

2.1. Movable Targets

Movable targets are stationed at fixed locations called bases. Associated with each base is an area of operation in which MTs are able to travel. Travel beyond these areas is arbitrarily denied because of communication and mobility constraints. The target area is defined as the union of the operating regions of all MTs.

During a crisis scenario, movable targets disperse and relocate to sites away from their respective bases. At the new locations, the MTs can set up and become operational. Later, they can disperse and relocate again. While relocating, any two MTs are assumed to stay at sufficient distance from each other to prevent their both being targeted by one RV and, thus, becoming a single target. Figure 1 shows a typical target area scenario.

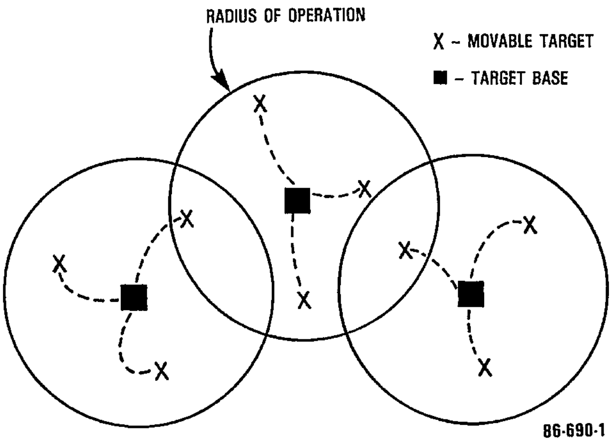

Figure 1: Movable Targets Operating Around Their Respective Bases

2.2. Missile Coverage Regions - Criteria For Automation

Each missile in FASIM is associated with a specific predetermined area within the total target area. This coverage area, called a grid, is a region where there is 100 percent confidence that all three MTs in a triplet are targetable. The advantage of using grids is twofold. First, any target located within the boundaries of a grid can be targeted with total surety with any missile assigned to that grid; second, grids are necessary for automation. If the target area is completely covered by grids, the mobility factor of the movable targets no longer poses a problem for force application. The MTs will be traveling in and out of grids, but will always be accessible by a missile.

The simulation modeled the grid structure to be either a square or a vertically oriented, downrange rectangle. The center of each grid is called the nominal aimpoint (NAP). A third grid structure modeled was a square grid overlaid with the downrange grid so that the two grids share the same aimpoint. Figure 2 illustrates the three grid structures.

To assure complete MT coverage within a target area, the entire target area is overlaid with grids, using only one of the

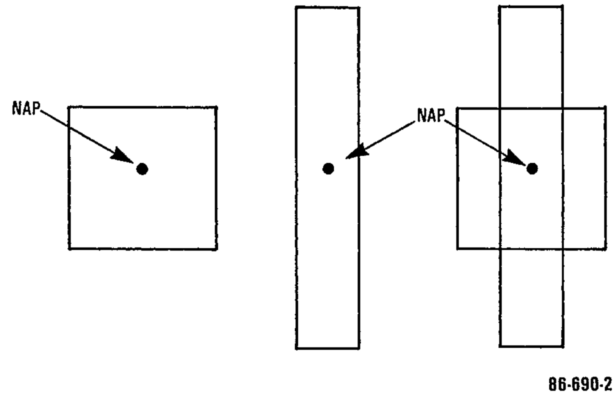

Figure 2: Square Grid, Downrange Grid, and Combined Square and Downrange Grid

three grid structures. The horizontal and vertical distances between the aimpoints, or offset, remain the same between all grids so that the grids are overlaid symmetrically. Depending on the offset, the number of grids required to cover a target area may vary. The smaller the offset, the greater the overlap, and the greater the number of grids needed to cover the target area. The larger the offset, the lesser the overlap, and the fewer the number of grids needed to cover the target area. Figure 3 shows how the offset can cause overlapping regions within grids.

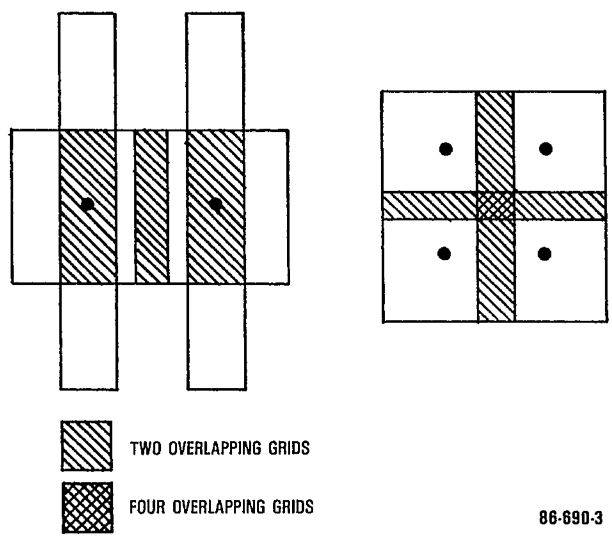

Figure 3: Overlapping Grids

3. THE MINIMIZATION ALGORITHM

Applying the minimization algorithm to any set of targets requires adherence to basic assumptions and constraints about grids and targets. The assumptions are: grid size is predetermined and constant for all grids; offset is such that every region in the target area is covered by at least one grid; a missile is accessible to more than one grid in the target area; any one grid may have more than one missile made accessible to it; the algorithm is applied to MTs once they become operational and the locations of the MTs are known; and an MT triplet formed within a grid is assigned to that grid. The major constraint of the algorithm is that all three MTs of a formed triplet must be contained in a single grid.

3.1. Triplet Creation

Grid overlap is the single most important factor in developing the minimization algorithm to create triplets. Greater overlap of grids allows an MT to be contained in several grids, thereby increasing the MT-to-grid assignment possibilities. The probability that a pure MT triplet is contained in one grid is also greatly increased. The trade-off is that grid overlap can greatly complicate the triplet creation process. More choices and decisions must be made to determine what MT is to be assigned to what grid, as shown in Figure 4, where targets X2, X3, and X4 can be assigned to either grid 1 or grid 2, and targets X6 and X7 can be assigned to either grid 2 or grid 3. A set of heuristic, judiciously chosen rules must be developed and the best combination of these rules found.

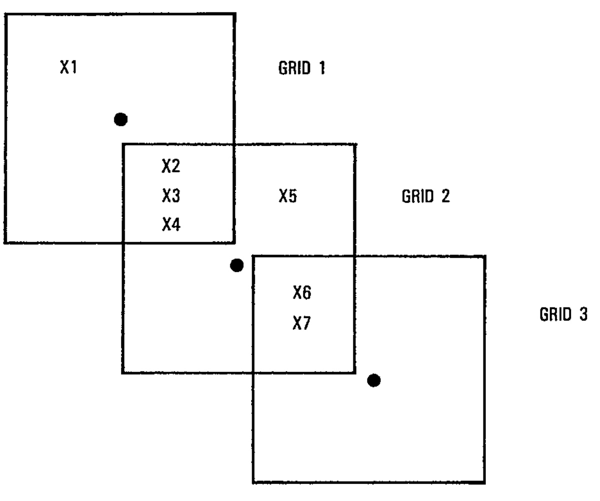

Figure 4: Target-to-Grid Relationships

Not all grids in a target area are assigned a missile, but just enough to guarantee that all MTs can be kept at risk. A logical first step in the algorithm is, therefore, to delete unnecessary grids from consideration, thereby reducing the number of possible MT-to-grid assignments. There are two variations for determining that a grid is unnecessary: first, if a grid's MT set is a subset of another grid's MT set and, second, if a grid contains only one or two MTs that are a subset of another grid's MT set. The latter variation does not deem a grid to be unnecessary if it contains three or more MTs because a perfect MT triplet can be formed within that grid. Grids that can be deleted from consideration are called redundant grids. The two grid redundancy variations as well as omitting grid redundancy were tested by the experimenters to determine which method maximized the number of triplets formed. In Figure 4 Grid 3 is redundant because its target set is a subset of the target set of Grid 2.

The next step in the algorithm is to determine which three MTs will be assigned and to what grid. Since overlap complicates this task, the relationships between MTs and grids must be examined. There are two categories of MTs: unique or common. An MT is unique if it is contained in only one grid; it is common if it is contained in more than one grid. Grids that contain unique MTs must be assigned a missile for the unique MT to be kept at risk; therefore, that grid should be selected first. Grids containing only common MTs are saved for last selection because common MTs have flexibility in their grid assignment. In Figure 4 target X1 is unique to Grid 1; targets X2, X3, and X4 are common to Grids 1 and 2; target X5 is unique to Grid 2; and targets X6 and X7 are common to Grids 2 and 3.

The algorithm proceeds as follows. Create a grid-MT relationship table, storing such information as the quantity and identity of common and unique MTs within each grid. Search the table and flag those grids that contain unique MTs. Once such grids are known, a criterion must be developed to determine which grid to select for assignment first. The first logical criterion is to select all grids that contain three or less MTs, at least one of which is unique. Since those grids must be selected for assignment, any common MTs should be included as part of the triplet. If this criterion fails, select grids having three or more unique MTs and assign a triplet MT set at random. If this criterion also fails, select all grids with two unique MTs, then all grids with one unique MT, and finally all grids with no unique MTs. If the next grid to be selected has no unique MTs, there are two variations for deciding which grid to select first: either select the grid with the least number of common MTs or select the grid with the largest number of common MTs. To determine which method maximized the number of triplets formed, the experimenters tested both variations.

When a grid is selected because it contains two or less unique MTs, one or two common MTs must be chosen to complete a triplet. There are two seemingly logical methods of selection: choose the common MT closest to the grid's aimpoint or choose the MT contained in the least number of grids. The former method is thought to make actual RV-to-target contact easier; the latter method is based on MT flexibility with respect to the number of grids containing it. To determine which method maximized the number of triplets formed, the experimenters tested both variations.

The force application algorithm is an iterative process that allows only one grid and one MT set to be assigned at each iteration. Each assigned MT is kept at risk and is not considered to be included in any other MT set, thus leaving fewer MTs to assign at each subsequent pass. For this reason, each iteration begins by updating the grid-MT table and checking each grid for grid redundancy. The grid-MT table is constantly changing, reflecting new grid-MT relationships due to the latest assignment. This process is repeated for all remaining grids. Each iteration ends when a specific criterion is met and the selected grid and triplet are assigned. The minimization algorithm stops when all MTs are assigned to grids.

4. DESIGN OF STATISTICAL EXPERIMENTS

4.1. Overview

The FASIM variations of triplet completion are heuristic in nature, consisting of competing versions or rules of grid selection and MT assignment. An objective method had to be developed for selecting the best combination of these rules. Since the force application algorithm is to be applied against targets that have a random distribution and density function, the final set of rules had to be the best in all situations.

Statistical experiments were devised to improve the algorithm's behavior in any possible situation. Random samples of potential target locations from a two-dimensional, uniform distribution were used to create test scenarios. To determine which of the competing versions of algorithmic variations was superior to the others, a succession of statistical experiments was conducted to compare the dependent variables, i.e., the number of missiles required. In this way the researchers developed an efficient technique for determining the optimum algorithmic combination of heuristic rules that make up the minimization algorithm.

For each basic experiment, the researchers identified controllable and uncontrollable variables, or factors, and their corresponding responses. A preliminary elimination of relatively unimportant factors resulted in a subset of factors to be scrutinized. Independent and dependent variables were then identified. Having accomplished this, an efficient experimental design was chosen. The randomized block factorial design was selected as a paradigm for two reasons: first, certain blocking variables needed to be controlled and, second, the effect of various offsets could be measured as factors. Since we were working with a computer simulation, different combinations of versions of the algorithm could be run under exactly the same target conditions. A sequence of experiments, each using the randomized block factorial design, was then conducted. Selection of versions for further testing was based on the results of the preceding experiment.

The same 20 runs were used for each individual experiment. The runs consisted of 5 random distributions of 75 targets, 5 random distributions of 60 targets, 5 random distributions of 42 targets, and 5 random distributions of 24 targets. Using these target distributions, competing variations of the computerized algorithm were compared at different levels of factors. Most frequently the variable, degree of offset (a measure of the amount each grid had been overlapped), was used as a factor. Levels of this factor were specified as .35, .50, .75, and 1.0. A factor of .35 indicates that 65 percent of a grid's area is overlapped with an adjacent grid; a factor of .50 indicates that 50 percent of the grid's area is overlapped with an adjacent grid; a factor of .75 indicates that 25 percent of the grid's area is overlapped with an adjacent grid; and a factor of 1.0 indicates no grid overlap.

For each individual experiment, the measurement of the significance of main effects (the competing versions of the heuristic algorithm) involved comparisons of the dependent variable, the number of missiles needed to cover all of the targets. In this manner the researchers selected the superior version from each experiment. Thus, after numerous individual experiments, the best combination of the various components of the algorithm was produced.

4.2. Factor or Variable Identification

The target location was chosen to be random for two reasons. First, these locations would not be controlled by the "targeter" in real life. Second, the researchers wished to infer which algorithmic version was the best in all possible target scenarios and did not want to bias the selection of the preferred version by unwittingly choosing target scenarios that were not general in nature.

Independent or controllable factors are those that can be controlled by a targeter, such as number and shape of grids, degree of offset, number of available missiles, number of missiles assigned to grids, and different versions of the algorithm.

It was determined that the dependent variable is the actual number of missiles needed to cover all of the targets involved in a particular target scenario. This variable is directly related to the number of completed triplets formed by the algorithm. One version of the algorithm is determined to be superior to another if it needs significantly fewer missiles to cover all of the targets involved in all of the different types of randomly generated scenarios.

4.3. Rationale for Selection of Randomized Block Design

The factorial arrangement used in this experiment not only gives information about the performance of each versional variation under different geometric conditions but also contrasts their performance over different target densities. Prior to the experiment it was not known whether some versions of the heuristic rules might produce uniform numbers of missiles over a range of geometric conditions and target densities while others might be unduly sensitive to changes in either geometries or target densities.

With the randomized block design, each version of the algorithm can be tried at two or more levels, such as offset, and at two or more values of blocking variables, such as target densities. In this way, interaction effects can be identified. If interactive effects are detected, the main effects are not investigated. If there is no convincing evidence of interaction, however, the researcher can proceed to investigate the main effects.

Randomness is an essential element of the randomized block factorial design experiments. To protect against systematic variation caused by uncontrollable experimental conditions, such as target location, we used the computer's random number generator to produce 20 sets of target distributions as test scenarios. To avoid having unnatural target conditions, such as two targets at an identical location, we incorporated into the target generation routine checks against this and other highly unlikely situations.

Since we were working with different versions of computerized algorithms, the same algorithm could be subjected to the 20 different test scenarios at all levels of blocking variables and factors without introducing error into the experiment. This aspect of the design reduced cost and contributed to the efficiency of the experiment.

In summary, the major advantages of using the randomized block factorial design are 1) the effects of the various versions of the algorithm on the number of missiles needed to cover all the targets are evaluated with the same precision as if the entire experiment had been devoted to exploring the effect of one of the versions alone, 2) the effects are evaluated over a wide range of experimental conditions with maximum efficiency, and 3) the design permits the investigation of interaction effects.

4.4. Layout and Computational Procedures

The randomized block factorial design paradigm in Figure 5 compares two competing versions of the algorithm. Five distributions — heavy, medium-heavy, medium, and light — are

	Version 1				Version 2			
	.35	.50	.75	1.0	.35	.50	.75	1.0
Lt. Dist.								
Med. Dist.								
Med-Hvy. Dist.								
Heavy Dist.								

86-165

Figure 5. A Randomized Block Factorial Design Paradigm

randomly generated, as discussed above. After the creation of the 20 test scenarios, version 1 was run 20 times with the grid offset set at 1.0. The number of missiles required to cover each of the target sets in each of the test scenarios was recorded in the appropriate cell. Then version 1 was run 20 times with the offset set to .75. This process was repeated until all cells in the paradigm were filled in with the number of missiles necessary to cover each of the test cases.

Once the paradigm was completed, analysis-of-variance (ANOVA) techniques were applied to the data to determine whether one version was significantly different from another. Computational tables, such as the one depicted in Figure 6, were used. These tables contained a record of the source of the variance, the sums of squares column (SS), a degree of freedom (df) column, a mean square (ms) column, and the F statistic derived from the F distribution for the degrees of freedom of the particular experiment. The sources of variance are identified as the treatment (algorithmic version), block (target density type), residual (variance due to error), and total variance.

In the usual manner, the sums of squares were divided by the corresponding degree of freedom, which resulted in the mean square. The F ratio was calculated by dividing the mean square of the residual effects into the mean square of the treatment or block effects.

Source	SS	df	MS	F	Table	P
Blocks	1925.475	19	101.34	230.3	3.00	.01
Treatment (B)	13.25	1	13.225	30.06	8.18	.01
Residual	8.275	19	.44			
Total	1946.975	39				

86-165

Figure 6: ANOVA Table

5. ANALYSIS OF RESULTS

The experiments conducted were a comparison of redundant grid methodologies; the effect on the algorithm using downrange grids, square grids, and combined square and downrange grids; grid selection techniques; and the comparison of two target selection methods when all MTs are common.

Using the randomized block factorial design, it was found that:

1. Deleting grids as redundant if their MT set is a subset of another grid's MT set is the superior method for minimizing the missiles required. This method proved to be superior at the 99 percent confidence level over implementing no redundancy and over treating grids that contain an MT subset of two or less MTs as redundant.
2. Using the square grid overlaid with a downrange grid to cover the target area is the superior method for minimizing the missiles required. This method proved to be superior at the 99 percent confidence level over using either grid separately.
3. When all targets are common and a grid must be selected, choosing the grid with the least number of targets is the superior method for minimizing the missiles required. This method proved to be superior at the 99 percent confidence level over choosing the grid with the largest number of targets.
4. When an MT must be selected within a grid, there is no significant difference between choosing the MT closest to the grid's aimpoint or choosing the MT contained in the least number of grids.

As a final measure of the effectiveness of the FASIM experiments, the original version of the algorithm was compared to the final version on 21 different, realistic test scenarios. A comparison of the two algorithms is shown in Figure 7. In sixteen of the 21 test scenarios, the final algorithm out-performed the original algorithm. In the remaining five the results were identical. In no test case did the original version out-perform the final version. Moreover, a close examination of Figure 7 shows that the final algorithm approaches and intersects the optimization curve a number of times.

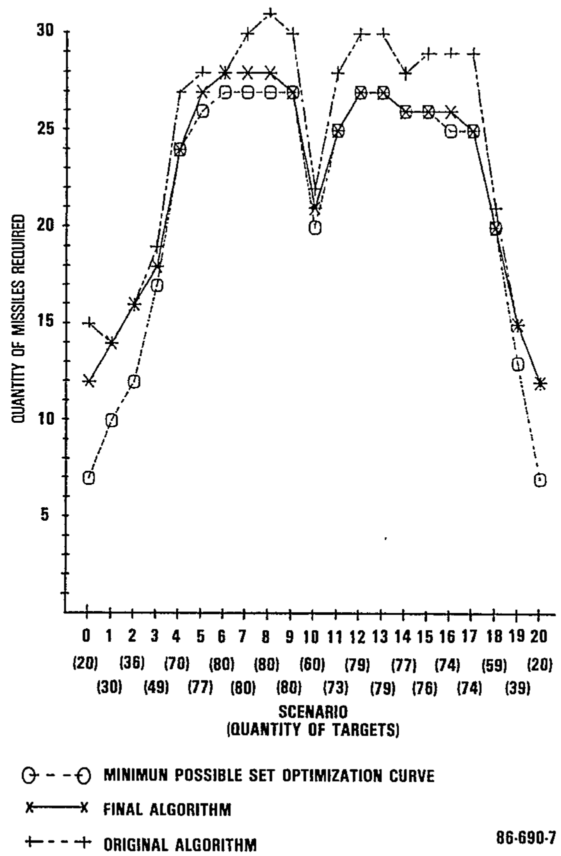

Figure 7: FASIM Improvements to the Triplet Minimization Algorithm

6. CONCLUSION

These results have led the FASIM research team to conclude that the experimental design technique perfected the algorithm to the point of near optimization. Moreover, the techniques outlined in this paper may be applied to other sets of heuristic rules, especially when there are two or more competing versions applied to the same problem.

ACKNOWLEDGEMENTS

The research on the FASIM project was supported by Independent Research and Development (IR&D) at GTE's Strategic Systems Division. The authors wish to thank Peter Himmelberger, the energetic and creative originator of the FASIM algorithm. They also wish to thank Edward Aron for his support and ideas on the project.

REFERENCES

Gillette, B., *Introduction to Operations Research: A Computerized Oriented Algorithmic Approach.* New York: McGraw-Hill, 1976.

Himmelberger, P. H., Morrissey, D. W., and Roberts, B. H., "FASIM: Force Application Simulation," TR.85.926.1, GTE Government Systems Corporation, Strategic Systems Division, Westborough, Mass., 1985.

Kirk, R.E., *Experimental Design: Procedures for the Behavioral Sciences.* Belmont, Calif.: Wadsworth Publishing Company, 1968.

Law, A.M. and Kelton, W.D., *Simulation Modeling and Analysis.* New York: McGraw-Hill, 1982.

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