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VALIDATION OF CORRELATION-INDUCTION STRATEGIES FOR SIMULATION EXPERIMENTS

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ABSTRACT

This paper develops a three-stage procedure for validating the use of the Schruben-Margolin correlation induction strategy together with the follow-up analysis of Nozari, Arnold, and Pegden in a simulation experiment designed to estimate a general linear model for the simulation response. Each stage of the procedure tests a key assumption about the behavior of the response across all points in the design. The first stage tests for multivariate normality, the second stage tests for the induced covariance structure postulated by Schruben and Margolin, and the third stage tests for the adequacy of the proposed linear model. Because the test in each stage presupposes the properties tested in previous stages, these diagnostic checks on the experimental design and analysis must be performed in the indicated order.

1. INTRODUCTION

The Schruben-Margolin correlation induction strategy [1] for the design of simulation experiments utilizes the variance reduction techniques of common random numbers and antithetic variates in a scheme based on the concept of blocking. This strategy has been shown to satisfy a variety of optimality criteria for a broad class of experimental designs [2]. Consider the situation in which each simulation run yields a univariate response, y. A particular run, called a *design point*, is identified by the settings of d factors or decision variables, denoted by η, that are used as inputs to the simulation model. In general, the relation of the response to the level of the d factors has the form:

$$y=\mu(\eta) + \epsilon \quad , \tag{1.1}$$

where ϵ represents the inability of μ to determine y. Schruben and Margolin assume μ is linear in the unknown parameters that relate the response to the factor settings. If m design points constitute the simulation experiment, then the linear model has the form:

$$y_i=\beta_0 + \sum_{k=1}^{p-1} \beta_k x_k(\eta_i) + \epsilon_i \quad \text{for } i=1, 2, \dots, m \quad , \tag{1.2}$$

where y_i is the response for the i'th design point, η_i is the setting of the d factors for the i'th design point, the $\{x_k : k=1, 2, \dots, p\}$ represent known functions of the factor settings, and the $\{\beta_k : k=0, 1, \dots, p-1\}$ are model parameters. Let $Y=(y_1, y_2, \dots, y_m)'$, $\beta=(\beta_0, \beta_1, \dots, \beta_{p-1})'$, $\epsilon=(\epsilon_1, \epsilon_2, \dots, \epsilon_m)'$; and define X to be the $(m \times p)$ matrix whose first column is all ones and whose (i,k+1) element is $x_k(\eta_i)$ (for $i=1, 2, \dots, m$ and $k=1, 2, \dots, p-1$). Thus the model in (1.2) can be written:

$$Y=X\beta + \epsilon \quad . \tag{1.3}$$

The variance of ϵ_i (for $i=1, 2, \dots, m$) is denoted by σ_i^2 and the variance-covariance matrix of Y is denoted by Σ. Schruben and Margolin [1] also assume that

$$\sum_{i=1}^{m} x_k(\eta_i)=0 \quad \text{for } k=1, 2, \dots, p-1 \quad .$$

A convenient notation is to write the design matrix X as $(1_m \; T)$, where 1_m is an $(m \times 1)$ column vector of ones.

The Schruben-Margolin correlation induction strategy is designed for the special case where X is orthogonally blockable into two blocks. The number of design points in each block is the *block size.* Suppose that the design matrix $X=(1_m \; T)$ satisfies $T'1_m=0_{p-1}$, a (p-1)-dimensional column vector of zeros. This design is orthogonally blockable into two blocks if there exists an $(m \times 2)$ matrix W of zeros and ones such that $T'W=0$ and $1_m'W=[m_1, m_2]$, where m_1 and m_2 are the respective block sizes. For this situation, Schruben and Margolin proposed the following assignment rule [1]:

> If the m-point experimental design admits orthogonal blocking into two blocks of sizes m_1 and m_2, preferably chosen to be as nearly equal in size as possible, then for all m_1 design points in the first block, use a set of pseudorandom numbers $R=(r_1, r_2, \cdots)$, chosen randomly, and for all m_2 design points in the second block, use $\overline{R}=(1-r_1, 1-r_2, \cdots)$.

To analyze the properties of this assignment rule, Schruben and Margolin made the following assumptions:

1. The response variance is constant across all points in the design, that is,

$$\sigma_i^2 \equiv \mathrm{Var}(y_i) = \mathrm{Var}[y_i(R)] = \sigma^2 \quad \text{for } i=1, 2, \ldots, m \ .$$

2. The responses y_i and y_j are uncorrelated if they are obtained with different random number streams:

$$\mathbf{R}, \mathbf{R}^* \text{ independent} \Rightarrow \mathrm{Corr}(y_i, y_j) = \mathrm{Corr}[y_i(\mathbf{R}), y_j(\mathbf{R}^*)] = 0 \ .$$

3. If y_i and y_j (for $i \neq j$) are obtained with the same random number stream, then

$$\mathrm{Corr}[y_i(\mathbf{R}), y_j(\mathbf{R})] = \rho_1, \quad 0 < \rho_1 < 1 \ .$$

4. If y_i and y_j (for $i \neq j$) are obtained from antithetic (complementary) random number streams, then

$$\mathbf{R}, \overline{\mathbf{R}} \text{ antithetic} \Rightarrow \mathrm{Corr}[y_i(\mathbf{R}), y_j(\overline{\mathbf{R}})] = \rho_2, \quad 0 < -\rho_2 < \rho_1 \ .$$

Let $\mathbf{X}_i$ (for $i=1, 2$) represent the design matrix for the i'th block. If the design points are so arranged that $\mathbf{X} = \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{bmatrix}$, then these assumptions lead to the following structure for the variance-covariance matrix, Σ:

$$\Sigma = \sigma^2 \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix} \ , \tag{1.4}$$

where Σ_{11} is an $(m_1 \times m_1)$ matrix with ones on the main diagonal and ρ_1 off the main diagonal, Σ_{12} is an $(m_1 \times m_2)$ matrix with all elements equal to ρ_2, $\Sigma_{21} = \Sigma_{12}'$, and Σ_{22} is like Σ_{11} except that it is $(m_2 \times m_2)$. Nozari, Arnold, and Pegden [3] derived appropriate methods for statistical analysis under the Schruben-Margolin correlation induction strategy when the block sizes are equal:

$$q \equiv m_1 = m_2 \Rightarrow m = 2q \ .$$

They considered separate and simultaneous inferences on β_0, and $(\beta_1, \beta_2, \ldots, \beta_{p-1})'$. In addition to the assumptions already mentioned, Nozari, Arnold, and Pegden assumed that r independent replications are made at each of the m design points. Let $\mathbf{y}_i = (y_{i1}, y_{i2}, \ldots, y_{im})'$ represent the m responses for the i'th replication. Then $\mathbf{Y} = (\mathbf{y}_1', \mathbf{y}_2', \ldots, \mathbf{y}_r')'$ is an $((rm) \times 1)$ dimensional vector of responses for the overall experiment. Nozari, Arnold and Pegden also assumed that the responses on each replication are jointly normal:

$$\mathbf{y}_i \sim N_m(X\beta, \Sigma) \quad \text{for } i=1, 2, \ldots, r \ . \tag{1.5}$$

2. THE VALIDATION PROCEDURE

In order to validate the use of the Schruben-Margolin correlation induction strategy and the follow-up analysis of Nozari, Arnold and Pegden, a preliminary (or pilot) experiment should be performed so that the following diagnostic checks can be made:

1. *Test for multivariate normality*:

$$\left.\begin{array}{l} H_0\colon \mathbf{y}_i \sim N_m(\mu_y, \Sigma_y) \text{ with} \\ \quad \mu_y \equiv E(\mathbf{y}_i),\ \Sigma_y \equiv \mathrm{Cov}(\mathbf{y}_i) \text{ unspecified;} \\ \text{versus} \\ H_1\colon \mathbf{y}_i \text{ has any m-dimensional distribution.} \end{array}\right\} \tag{2.1}$$

2. *Test for the Induced Covariance Structure*:

$$\left.\begin{array}{l} H_0\colon \mathrm{Cov}(\mathbf{y}_i) = \Sigma \text{ as in (1.4)} \\ \quad \text{with } \sigma^2, \rho_1, \rho_2 \text{ unspecified;} \\ \text{versus} \\ H_1\colon \mathrm{Cov}(\mathbf{y}_i) \text{ is positive definite.} \end{array}\right\} \tag{2.2}$$

3. *Test for Lack of Fit in the Linear Model*:

$$H_0\colon E[\mathbf{y}] = X\beta \ \text{ vs } \ H_1\colon E[\mathbf{y}] \neq X\beta \ . \tag{2.3}$$

If all three null hypotheses are accepted, then we have some basis for designing the main experiment with the Schruben-Margolin correlation induction strategy and for using the follow-up analysis of Nozari, Arnold and Pegden. If one or more of the tests leads to rejection of the corresponding null hypothesis, then this information can be used to take suitable corrective action. For example, departures from normality and/or the assumed covariance structure may indicate the need for an appropriate transformation of the original observations [4]. Moreover, lack of fit in the postulated linear model may call for the inclusion of higher-order terms in the model. We now discuss each of these tests in detail.

2.1 Test for Multivariate Normality

Because the Shapiro-Wilk test has proved to be a superior omnibus test for univariate normality [5], we sought an appropriate multivariate extension of this procedure. Our computational experience with versions of the multivariate Shapiro-Wilk test due to Malkovich and Afifi [6, 7] and Royston [8] led us to adopt the former procedure as described below.

In terms of the sample statistics

$$\overline{\mathbf{y}} = r^{-1}\sum_{i=1}^{r} \mathbf{y}_i \ , \quad \mathbf{A} = \sum_{i=1}^{r} (\mathbf{y}_i - \overline{\mathbf{y}})(\mathbf{y}_i - \overline{\mathbf{y}})' \ , \tag{2.4}$$

we identify the observation $\mathbf{y}^\dagger \in \{\mathbf{y}_i \ : \ i=1, 2, \ldots, r\}$ for which

$$(\mathbf{y}^\dagger - \overline{\mathbf{y}})'\mathbf{A}^{-1}(\mathbf{y}^\dagger - \overline{\mathbf{y}}) = \max\left\{ (\mathbf{y}_i - \overline{\mathbf{y}})'\mathbf{A}^{-1}(\mathbf{y}_i - \overline{\mathbf{y}}) : i=1, 2, \ldots, r \right\} \ . \tag{2.5}$$

We then compute the variates

$$Z_i \equiv (y^{\dagger}-\bar{y})'A^{-1}(y_i-\bar{y}), \quad i=1, 2, \ldots, r \qquad (2.6)$$

and let

$$Z_{(1)} < Z_{(2)} < \ldots < Z_{(r)}$$

denote the corresponding order statistics. Let $\{a_i : i=1, 2, \ldots, r\}$ denote the coefficients of the univariate Shapiro-Wilk test for a random sample of size r. The $\{a_i\}$ can be obtained by calling subroutine WCOEF described in [5]. The statistic for the multivariate Shapiro-Wilk test is then given by

$$W^* = \frac{\left[\sum_{i=1}^{r} a_i Z_{(i)}\right]^2}{(y^{\dagger}-\bar{y})'A^{-1}(y^{\dagger}-\bar{y})} \quad . \qquad (2.7)$$

The computed W^* is referred to a table of critical values given in [6].

2.2 Test for the Induced Covariance Structure

Consider the hypotheses

$$H_0: \mathrm{Cov}(y_i)=\Sigma \text{ vs } H_1: \mathrm{Cov}(y_i) \text{ is positive definite} \ .$$

Let Γ denote the following $(m\times m)$ matrix:

$$\Gamma=\begin{bmatrix} m^{-\frac{1}{2}}1_q' & m^{-\frac{1}{2}}1_q' \\ m^{-\frac{1}{2}}1_q' & -m^{-\frac{1}{2}}1_q' \\ C_q' & O \\ O & C_q' \end{bmatrix} , \qquad (2.8)$$

where C_q is a $(q\times(q-1))$ matrix such that the $(q\times q)$ matrix $[q^{-\frac{1}{2}}1_q\, C_q]$ is orthogonal [3]. Also, let $y_i^*=\Gamma y_i$ (for $i=1, 2, \ldots, r$). If the null hypotheses (2.1) and (2.2) are both true, then

$$\left.\begin{array}{l} y_i^* \sim N_m(\mu^*, \Sigma^*) \text{ for } i=1, 2, \ldots, r \ , \\ \text{where } \mu^*=\Gamma E(y_i) \text{ is unspecified but } \Sigma^*=\Gamma\Sigma\Gamma'. \end{array}\right\} \qquad (2.9)$$

Nozari, Arnold, and Pegden [3] note that, since the transformation from **y** to **y*** is invertible and does not involve any unknown parameters, any hypothesis-testing procedure based on **y*** which is optimal among procedures based on **y*** is also optimal among hypothesis-testing procedures based on **y**.

Under the null hypotheses (2.1) and (2.2), it can be shown [3] that

$$\Sigma^*=\begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 I_{m-2} \end{bmatrix} , \qquad (2.10)$$

where

$$\lambda_1=\sigma^2[1 + (q-1)\rho_1 + q\rho_2] \ ,$$
$$\lambda_2=\sigma^2[1 + (q-1)\rho_1 - q\rho_2] \ ,$$
$$\lambda_3=\sigma^2(1 - \rho_1) \ .$$

In view of (2.9), the maximum likelihood estimators of λ_1, λ_2, and λ_3 are

$$\hat{\lambda}_1=r^{-1}A_{11}^* \qquad (2.11)$$

$$\hat{\lambda}_2=r^{-1}A_{22}^* \qquad (2.12)$$

$$\hat{\lambda}_3=[r(m-2)]^{-1}\mathrm{tr}(A_{33}^*) \qquad (2.13)$$

where

$$A^*=\begin{bmatrix} A_{11}^* & A_{12}^* & A_{13}^* \\ A_{21}^* & A_{22}^* & A_{23}^* \\ A_{31}^* & A_{32}^* & A_{33}^* \end{bmatrix} ;$$

and

$$A^*=\Gamma A\Gamma'=\Gamma\left[\sum_{i=1}^{r}(y_i-\bar{y})(y_i-\bar{y})'\right]\Gamma' \ . \qquad (2.14)$$

Thus the likelihood ratio statistic for testing H_0 versus H_1 in (2.2) is:

$$L=\left[\frac{\det(r^{-1}A^*)}{\hat{\lambda}_1\hat{\lambda}_2\hat{\lambda}_3^{m-2}}\right]^{\frac{r}{2}} \ . \qquad (2.15)$$

If H_0 is true and r is large, then

$$-2\ln(L)\sim\chi^2[m(m+1)/2 - 3] \qquad (2.16)$$

(see [9]). Thus, we reject H_0 in (2.2) at the significance level α if $-2\ln(L)$ exceeds the $(1-\alpha)^{th}$ quantile of the chi-square distribution with $m(m+1)/2 - 3$ degrees of freedom.

This test can be written in terms of the original responses, y_{ijk}, where y_{ijk} is the k'th response in the j'th block of the i'th replication (for $i=1, 2, \ldots, r$; $j=1, 2$; $k=1, 2, \ldots, q$). Define

$$\bar{y}_{ij.}=q^{-1}\sum_{k=1}^{q} y_{ijk} \ , \quad \bar{y}_{i..}=2^{-1}\sum_{j=1}^{2}\bar{y}_{ij.} \ , \quad \bar{y}_{...}=r^{-1}\sum_{i=1}^{r}\bar{y}_{i..} \qquad (2.17)$$

The MLE of λ_1, λ_2, and λ_3, in terms of the original responses, are:

$$\hat{\lambda}_1=r^{-1}m\sum_{i=1}^{r}(\bar{y}_{i..}-\bar{y}_{...})^2 \ , \qquad (2.18)$$

$$\hat{\lambda}_2=(2r)^{-1}q\sum_{i=1}^{r}[(\bar{y}_{i1\cdot}-\bar{y}_{i2\cdot})-(\bar{y}_{\cdot 1\cdot}-\bar{y}_{\cdot 2\cdot})]^2 \ , \qquad (2.19)$$

$$\hat{\lambda}_3=\frac{[m(r-1)]\hat{\sigma}^2 - r(\hat{\lambda}_1+\hat{\lambda}_2)}{r(m-2)} \ , \qquad (2.20)$$

where

$$\hat{\sigma}^2=[m(r-1)]^{-1}\sum_{i=1}^{r}\sum_{j=1}^{2}\sum_{k=1}^{q}(y_{ijk}-\bar{y}_{\cdot jk})^2 \ . \qquad (2.21)$$

Equations (2.18) and (2.19) follow directly from results given in [3]. Equation (2.20) defines a *pure error* sum of squares that is analogous to a quantity derived in [3] but that is free of any bias due to inadequacy of the postulated linear model (1.5). Thus, we have removed any effects due to lack of fit from the test for the induced covariance structure.

2.3 Test for Lack of Fit in the Linear Model

Consider the test H_0: $E[y]=X\beta$ vs H_1: $E[y]\neq X\beta$, where X has rank p ($\leq$m). This is equivalent to testing for

$$H_0\colon E[y_i^*]=\Gamma X\beta \ \text{ vs } \ H_1\colon E[y_i^*]\neq\Gamma X\beta \ . \qquad (2.22)$$

If we write

$$y_i^*=\begin{bmatrix} y_{i1}^* \\ y_{i2}^* \\ y_{i3}^* \end{bmatrix}, \text{ where } y_{i3}^* \text{ is } ((m-2)\times 1), \ i=1,..,r \qquad (2.23)$$

and $\beta\equiv[\beta_0, \beta_1']'$ where β_1 is $((p-1)\times 1)$, then testing (2.22) is in turn equivalent to performing the following two tests independently:

$$H_0\colon E[y_{i2}^*]=0 \ \text{ vs } \ H_1\colon E[y_{i2}^*]\neq 0 \ ; \qquad (2.24)$$

$$\left.\begin{array}{l} H_0\colon E[y_{i3}^*]=\begin{bmatrix} C_q' & 0 \\ 0 & C_q' \end{bmatrix}\mathbf{T}\beta_1 \\ \text{versus} \\ H_1\colon E[y_{i3}^*]\neq\begin{bmatrix} C_q' & 0 \\ 0 & C_q' \end{bmatrix}\mathbf{T}\beta_1 \end{array}\right\} \ . \qquad (2.25)$$

To achieve an overall level of significance α in testing for (2.22), we perform the tests (2.24) and (2.25) each at the significance level

$$\delta = 1 - (1-\alpha)^{\frac{1}{2}} \ . \qquad (2.26)$$

In terms of the original data, we reject H_0 in (2.24) if

$$\frac{|\bar{y}_{\cdot 1\cdot}-\bar{y}_{\cdot 2\cdot}|}{\left\{[r(r-1)]^{-1}\sum_{i=1}^{r}[(\bar{y}_{i1\cdot}-\bar{y}_{i2\cdot})-(\bar{y}_{\cdot 1\cdot}-\bar{y}_{\cdot 2\cdot})]^2\right\}^{\frac{1}{2}}} > t_{1-\delta}(r-1) \qquad (2.27)$$

where $t_{1-\delta}(r-1)$ is the $(1-\delta)^{th}$ quantile of the student-t distribution with r-1 degrees of freedom. To test for lack of fit in (2.25), we take

$$SS_E\equiv\sum_{i=1}^{r}||y_i-X\beta||^2 \ , \qquad (2.28)$$

$$SS_E^*=SS_E - q\sum_{i=1}^{r}\sum_{j=1}^{2}(\bar{y}_{ij\cdot}-\bar{y}_{\cdots})^2 \ , \qquad (2.29)$$

$$df_E^*=mr-p-2r+1 \ , \qquad (2.30)$$

$$SS_{PE}^*=r(m-2)\hat{\lambda}_3 \ , \text{ and} \qquad (2.31)$$

$$df_{PE}^*=2(q-1)(r-1) \ ; \qquad (2.32)$$

then we reject H_0 in (2.25) if

$$\frac{(SS_E^*-SS_{PE}^*)/(df_E^*-df_{PE}^*)}{SS_{PE}^*/df_{PE}^*} > F_{1-\delta}(df_E^*-df_{PE}^*, df_{PE}^*) \qquad (2.33)$$

where $F_{1-\delta}(df_E^*-df_{PE}^*, df_{PE}^*)$ is the $(1-\delta)^{th}$ quantile of the F-distribution with $df_E^*-df_{PE}^*$ and df_{PE}^* degrees of freedom.

3. EXAMPLE

Consider the job shop example given by Nozari, Arnold, and Pegden [3] consisting of eight design points and ten replications. The (8×8) design matrix is:

$$\mathbf{X}=\begin{bmatrix} 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & -1 & 1 & -1 & -1 \end{bmatrix} \qquad (3.1)$$

This design matrix has rank 7 and is orthogonally blockable into two blocks of size 4. Therefore, in obtaining the data, Nozari, Arnold and Pegden employed the Schruben-Margolin correlation induction strategy. The responses are given in the following table:

Table 1: Observed Values of the Response

				Replication (i)									
Block (j)	X_1	X_2	X_3	1	2	3	4	5	6	7	8	9	10
1	-1	-1	1	17	8	23	34	30	43	12	11	17	28
1	-1	1	-1	6	9	16	29	21	44	7	13	18	19
1	1	-1	-1	15	5	19	32	28	41	8	11	17	25
1	1	1	1	0	0	12	27	17	27	0	0	8	15
2	-1	-1	-1	27	27	19	16	14	11	17	23	25	27
2	-1	1	1	15	20	10	6	5	2	12	13	14	15
2	1	-1	1	19	25	16	13	7	9	15	21	21	23
2	1	1	-1	24	18	10	9	9	0	8	8	10	19

We note that, for this example, r=10, m=8, and p=7. The least squares estimate of β is:

$$\hat{\beta}=(\hat{\beta}_0, \hat{\beta}_1, ..., \hat{\beta}_6)'=$$
$$=(28.125, -3.150, -1.250, 1.550, -.700, .600, -3.200)' .$$

We now apply the validation tests to this example.

3.1 Test for Multivariate Normality

Because Malkovich's tabulation [6] of critical values for the multivariate Shapiro-Wilk test statistic (2.7) is limited to dimensionalities from 2 to 5, we performed this test using only 2 design points from each block as shown in Table 2.

Table 2: Design Points Used in the Test for Multivariate Normality

Block	X_1	X_2	X_3
1	-1	1	-1
1	1	1	1
2	-1	-1	-1
2	1	-1	1

The computed value

$$W^* = 0.6945$$

is not significant even at the α=0.25 level. Thus, we concluded that there is no significant departure from multivariate normality in the simulation response and that it is reasonable to proceed with the subsequent steps of the validation procedure.

3.2 Test for the Induced Covariance Structure

Since the test for normality was accepted, we then performed the test for the induced covariance structure as given in section 2.2. Equation (2.21) yielded

$$\hat{\sigma}^2=81.783 .$$

Then, from equations (2.18), (2.19), and (2.20), we obtained:

$$\hat{\lambda}_1=113.330, \quad \hat{\lambda}_2=437.200, \quad \hat{\lambda}_3=6.385 .$$

From the sample covariance matrix given in [3] we calculated that

$$\det(r^{-1}A^*)=\det(r^{-1}A)=122,769.248 .$$

Substitution of these values into (2.15) gave $L=6.5388\times10^{-23}$, which, in turn, resulted in the following value for the test statistic

$$-2\ln(L)=102.1634 .$$

Now the quantile of order 0.9995 for a chi-square distribution with 33 degrees of freedom is

$$\chi^2_{0.9995}(33) = 66.4 ;$$

thus we conclude that the assumptions regarding the induced covariance structure are not satisfied. Before further analysis can be performed, an appropriate variance-stabilizing transformation [4] should be applied to the original data.

4. CONCLUSIONS

The validation procedure developed in this paper provides practitioners with standard statistical measures of the extent to which a proposed simulation experiment is amenable to the application of the Schruben-Margolin correlation induction strategy together with the follow-up analysis of Nozari, Arnold and Pegden. Failure of any of the proposed diagnostic checks may indicate the need for an appropriate corrective action before further development of the experimental design.

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