CLOSED LOOP METHODOLOGY APPLIED TO SIMULATION

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By using modern system theory to model physical systems, questions concerning the design, analysis, test, and evaluation of such systems can be answered. This tutorial will compare the traditional open loop methodology of simulation to the modern closed loop methodology. Advantages and disadvantages of each methodology will be discussed. Systems will be typed as being deterministic or stochastic. The use of partial differential equations, ordinary differential equations, and algebraic equations to describe the system will be discussed. In order to solve system questions, the need to solve the estimation, identification, and control (EIC) problems will be motivated. The use of the EIC solutions with respect to the traditional open loop methodology (OLM) and the modern control loop methods (CLM) will be examined.

1. INTRODUCTION

The analysis of any physical system must begin by characterizing the specific process to be modeled. This characterization must consider (1) the system type, (2) governing equations of the process, and (3) system questions to be answered. Table 1 illustrates these notions.

- **Types of Systems**
  - Deterministic, i.e., noise-free system
  - Stochastic, i.e., systems with noise

- **System Description**
  - Ordinary differential equations
  - Partial differential equations
  - Algebraic Equations

- **System Questions to be Answered**
  - Solution to governing equations
  - Estimation
  - Identification
  - Control

<table>
<thead>
<tr>
<th>Table 1: Process Characterization</th>
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<td>Systems may be typed as being either deterministic or stochastic. A deterministic system is defined as one that incorporates no uncertainty and the stochastic type as one that includes uncertainty in the model. True systems may be represented by physical laws expressed by partial differential equations (PDE), ordinary differential equations (ODE), and algebraic equations (AE). Hence, the governing equations of the process must be given by PDE, ODE, AE, or a combination of the three. In the case of the deterministic type system, the solutions may be easily obtained. The stochastic system presents additional concerns. The solution of a stochastic systems may be obtained by defining and solving the estimation, identification, and control (EIC) problems associated with the given process.</td>
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Open-loop methods (OLM) and closed-loop methods (CLM) are considered. OLMs are described generally by a system that lacks feedback from the operations that are occurring. An OLM is presented in Figure 1.
The open-loop methodology in Figure 1 uses an a priori data base to provide predictions of outputs which are compared with measured data from the test cell. Results range from complete agreement to no agreement at all. In addition, the confidence level of results of a given test is typically low until a large statistical data base indicates the attributes of the samples. Typically, as one begins to accumulate test data, the predictions are adjusted to accommodate these data. This "knob tweaking" is usually conducted by methods that are far from being mathematically rigorous.

Because OLMs have considerable shortcomings, the need for a more systematic approach has become evident in recent years. Modern system theory provides such an approach based on CLMs that form the bases for most applications where feedback is used to control the system. The CLM is shown in Figure 2 by using adaptive procedures that provide a model with quantified confidence levels. Modern system theory recognizes the potential differences in predictions and results that can be attributed to forcing function uncertainty, incorrect estimates of constituents in the governing equations, and the possibility that the model order is insufficient to describe these systems. The adaptive processor design attempts to accommodate these uncertainties and to systematically drive the difference between predictions and test data to a minimum.

Acceptance of this thesis implies that the solution of the control problem is imperative to model the system.

In presenting the thesis, the term "classification set" has been introduced. Since the set has significant impact, it needs further clarification at this time.

The notion of a set is generally defined in mathematical terms. In particular, this set may be given as the set that contains the following elements: independent variables, dependent variables, coefficients of the governing equations, and the sensor output.

Since this definition for the classification set holds for the propagation of a signal through any system, the elements of this set will be obtained from the equations that govern the evolution of a specific phenomena. Prior to doing this, the relationships of estimation, identification, and control (etc) to the classification set will be given. For example, a solution to the estimation problem will yield estimates of the independent and dependent variables. Similarly, the solution to control encompasses the solution of both estimation and identification, in addition to providing an estimate of sensor output.

Having stated that OLM and CLM are essentially techniques for controlling the comparison error, the discussion will now turn toward the theoretical aspects required to obtain the systematic results of the CLM. This is introduced by presentation of the thesis:

"Control of the error which results from the comparison of data obtained from the true system to that produced by an a priori data base will result in:

a) obtaining a classification set, which implies the system can be modeled. This in turn implies that the system equations can be solved, or

b) denial of the classification set, which implies that the system cannot be modeled."

Before further clarification of the classification set is feasible, the basic elements of the physical process must be considered. To do this for the OLM, a lower layer of Figure 1 must be produced. This second layer is shown in Figure 3. From this figure, two basic elements may be defined as being (1) the subsystem under evaluation, and (2) a measurement device. The subsystem under investigation produces a measurable set, M, and the measurement device acting on this set produces the output, z. In addition to illustrating the basic elements, Figure 3 shows the basis for their physical makeup.
At this point, the subsystem under evaluation can be represented in state variable format by:

\[(1) \quad \dot{x}(k+1) = A(k+1, k) x(k) + w(k)\]

where \(x\) is defined to be a \(n \times 1\) state vector; \(A\) is the \(n \times n\) system dynamics matrix; and \(w\) accounts for random uncertainties in propagation of the state.*

In addition, the measurement device may be modeled by:

\[(2) \quad z(k+1) = m(k+1) + v(k+1)\]

where the sensor output, \(z\), is given to be a \(m \times 1\) vector sum of the measurement set, \(m\), and the random sensor uncertainty, \(v\). Since the sensor output can be considered to be a linear combination of the state, \(x\), the expression:

\[(3) \quad \dot{x} = \frac{\partial}{\partial t} \frac{\partial}{\partial t} T\]

where \(\hat{\mathbf{r}}\) is the total angular momentum vector and \(\dot{T}\) is the vector sum of the external torques. Since spacecraft orientation is concerned only with directional changes of \(\hat{\mathbf{r}}\) and not magnitude, equation (1) can be replaced by the two non-dimensional scalar equations

\[(4) \quad \dot{x}_1 = \frac{T_1}{H_0} \quad \dot{x}_2 = \frac{T_2}{H_0}\]

where the left hand side of the equations represent time translational changes of \(x_1\) and \(x_2\) respectively, \(T_1\) and \(T_2\) are respective components of \(T\) directed along \(x_1\) and \(x_2\), and \(H_0\) is the magnitude of the nominal momentum. Assume that the time derivative can be approximated by

\[\frac{dx}{dt} = \frac{x(k+1) - x(k)}{\Delta t}\]

where \(x(k+1)\) is the value of \(x\) at time \(k+1\), \(x(k)\) the value of \(x\) at time \(k\), and \(\Delta t\) the time interval between time \(k+1\) and \(k\). Then equation (4) can be written as

\[(5) \quad x(k+1) = \phi(k+1, k) x(k) + \theta(k+1, k) w(k)\]

where the transition matrix, \(\Phi\), is of dimension \(2 \times 2\); the matrix torque modifier, \(\Theta\), is of dimension \(2 \times 2\); and

\[x^T = (x_1, x_2); \quad u^T = (T_1, T_2)\]

Having discussed the equations of motion, consider the measurement source, i.e., the measurement is given as a nonlinear function of the state plus noise

\[z(k+1) = f(x(k+1)) + v(k+1)\]

Expansion of the function \(f\) in a Taylor series about a nominal, \(x^*\), yields

\[z(k+1) = z^*(k+1) + H(k+1)\delta x(k+1) + v(k+1)\]

or

\[(6) \quad r(k+1) = H(k+1)\delta x(k+1) + v(k+1)\]

where the residuals, \(r\), are defined as \(z\) minus \(z^*\), and \(H\) is the partial of the vector function \(f\) with respect to \(x\).

Since propagation of the perturbed state must obey (5), the system to be considered is described by the residuals, (6), and the perturbed state \(x\), i.e.,

\[(7) \quad \delta z(k+1) = \phi(k+1, k)\delta x(k) + \theta(k+1, k) u(k) + w(k)\]

The sequence, \(w(k)\), describes the random uncertainties associated with \(T_1\) and \(T_2\). These uncertainties are characterized by the covariance matrix, \(Q(k)\). Errors in the torques are represented by \(u(k)\).

2. APPLICATION

As shown in Table 2, the application of the CLM technique can be used to address a wide range of problems. For example, consider the application of the methodology to the attitude determination problem. This problem may be considered to belong to a set of problems associated with the broad category of guidance and control. On the other hand, application of CLM to the combat vehicle support plan illustrates its use to a problem in operational research. This section will detail the first problem.

- Attitude control systems
- Attitude determination
- Design of a ground control station
- Control of large space structures
- Robotics development
- Magnetic dipole discrimination
- Combat vehicle support plan
- Calibration of system/sensor
- Heat diffusion identification
- IR signal propagation
- Soil-structural dynamics
- Acoustic damping
- B-52 navigational system assessment

Table 2: CLM Applications

Consider the problem of attitude determination of a spinning satellite. The equations which govern the attitude of the spacecraft in the presence of external torque are given by the vector equation

\[\frac{\partial}{\partial t} \frac{\partial}{\partial t} T\]

*The notation \((k)\) implies that the equations are evaluated at time \(t_k\).*
At this point, one can apply the Kalman filter equations to the system described by (6) and (7), i.e., a sequential estimate of the perturbed state, \( \hat{x} \), can be obtained utilizing the residuals \( r \). Having obtained \( \hat{x} \), the absolute value of \( \hat{x} \) is given by

\[
\hat{x}(k+1) = \hat{x}(k+1) + \hat{S}_x(k+1)
\]

The state obtained in (8) can now be compared to the state obtained by integration of the nominal torques, i.e., equation (5). If there is poor agreement between \( \hat{x} \) and \( x \), one would expect to see trends in the residual data. Trends in the residuals are illustrated in Figures 4 and 5. Since the residuals should have noise characteristics, these trends are undesirable.

\[
\begin{align*}
\theta(k+1) &= \theta(k+1) + \Delta \theta_k \Delta x(k+1) + \hat{S}_\theta(k+1) \\hat{u}_k + w(k+1) \\
\end{align*}
\]

where \( \hat{u}_k \) is the nominal value of the attitude and \( \Delta \theta_k \) are nominal values of the attitude and \( \Delta x \) are the residuals respectively. However, utilization of (9) requires \( \hat{u}_k \) to be known.

The perturbed value of the torques, \( \hat{u}_k \), can be obtained from minimization of an error function, \( E \). This cost function is given by

\[
E = (\hat{x}(k+1) - x(k+1))^T P^{-1} (\hat{x}(k+1) - x(k+1)) + \hat{u}_k^T Q^{-1} \hat{u}_k
\]

where the matrix, \( P \), is the covariance matrix of \( \hat{x} \) obtained via the Kalman filter algorithms, \( Q \) is the covariance of the noise sequence, \( \hat{w} \).

Minimization of (10) results in

\[
\hat{S}_x(k) = (\hat{S}_x(k+1) - \hat{S}_x(k) \theta^T(k+1,k))^{-1} \theta(k+1,k) + Q^{-1} \hat{u}_k
\]

Having obtained a value for the bias torques, (11) can then be inserted into (9) to yield the new integrated state.

The recent discussion addressed torques of a general nature. Specifically, the main source of the perturbing torques which tend to alter the orientation of a satellite vehicle result from: 1) gravity gradient torques, 2) solar radiation pressure, and 3) interaction of the spacecraft’s magnetic moments with the magnetic field of the earth. Torques resulting from atmospheric pressure are negligible compared to the above mentioned forcing functions.

The torques, \( \tau_1 \) and \( \tau_2 \), discussed previously are given explicitly as

\[
\begin{align*}
\tau_1 &= T_{ST} + T_{MAG} + T_{GG} \\
\end{align*}
\]

where \( T_{ST} = T_1 + T_2 \)

\[
T_{MAG} = \text{sum of magnetic torques}
\]

\[
T_{GG} = \text{gravity gradient torque}
\]

The solar and magnetic torques can be written as

\[
\begin{align*}
T_{ST1} &= a \cdot T(1,1) + b \cdot T(2,1) \\
T_{ST2} &= a \cdot T(1,2) + b \cdot T(2,2) \\
T_{MAG1} &= c \cdot T(3,1) + d \cdot T(4,1) + e \cdot T(5,1) \\
T_{MAG2} &= c \cdot T(3,2) + d \cdot T(4,2) + e \cdot T(5,2)
\end{align*}
\]

where the variables, \( a \) and \( b \), are nominal parameters which modify the respective solar torques.

Magnetic torques are computed by

\[
T = MB
\]

where

\[
M = \begin{pmatrix}
\cos \theta & -\sin \theta & 0 & c \\
\sin \theta & \cos \theta & 0 & d \\
0 & 0 & 1 & e
\end{pmatrix}
\]

The angle, \( \theta \), is the azimuth which relates a platform frame to the body reference frame. This
platform reference frame is necessitated by the fact that the vehicle magnetic moments, \( c, d, \) and \( e \), are measured in a platform frame. The magnetic field of the earth is represented by the vector, \( \mathbf{B} \). With some manipulation, (15) and (16) can be obtained from (17).

The gravity gradient torques, \( T_{GG1} \) and \( T_{GG2} \), are obtained from

\[
(18) \quad T = \frac{\mu g^2}{r^3} \mathbf{I} \times (I \cdot \mathbf{r})
\]

where \( \mu = \) gravitational constant

\( r = \) magnitude of the position vector from the center of the earth to the spacecraft

\( \mathbf{r} = \) unit vector directed along the position vector

\( I = \) inertia dyadic of the vehicle

Having defined the perturbing torques, the problem can be manipulated into the form given by (5).

Examination of Figures 4 and 5 imply that the nominal torque profiles currently employed are incorrect. These torques can be corrected utilizing an estimate of the attitude, and minimizing the weighted difference between the estimate and its corresponding nominal. These new torques can then be inserted to yield a modified attitude. Figures 6 and 7 yield a comparison of the estimated spacecraft orientation with its nominal. Figures 8 and 9 illustrate the comparison of the corrected attitude with its estimated orientation.

Nominal values for the solar torque modifiers and the magnetic components are given in Table 3. Utilizing these nominals, the error in \( x_1 \) was found to be primarily due to an error in \( T(1,1) \)—see Figure 10. The discrepancy in \( x_2 \) appears to be primarily a function of the error in \( T(1,2) \)—see Figure 11.
### Table 3: Nominal Torque Modifiers

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>NOMINAL VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>1.0</td>
</tr>
<tr>
<td>b</td>
<td>1.0</td>
</tr>
<tr>
<td>c</td>
<td>200.0</td>
</tr>
<tr>
<td>d</td>
<td>-50.0</td>
</tr>
<tr>
<td>e</td>
<td>-200.0</td>
</tr>
</tbody>
</table>

In conclusion, it can be said that trends are indicative of modeling errors. In this case, trends are due to the modeling of the external torques. This paper illustrates the use of the Kalman filter algorithms to obtain an estimate of the attitude based on measurement information. Having an estimate of the state, the uncertainties in the torque models can be determined in order to minimize the difference between the estimate and the corresponding integrated state. These biases can then be added to the nominal attitude to produce a corrected spacecraft orientation.

### REFERENCES


