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RANKING AND SELECTION PROCEDURES USING STANDARDIZED TIME SERIES

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ABSTRACT

We study the problem of determining that one of k stationary simulated processes which has the largest mean. We adapt for use in the simulation environment a ranking and selection procedure due to Dudewicz and Dalal (1975). In order to implement this procedure, it is necessary to estimate the process variance of each of the k simulated systems; variance estimators arising from the theory of standardized time series are used for this purpose.

1. INTRODUCTION

We consider the *ranking and selection* problem of determinating the "best" of k competing populations or systems or processes. "Best" is used with respect to that population criterion which is thought to be the most important. In this paper, we shall focus on the problem of finding that one of k stationary stochastic processes which has the largest mean.

Virtually all of the existing ranking and selection methods assume that the observations taken from a particular population are realizations of independent and identically distributed random variables. Since the random variables resulting from a simulation process are almost never i.i.d., we cannot directly use the existing ranking and selection procedures in the simulation environment. The main roadblock here involves the fact that the existing procedures require estimation of each population's process variance.

In Section 2 of this paper, we present a concise description of the problem under study. Further, process variance estimators appropriate for use with arbitrary stationary stochastic processes are provided. In Section 3, we outline an adaptation of an existing ranking and selection method for use in simulations.

2. BACKGROUND

2.1 Ranking and Selection

We will have reason to concentrate primarily on the so-called *normal means problem*: Suppose there are k independent normal populations, $\Pi_1, \Pi_2, \ldots, \Pi_k$, where $\Pi_i \sim \text{Nor}(\mu_i, \sigma_i^2)$, $i=1,\ldots,k$. The μ_i's and σ_i^2's are assumed to be unknown. We also denote the ordered but unknown means as $\mu_{[1]} \leq \mu_{[2]} \leq \cdots \leq \mu_{[k]}$. The rough goal is to find the Π_i corresponding to $\mu_{[k]}$, the largest of the k means.

A typical normal means ranking and selection procedure dictates that the experimenter take a certain number of independent observations from each of the normal populations. Based on these observations, the procedure then tells the experimenter which one of the k populations most likely has the largest true mean. If the experimenter selects the population that is associated with $\mu_{[k]}$, we say that a *correct selection* (CS) has been made. Our formal goal is to satisfy the following *probability requirement* (PR):

$$P\{ \text{CS} \mid \mu_{[k]} - \mu_{[k-1]} \geq \delta^* \} \geq P^*.$$

(We take $1/k < P^* < 1$ and $\delta^* > 0$ for the obvious reasons.)

$\Omega_{\delta^*} \equiv \{ \underset{\sim}{\mu} \mid \mu_{[k]} - \mu_{[k-1]} \geq \delta^* \}$ is called the *preference zone*, and its complement is the *indifference zone* - the PR does not involve any $\underset{\sim}{\mu} \in \Omega^c_{\delta^*}$. Procedures which satisfy the PR are called indifference zone ranking and selection procedures [Bechhofer (1954)]. For a more complete discussion concerning such procedures, the reader should see Gupta and Panchapakesan (1979)

or Gibbons, Olkin, and Sobel (1977). Goldsman (1983) gives an elementary ranking and selection tutorial.

2.2 Simulation

As we have already pointed out, the random variables associated with a simulation process are rarely i.i.d.; so estimation of the process variance is sometimes difficult. This variance estimation problem is an active area of research. A number of variance estimation techniques have been studied; see any recent simulation text for the pertinent references. In this paper, we will work with the batched means and standardized time series methodologies. (The method of independent replications is asymptotically equivalent to batched means, so it will not be discussed here.)

Consider a stationary stochastic process $X_1, X_2, \ldots, X_n$. We shall describe various estimators for the process variance, $\sigma^2 \equiv \lim_{n\to\infty} n\mathrm{Var}(\bar{X}_n)$, where $\bar{X}_n \equiv \Sigma X_i/n$, the grand sample mean.

Batched Means

Divide the stream into b contiguous, nonoverlapping *batches*, each consisting of m observations (n = bm). The random variable corresponding to the j—th observation from batch i is $X_{(i-1)m+j}$, $i=1,\ldots,b$; $j=1,\ldots,m$. Define the *batched mean* from the i—th batch by:

$$\bar{X}_{i,m} \equiv \frac{1}{m} \sum_{j=1}^{m} X_{(i-1)m+j}.$$

Then the *classical batched means* estimator for σ^2 is given by:

$$V_{0,b} \equiv \frac{m}{b-1} \sum_{i=1}^{m} \left[\bar{X}_{i,m} - \bar{\bar{X}}_n \right]^2 \approx \frac{\sigma^2 \chi^2(b-1)}{b-1},$$

where "$\approx$" is read "is approximately distributed as".

Standardized Time Series

A stationary simulation process can (asymptotically) be standardized into a so—called Brownian bridge. Properties of Brownian bridges are then exploited in order to obtain variance estimators. [cf. Kang and Goldsman (1985) (in these *Proceedings*), Schruben (1983), or Goldsman (1984) for additional motivation and details.]

Again suppose that the X_i's are divided into b batches, each of size m. Denote the j—th cumulative mean (j=1,...,m) from batch i as:

$$\bar{X}_{i,j} \equiv \frac{1}{j} \sum_{t=1}^{j} X_{(i-1)m+t}.$$

Finally, define for all batches,

$$\hat{A}_i \equiv \sum_{j=1}^{m} j\left[\bar{X}_{i,m} - \bar{X}_{i,j} \right].$$

Schruben derives the following estimators for σ^2:

Area estimator -

$$V_{1,b} \equiv \frac{12}{(m^3-m)b} \sum_{i=1}^{b} \hat{A}_i^2 \approx \frac{\sigma^2 \chi^2(b)}{b}.$$

Combined classical-area estimator -

$$V_{2,b} \equiv \frac{(b-1)V_{0,b} + bV_{1,b}}{2b-1} \approx \frac{\sigma^2 \chi^2(2b-1)}{2b-1}.$$

3. A RANKING AND SELECTION PROCEDURE

We adapt for use in the simulation environment a normal means ranking and selection procedure due to Dudewicz and Dalal (1975) (D—D). Their procedure is designed to find that one of k independent normal populations which has the largest mean; the means and variances of these populations are assumed to be *completely unknown*. The D—D procedure is a *two stage* indifference zone procedure. In the first stage of sampling, the variance of each normal population is estimated. These estimates are then used to determine how many additional observations must be taken from each population in the second stage.

After the second stage of sampling is completed, the procedure tells the experimenter which of the populations is most likely to have the largest mean.

The problem of interest here is to find that one of k stationary *simulation* processes which has the largest mean. In this case, it is generally not possible to directly obtain the i.i.d. normal observations which the D—D procedure requires. However, if we choose large enough batch size m, a central limit theorem allows us to treat the batched means from a particular simulation process as if they were i.i.d. normal (albeit with unknown mean and variance). The D—D procedure also requires estimation of each normal population's variance; the analogous task in the simulation environment can essentially be performed by the process variance estimators described in the previous section.

The following procedure is paraphrased from Goldsman and Nozari (1985):

STAGE 1

1-0 Specify $\{P^*, \delta^*\}$.

1-1 Make an initial run of the k simulation processes, $\pi_1, \ldots, \pi_k$.

Process 1: $Y_{1,1}, Y_{1,2}, \ldots, Y_{1,mn_0}$

Process 2: $Y_{2,1}, Y_{2,2}, \ldots, Y_{2,mn_0}$

$\vdots$

Process k: $Y_{k,1}, Y_{k,2}, \ldots, Y_{k,mn_0}$

For ease of exposition, we have assumed that each of the k simulations has been run for exactly mn_0 units of simulated time. m and n_0 are specified *a priori* by the experimenter. m must be large enough so that the necessary assumptions underlying the theory of standardized time series [cf. Schruben (1983)] are approximately valid.

1-2 Calculate the (first stage) sample mean from each of the k processes.

The sample mean from process i is given by:

$$\bar{Y}_i^{(1)} \equiv \sum_{t=1}^{mn_0} Y_{i,t}/mn_0 .$$

1-3 Divide each of the k runs into n_0 batches, each of size m.

1-4 Calculate the cumulative means from each of these batches.

The t-th cumulative mean from batch j of process i is given by:

$$\bar{Y}_{i,j,t} \equiv \frac{1}{t} \sum_{p=1}^{t} Y_{i,(j-1)m+p} ,$$

where i=1,...,k; j=1,...,n_0; t=1,...,m.

Note that $\bar{Y}_{i,j,m}$ is the j-th batched mean from π_i.

1-5 Estimate the variance of each π_i.

Define the variance of process i by:

$$\sigma_i^2 \equiv \lim_{n\to\infty} n\mathrm{Var}\left(\bar{Y}_{i,t}/n \right) .$$

We can use the classical batched means, area, or combined methods in order to estimate σ_i^2. To illustrate, we consider the area estimator:

$$V_{i,1,n_0} \equiv \frac{12}{(m^3-m)n_0} \sum_{j=1}^{n_0} \hat{A}_{i,j}^2 ,$$

where

$$\hat{A}_{i,j} \equiv \sum_{t=1}^{m} t\left[\bar{Y}_{i,j,m} - \bar{Y}_{i,j,t} \right] .$$

STAGE 2 (We continue to use the area estimator for illustrative purposes.)

2-1 Determine the number of observations to be taken in the second stage.

Calculate for i=1,...,k:

$$n_i \equiv \max \left\{ n_0+1, \left\| \frac{1}{m} V_{i,1,n_0} (h/\delta^*)^2 \right\| \right\} ,$$

where $\|.\|$ is the "ceiling" function and $h > 0$ is the unique solution of:

$$\int_{-\infty}^{\infty} \left[F_{n_0}(x+h) \right] dF_{n_0}(x) = P^* ,$$

F(.) being the cdf of the t-distribution with the degrees of freedom of the variance estimator. [h tables can be found in D-D and Koenig and Law (1982).]

2-2 Take $m(n_i - n_0)$ additional observations from each π_i, i=1,...,k.

2-3 Calculate the additional batched means from each π_i:

$$\bar{Y}_{i,j,m} \equiv \frac{1}{m} \sum_{p=1}^{m} Y_{i,(j-1)m+p} ,$$

i=1,...,k; j=n_0+1,...,n_i.

2-4 For each process, calculate the "pseudo-average":

$$\bar{\bar{Y}}_i \equiv \frac{1}{n_i} \sum_{j=1}^{n_i} a_{i,j} \bar{Y}_{i,j,m} ,$$

where a formula for the $a_{i,j}$'s can be found in D-D.

2-5 Declare that π_i associated with the largest pseudo-average as having the largest mean.

Remarks

(1) It turns out that the $a_{i,j}$'s are $\approx 1/n_i$; so the procedure is intuitvely appealing.

(2) Law and Kelton (1982) and Goldsman (1983) show how to implement the D–D procedure using the classical batched means variance estimator.

(3) Nozari and Morris (1984) present an interesting application of the D–D procedure in a simulation context.

(4) It is straightforward to adapt other ranking and selection procedures for application in simulations.

(5) Iglehart (1977) and Sullivan and Wilson (1984) give ranking and selection procedures using variance estimators arising from regeneration and spectral analysis, respectively.

4. SUMMARY

We have shown that it is relatively easy to adapt many ranking and selection procedures for use with computer simulations. Specifically, we adapted the Dudewicz and Dalal normal means procedure for the purpose of finding that one of k stationary simulation processes which has the largest mean.

When they are applicable, ranking and selection methods are more parsimonious with observations than other methods of classical statistics; therefore, use of the appropriate ranking and selection procedures can frequently save the experimenter time and expense. It is hoped that the value of such methods will stimulate further investigation.

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