EVALUATION OF A METRIC INVENTORY SYSTEM USING SIMULATION

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Traditional measures obtained from the application of theoretical inventory models do not provide detailed information on the ability of multi-echelon, repairable item inventory systems to maintain their supported machines in an operational status. Monte Carlo simulation techniques are applied to obtain estimates of the operational status actually attained. An advantage of this simulation technique is that they do not require all the strict assumptions of the theoretical models. The SIMAN simulation language developed by Fritscher and Poggen is used for this analysis.

1. INTRODUCTION

The objective of this paper is to determine the operational status of a large-scale repairable item system where the spare levels are established by the Multi-Echelon Technique for Recoverable Item Control (METRIC). The use of METRIC to establish the level of spares needed does not ensure that the number of supported machines remains at the established level. The traditional measures provided in inventory analysis (expected shortages, etc.) are indices of supply behavior and do not directly answer how well the maintained machines are supported. Additionally, certain assumptions required by METRIC models are not met in operational systems. Monte Carlo simulation is applied to statistically test the assumptions required by METRIC models to actually attain within an operational inventory and repair system to determine the effects on the operational status of relaxing strict assumptions of the theoretical models.

The paper consists of four sections. In the first section, the conceptual model of the system is presented and analyzed using the theoretical models cited in the literature. Based on the conceptual model, a network simulation model is developed and used to analyze the operational status of the system. Next, the results of the simulation analysis are analyzed to test the hypothetical effects of relaxing theoretical assumptions. The final section presents conclusions and suggestions for future research.

2. CONCEPTUAL MODEL

The system used as the basis for this study supports the inventory and repair of the missile guidance sets (MGSs) for the Minuteman III missile force. The MGSs are characteristic of repairable items -- they may be repaired upon failure1 and returned to a serviceable condition. In fact, this is a closed system, since no new MGSs are being purchased. The system is illustrated in Figure 1 as a hierarchical maintenance and supply support system where defined locations have both repair and stockage capabilities.

The lower echelon in Figure 1 represents Strategic Missile Wings, which will also be referred to as base-level facilities. Each wing is normally composed of three squadrons, with each squadron containing 50 missiles. The operations of a base-level facility are detailed in Figure 2. Note that only one of the three squadrons is depicted. The other squadrons have identical operations.

The monitoring of the MGSs is performed at a squadron launch facility. When a failure is detected within a MGS, a maintenance crew is dispatched for on-site inspection of the missile. Ten percent of the failed MGSs are returned to operational status within one to two and a half hours. Ninety percent of the failed MGSs require replacement with an operational unit. Operational MGSs are obtained from a base supply and the defective unit is removed from the missile and sent to base maintenance. The total time necessary to requisition, transport, and transport defective units ranges from ten to eighteen hours.

At the base maintenance facility, defective MGSs are inspected. Minor repair can be performed at the base-level facility, with the MGS returned to supply within 1 to 1.5 days. However, 97 percent of the MGSs cannot be repaired at the base-level, and must be forwarded to depot-level maintenance facilities where more sophisticated equipment and highly specialized technicians are available. The time for storing and transporting inspected MGSs to the depot ranges from ten to twenty-five days. When a MGS must be sent to the depot for

1. The term "failure" includes degraded performance.

Figure 1. General Flow of Missile Guidance Sets Through the Multi-Echelon Maintenance and Supply Support System.
repair, a replacement unit is requisitioned from the depot-level supply facility. Approximately five to ten days are required to ship the replacement MGS to the base supply facility.

The upper echelon in Figure 1 represents the depot-level maintenance and supply facilities. The operations of the depot-level facilities are depicted in Figure 2. Approximately ten percent of the defective MGSs may be returned to operational status after major adjustments requiring three to seven days. Seventy percent of the MGSs are classified as "repeat failures", requiring ten to fourteen days repair time. Ten percent of the MGSs are new or infrequent problems, and require fourteen to twenty-one days repair time. The repair times for the latter categories assume that replacement parts are in stock. If one or more parts are not available, an extra repair time of fifteen to twenty days is required to order and receive the parts. Approximately ten percent of the MGSs requiring repair incur the extra repair time. Since the depot maintenance facility and the depot supply facility are geographically separated, an additional ten to fourteen days is required to ship the adjusted and repaired MGSs to supply.

Each missile squadron is required to maintain 49 missiles in an operational mode to maintain its operational status. Hence, the measure of effectiveness for this system is the operational status of the squadrons. The inventory problem is to determine the minimum stock levels for spare MGSs at each base supply facility and at the depot supply facility that maintains a status of 49 operational missiles.

The MRTRIC provides a methodology for computing optimal stock levels in a two-echelon inventory and repair system. The MRTRIC was originally developed by Sherbrooke and is discussed extensively in the literature. This model requires the following assumptions:

a. The demand for MGSs at base $j$ follows a compound poisson process with rate $\lambda_j$. The demand at each base is assumed to be independent and identically distributed according to a compound distribution with mean $f_j$.

b. The probability that a defective MGS can be repaired at the base facility is $r_j$. Hence, the probability that a defective MGS must be sent to the depot facility for repair is $(1-r_j)$.

c. $\lambda_j$ is the expected repair time for the

2See Clark, 1972; Clark and Scarf, 1960; Denny and Pressuti; and Nahmias.
A : Base Repair Time
Item                          Avg.  B(x)
Travel to site               .075  .075
On-Site Inspection           .075  .075
Requisition NGS              .095  .095
NGS Replacement              .420  .420
Base Repair Time             1.250 1.250
Estimated Base Repair Time   2.000

D : Depot Repair Time
Item       Prob   Avg.  B(x)
Major Adjustments            .10   5.0  .500
Repeat Failures              .70  12.0  7.500
New Failures                 .07  29.0  2.069
New Failures                 .18  17.0  2.000
New with Parts Delay         .02  35.0  .700
Depot Maintenance Time       13.975
Travel to Depot              17.500
Travel to Depot Supply       12.000
Estimated Depot Repair Time  43.475

Qj : Order and Ship Time at Base j 7,500

MTTR : Mean-Time-To-Repair at Squadron
Item       Prob  Avg.  B(x)
Travel to Site                1.00  .075  .0750
Inspect NGS                   1.00  .095  .0950
Requisition NGS              .90  .095  .0895
NGS Replacement               .90  .430  .3870
Estimated MTTR               .6225

B(So|Ao D) : Expected Backorders at Depot with Stock Level So
= \sum_{x} (x-So) p(x|Ao D)

S(So|D) : Expected Waiting Time per Demand
= \left[ B(So|Ao D) * S \right] / B(\theta|Ao D)

Tj(So) : Expected Lead Time at Base j
= rj Aj + (1-rj)(Oj + S(So|D))

Bj(So,Sj) : Expected Backorders at Base j
= \sum_{x} (x-Sj) p(x|Aj Tj(So))

Table 1. METRIC Equations for Calculating Expected Backorders.

METRIC Inventory System

b. The theoretical models do not allow priorities in the requisition and resupply process. In the conceptual system, priorities are allowed between the depot and base supply facilities. When backorders exist at the depot, repaired NGSs are shipped to the base with the lowest stock level.

These deficiencies in the theoretical models may affect the operational status that can be maintained. In the next section, a network simulation model of the conceptual system will be developed and used to assess the operational status attained using actual stock levels of twelve NGSs per supply facility. The effect of imposing realistic limits on the number of servers and of allowing resupply priorities will also be analyzed.

3. NETWORK SIMULATION MODEL

3.1 Description of the Model

The network simulation model is depicted in Figure 6. The failed NGSs are emulated as transactions that flow through a series of service activities. Resources are used to model operational missiles, squadron maintenance crews, base stock levels of NGSs, depot stock levels of NGSs, and depot service channels. Fifty missile resources are allocated to each squadron, and twelve NGS resources are allocated to each supply facility. Maintenance crew and depot service channel resources are controlled variables.

The missile resource availability at each squadron is used to measure the operational status maintained. If the number of non-operational missiles (NOM) increases above one at any squadron, then the desired 98 percent
Figure 6. SIM Network Model of Maintenance Supply and Support System

operational status has not been maintained. Two surrogate measures are used to assess operational status: (a) NOR time, which is the time weighted average of the number of days in which each of the squadrons had less than 49 operational missiles, and (b) NOR count, which is the total number of missiles included in the NOR time.

Other surrogate measures are used to provide insight into the flow of MGS through the system and to allow comparison of the simulation model to the theoretical models. These include: (a) server wait time, which is the waiting time at the depot for a service channel, (b) depot repair time, which is the average time to repair a failed MGS at the depot and ship it to the depot supply facility, (c) the average waiting time at the depot for requisitioned MGS to be shipped, (d) the average backorders at depot supply, and (e) the average backorder time for each base to obtain a replacement MGS from depot supply.

3.2 Experimental Design

A randomised block factorial design was used to test the effects of the controlled treatments on the surrogate measures of effectiveness. Three treatments were controlled in the parametric model.

Priorities Priorities were either used or not used in the resupply process between the depot and base supply facilities. It is expected that the use of priorities will reduce the NOR time and base lead times. Altering the resupply process should not affect the depot repair cycle, the number of backorders, or the depot waiting time. The base stock levels will be affected only when one base has a lower stock level than the other bases. Since the bases are expected to have similar stock levels, the use of priorities should not significantly impact base stock levels. The NOR count will be reduced only in those situations when the NOR status of the missiles is extended due to stockouts at base supply.

Depot Servers The number of MGS that may be concurrently serviced at the depot maintenance facility may be unrestricted or restricted to nine. Restricting the depot servers should increase the number of MGS queued for depot maintenance, thereby increasing the depot repair cycle. This, in turn, should produce increases in the base lead times, the number of backorders, and the depot waiting time while decreasing the base stock levels. The NOR count should not be affected and the NOR time would increase only if base stockouts increase.

Maintenance Crews The number of maintenance crews assigned to each squadron may be unrestricted or restricted to two crews on duty.
at any point in time. Restricting the number of maintenance crews should increase the mean-time-to-repair at the squadron level. This, in turn, should increase the NOR time, the depot cycle time, depot wait time and the number of backorders. Base lead times should also increase resulting in lower base stock levels. The NOR count would be affected only if the above changes result in increased stockouts at the base level.

3.3 Model Verification

Internal verification of the model was performed with the trace option, permitting a step by step analysis of the trace. The trace was continued until all branches within the model were taken at least once. External verification was performed by running 500 transactions through the system to check transaction passages through those nodes requiring probabilistic branching. The percentage of total transaction passing through the nodes approximated the probabilities associated with the node.

3.4 Model Validation

Validation of the simulation model was also accomplished. This process involved using constant values equal to the expected durations in lieu of actual distributions. Since the simulation model replicates the theoretical model, similar results should be obtained. A comparison of the theoretical and simulated results are presented in Table 2. All simulated results were within one standard deviation of the expected theoretical results except for the probability that the defective MGS is repaired on-site. This discrepancy is attributed to the relatively small number of failed MGSs which were generated at each squadron during the validation run.

3.5 Starting Conditions

Starting conditions are especially important since a continuous steady state system is being modeled. It is not reasonable to begin measuring the operational status until the simulation model achieves a steady state condition. Starting conditions were determined by making simulation runs of increasing time durations, with five replications per run, and assessing the stability of the average number of backorders. The number of backorders is sensitive to fluctuations in squadron, base and depot operations. Hence, stability in this measurement is a good indicator of achieving system steady state. The number of backorders from three simulation runs are plotted in Figure 7. These results confirm that relative stability is achieved after 300 time units. Hence, collection of statistical data will be delayed until the simulation model has run 300 days.

3.6 Sample Size

The duration and number of simulation runs was calculated using the sample size derivations presented in Hanks. An initial pilot run was made to obtain an estimate of the variance associated with the NOR time (0.1736) and the NOR count (38.375). Alpha risk was set at five percent and beta risk was set at thirty percent for all calculations. Using these specifications, at least 52 MGSs must be processed through each squadron to be 95 percent confident of detecting a shift of two missiles in the NOR count. With a squadron NMBB of fifteen days, it will require approximately 750 days to model the failure of 52 units at the squadron level. This equates to over two years of simulated activities after the system has attained steady state conditions. Ten replications of each simulation run must be performed to be 95 percent confident that the variance in the NOR time of 0.25 will be detected as statistically significant in the randomized block factorial model.

3.7 Variance Reduction Techniques

Common random number streams were used for each combination of treatment levels (blocks). A different seed was applied to each of the random number streams used across all replications within each block. Across blocks, the same seed for the random number streams was used. The application of common random number streams across blocks should reduce the total variance for each surrogate measure.

To further reduce variation in the simulation model, a separate random number stream was used to generate durations within each squadron. This produces the same sequence of MGS failures across the blocks for each of the ten replications. Comparisons across the blocks are enhanced by limiting the sources of variation to treatment effects.

4. ANALYSIS OF RESULTS

The results of the network simulation model were analyzed using multivariate analysis of variance (MANOVA). Bartlett's test for homogeneity of variance indicates that the variance due to experimental error within each treatment population is homogeneous. It is also assumed that the errors are normally distributed for each treatment population.

A three-way MANOVA model was applied to the surrogate measures of effectiveness. Each of the three base lead times and base stock levels were tested jointly to determine if the treatments had similar effects on all three of the independent measurements. All measurements were tested both separately and jointly to determine overall treatment effects. The results of the MANOVA are presented in Table 3. Table 4 standardizes the averaged measurements within each block, and provides the overall average and standard deviation for each surrogate measure of effectiveness. Table 4 provides the statistical significance of trends in the data resulting from each treatment effect, while Table 4 provides an indication of the direction of the trends. Each hypothesis test listed in Table 3 will be discussed below.

4.1 Effect of Depot Priority

Allowing the use of priorities between depot and base supply facilities should reduce the occurrence of stockouts at the base level. This treatment will only have an effect during the periodic high demand at a particular base. The possibility that all three bases experience high demand simultaneously is low. The results presented in Table 3 indicate that only NOR time and base lead time are significantly affected by
allowing the use of priorities. NOR time would be reduced since the likelihood of stockouts are reduced. Base lead time is reduced during periods of high demand, and the aggregate effect of this reduction is significant.

4.2 Effect of Restricting Depot Servers

Limiting the number of concurrent parallel servers at the depot maintenance facility increases the waiting time for servers at the depot. As a result, the depot repair time is increased, thereby increasing the depot repair time and depot backorders. This increases the lead time for satisfying base requisitions resulting in lower base stock levels. Lower base stock levels increase the occurrence of stockouts.

The NOR time and NOR count will only be affected during stockouts at the base supply facility. Restricting depot servers should increase the likelihood that stockouts will occur. Plots of base stock levels over a 200-day period are presented in Figure 8. This data verify that stockouts do occur when depot servers are restricted. However, the aggregate effect of base stockouts on NOR time and NOR count is not significant.

4.3 Effect of Limiting Maintenance Crews

Restricting the number of maintenance crews available at the squadron level should increase the amount of time required to service missiles when more than two MDSs malfunction simultaneously. Within the modeled system, this occurs only when base stockouts cause a delay in the MDS repair time. In these situations, the mean-time-to-repair (MTTR) should increase. Increasing MTTR delays the transfer of failed MDSs to the depot. Since it takes longer for failed MDSs to arrive at the depot, the queue for depot servers is decreased, thereby decreasing the depot server waiting time. As a result, the depot repair time is decreased.

The waiting time to satisfy base requisitions at depot supply also appears reduced. An increase in MTTR delays the submission of requisitions, which reduces the number of backorders and the delay in satisfying backorders. Since total system time from base satisfying base requisitions remains constant, increasing MTTR reduces the lead time in both the depot waiting time and base lead time. The reductions in base lead time were not statistically significant. Since restricting the number of maintenance crews did not significantly decrease the base stock levels, the NOR count should not increase (NOR count is only affected by stockouts at the base level).

4.4 Effect of Priority by Depot Server Interaction

Significant changes occur in the trends of NOR time from the combined effect of using priorities and limiting depot servers. When priorities are not used, restricting depot servers causes an increase in base stockouts and a subsequent increase in NOR time. When priorities are used, the likelihood of base stockouts are reduced thereby causing a decrease in NOR time. Reference to Figure 8 confirms that the use of priorities reduces the likelihood of stockouts.

4.5 Effect of Priority by Maintenance Crew Interaction

The combined effect of using priorities and restricting the number of maintenance crews causes significant changes in the trends for all surrogate measures, except waiting time for depot servers. Table 4 reveals that the maintenance crew treatment has no effect when the number of depot servers remains unrestricted. The significance of this treatment is attributed to

<table>
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<th>Surrogate Measures of Effectiveness</th>
<th>No: No Effect from Treatments</th>
<th>No: No Interaction Between Treatments</th>
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<tr>
<td>NOR Time</td>
<td>Reject</td>
<td>Reject</td>
</tr>
<tr>
<td>NOR Count</td>
<td>Reject</td>
<td>Reject</td>
</tr>
<tr>
<td>Server Wait Time</td>
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<td>Reject</td>
</tr>
<tr>
<td>Depot Repair Time</td>
<td>Reject</td>
<td>Reject</td>
</tr>
<tr>
<td>Depot Wait Time</td>
<td>Reject</td>
<td>Reject</td>
</tr>
<tr>
<td>Backorders</td>
<td>Reject</td>
<td>Reject</td>
</tr>
<tr>
<td>Base Lead Time</td>
<td>Reject</td>
<td>Reject</td>
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<td>Base Stock Level</td>
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<tr>
<td>Joint Effect</td>
<td>Reject</td>
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Table 3. Statistical Significance of Treatments on Surrogate Measures of Effectiveness

<table>
<thead>
<tr>
<th>Priorities</th>
<th>Not Used</th>
<th>Used</th>
<th>Overall</th>
<th>Std. Dev.</th>
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<td>2</td>
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<td>NOR Count</td>
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<tr>
<td>Server Waiting Time</td>
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<td>.899</td>
<td>.879</td>
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<tr>
<td>Depot Repair Time</td>
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<tr>
<td>Depot Wait Time</td>
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<td>.281</td>
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<tr>
<td>Backorders</td>
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<td>.232</td>
<td>.304</td>
<td>.304</td>
</tr>
<tr>
<td>Base Lead Time</td>
<td>.548</td>
<td>.548</td>
<td>.361</td>
<td>.361</td>
</tr>
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</table>

Table 4. Standardized Measurements Within Each Treatment Block
the changes in trends when depot servers are restricted. When priorities are not used, restricting maintenance crews increases the MRTT. As a result, delays of MROs to the depot which were restricted to the depot decreases the queue for depot services. The waiting time for depot services, and the depot wait time for spares, are reduced. When priorities are used, the increase in events decreases the depot repair time. However, the priority system is given to filling requirements from bases with the lowest stock levels, higher priority requirements spend additional waiting time at the depot. Base lead time decreases since the lower priority requirements remain in backorders status and are not filled. Hence, the lead time statistic is downward biased. Base stock levels decrease as a result of increased backorders.

4.6 Effect of Depot Server by Maintenance Crew Interaction

When depot servers are not restricted, there is no change across the maintenance crew treatment. When depot servers are restricted, restricting maintenance crews produces different trends across the two priority levels: a significant three-way interaction occurs. Averaging the effect of changing priority levels across both measures except OR time and server wait time.

4.7 Three-Way Interaction Effect

The three-way interaction effect is discussed in section 4.6.

5 CONCLUSIONS AND RECOMMENDATIONS

This analysis indicates that within the modeled multi-echelon inventory system, squadrons drop into an MRO status approximately 13.43 (± 7.6) times per year. The average duration of this MRO status is 8.5 hours (± 7.4). Thus, the operational status is maintained, on average, for 78,726 of the 78,824 available hours per year. This suggests that the 96 percent limitation of operational status derived from the optimal inventory model is quite conservative. In fact, the goal of maintaining 98 percent of all squadron missions in an MRO status is achieved 98.86 percent of the time.

The major cause of the MRO status appears to be the MRTT at the squadron level. Note that the MRTT would be increased by over 2 hours, on average, if the maintenance crews could carry the spare MROs when they are dispatched to inspect a malfunctioning weapon. The model simulation model was modified to eliminate separate MRO travel. The travel time for the maintenance crew was increased to account for the additional time needed to pick up the MRO prior to departing for the missile site. The operational system was simulated, meaning that priorities at the depot were allowed and that both depot servers and maintenance crews were restricted in number. Ten replications of this revised model produced an average NRT time of 7.68 hours and an average MRO time of 12.22 hours per year. The total MRO time for a year was reduced from 114.16 hours to 35.35 hours per year by performing separate travel for the replacement MROs.

To confirm that increasing the MRO stock levels has little effect on MRO status, both the basic and revised (eliminating separate travel) were executed with fourteen MROs per supply system. This produced total MRO times of 107.82 and 58.07, respectively. Eliminating separate MRO travel and increasing MRO stock levels to 14 produces an average NRT of 11.59 and MRO time of 7.81. Hence, the system appears relatively insensitive to increases in the MRO stock levels.

The network simulation model could be extended by adding cost data, and comparing alternatives of stocking additional MROs, adding additional servers at the depot repair facility, or increasing the number of maintenance crews. These extensions were not performed due to the non-availability of cost data.

In summary, this paper has demonstrated the use of a network simulation model to evaluate a multi-echelon inventory system. Theoretical models do offer a good starting point for evaluating such systems. However, they do not provide a detailed analysis of the operations which occur within the system. The use of a simulation model can provide this form of analysis to the decision maker, offering insights into the synergistic effects of varying system parameters. Also, the simulation model may be extended to evaluate expected changes in system performance from the introduction of new policies.

REFERENCES


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