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APPLIED VARIANCE REDUCTION – SOME CONCEPTS AND EXAMPLES

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Given virtually infinite computational speed and resources – in terms of both people and machines – we would all carefully structure deterministic methods. The absence of such capabilities is what motivates Monte Carlo simulation; it should not, however, cause us to abandon systemic approaches. Unfortunately, in most applications of Monte Carlo simulation, the judicious selection of sampling techniques is ignored, often at great expense. In addressing this point, the "benchmark" solution to an apparently simple (but delusive) problem provides a method of gauging the merits of some basic sampling tactics. Parallel-synchronous techniques substantially reduce the sample size needed to compare alternatives on a relative basis but do little for absolute measures. On the other hand, perfecting the distribution of a short-term sequence of psuedorandom numbers can meaningfully improve the rate of convergence of estimates of the mean. These techniques can also contribute to the efficiency of large simulation models and find utility in (strict) Monte Carlo applications.

APPLIED VARIANCE REDUCTION –
SOME CONCEPTS AND EXAMPLES

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GENERAL DYNAMICS
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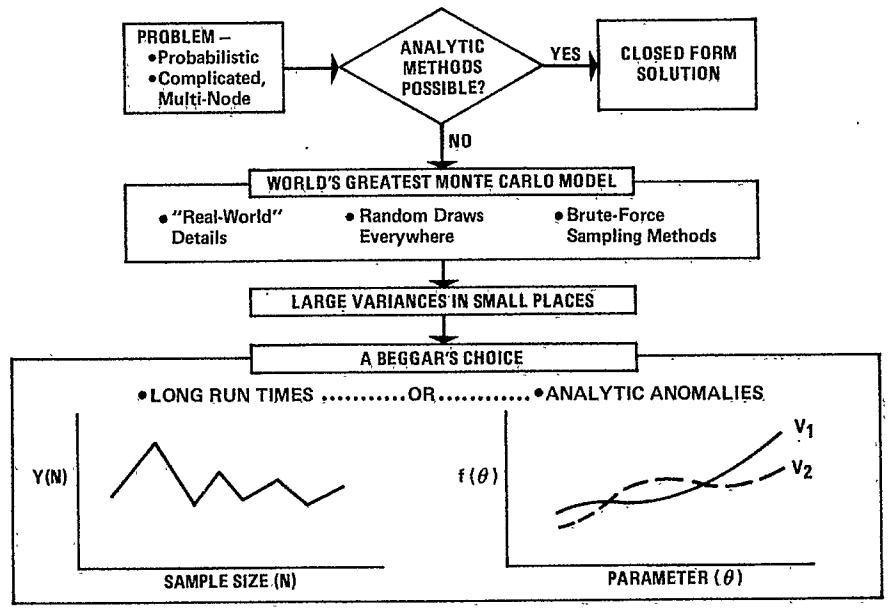
TOPICS ADDRESSED

- PERSPECTIVES AND MOTIVATION
- ILLUSTRATIVE PROBLEM AND BENCHMARK SOLUTION
- CONVERGENCE UNDER RAW SAMPLING TECHNIQUES
- CONVERGENCE UNDER STRUCTURED TECHNIQUES
  - Parallel-Synchronous (P-S) Sampling
  - P-S Sampling of "Perfect" Distributions
- REFLECTING INSIGHT: A NOTE ON MODELING TACTICS
- ANALOGOUS APPLICATION TO (Strict) MONTE CARLO
- RETROSPECTIVE

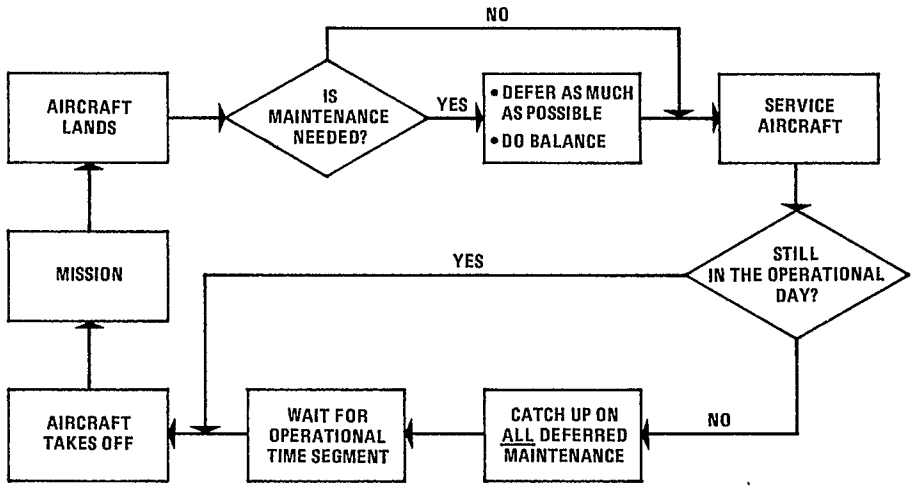
SOME MODELS GROW LIKE TOPSY



THE TASK IS TO ISOLATE RELEVANT VARIANCE

IRRELEVANT VARIANCE IS THAT WHICH . . .	RELEVANT VARIANCE IS THAT WHICH . . .
. . . IS PERIPHERAL TO YOUR STUDY.	. . . IS FOCAL TO YOUR STUDY.
. . . OBFUSCATES, OBSTRUCTS, AND REMAINS BECAUSE THE SYSTEM IS NOT UNDERSTOOD OR SUITABLY REPRESENTED.	. . . IS NEEDED TO GAIN INSIGHT AND CAN BE USED TO ADVANCE SYSTEM REPRESENTATION.
. . . REMAINS WHEN THE DISTRIBUTIONS OF THE SAMPLES DO NOT YET APPROXIMATE THE TRUE DISTRIBUTIONS.	. . . REMAINS WHEN THE DISTRIBUTIONS OF THE SAMPLES CLOSELY APPROXIMATE THE TRUE DISTRIBUTIONS.
. . . SHOULD BE CONSTRAINED.	. . . SHOULD BE GIVEN FREE REIN.

ILLUSTRATIVE CASE: THE AIRCRAFT MISSION CYCLE



PROBLEM DEFINITION

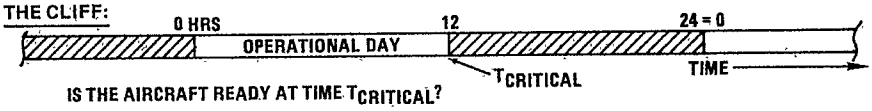
TO ESTIMATE:

1) EXPECTED MISSION RATE (Takeoffs Per Day)
2) SENSITIVITY TO MAINTENANCE DEFERRAL

THE INPUTS:

	BASELINE	RANGE
PROBABILITY, MAINT. NEEDED, PER MISSION	0.40	
MEAN MAINTENANCE TIME (Exponential Distribn.)	6.0 Hrs	–
FRACTION OF MAINTENANCE DEFERRABLE	0.0	0.0-0.6
SERVICE TIME (Fuel, Prep for Takeoff, Etc.)	1.0 Hrs	–
MISSION TIME (Takeoff to Landing)	1.5 Hrs	–
LENGTH OF OPERATIONAL DAY (Takeoff Allowed)	12. Hrs	–

THE CLIFF:

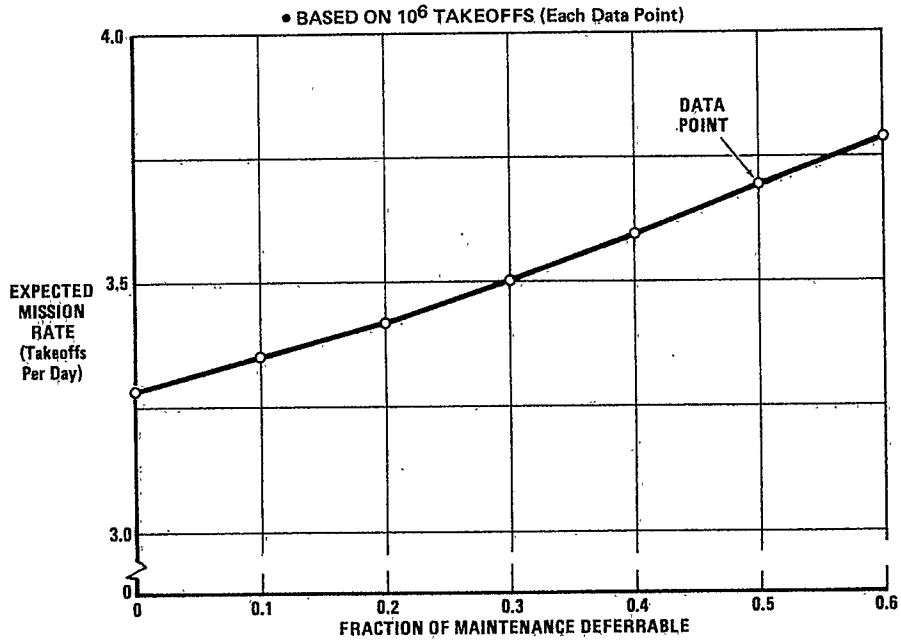


IS THE AIRCRAFT READY AT TIME $T_{CRITICAL}$?

- Yes? Take Credit for a Full Mission.
- No? Grounded. Only Maintenance, Service Allowed

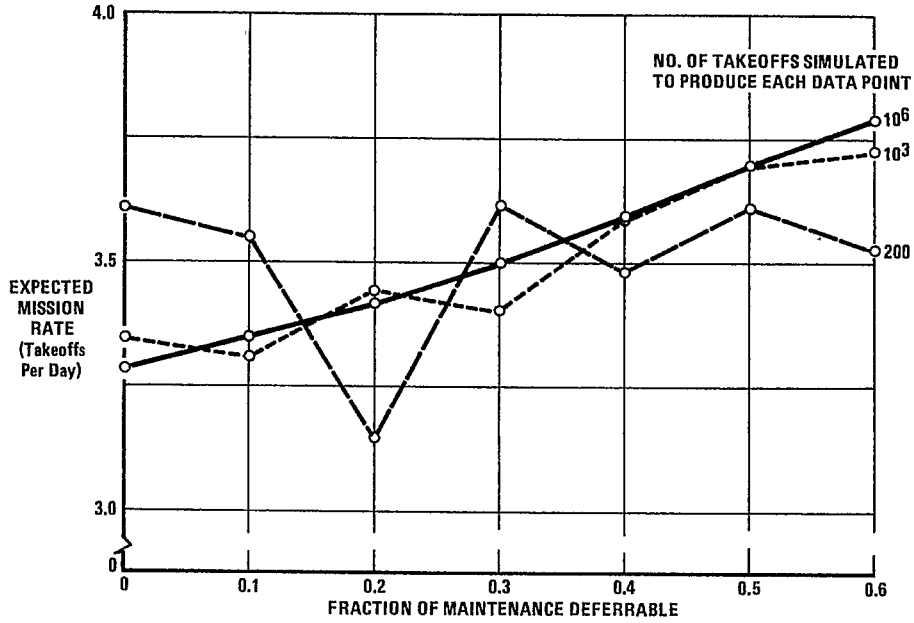
WE ASSUME THIS TO BE OUR "TRUTH" MODEL

- RAW SAMPLING METHODS
- BASED ON 10^6 TAKEOFFS (Each Data Point)

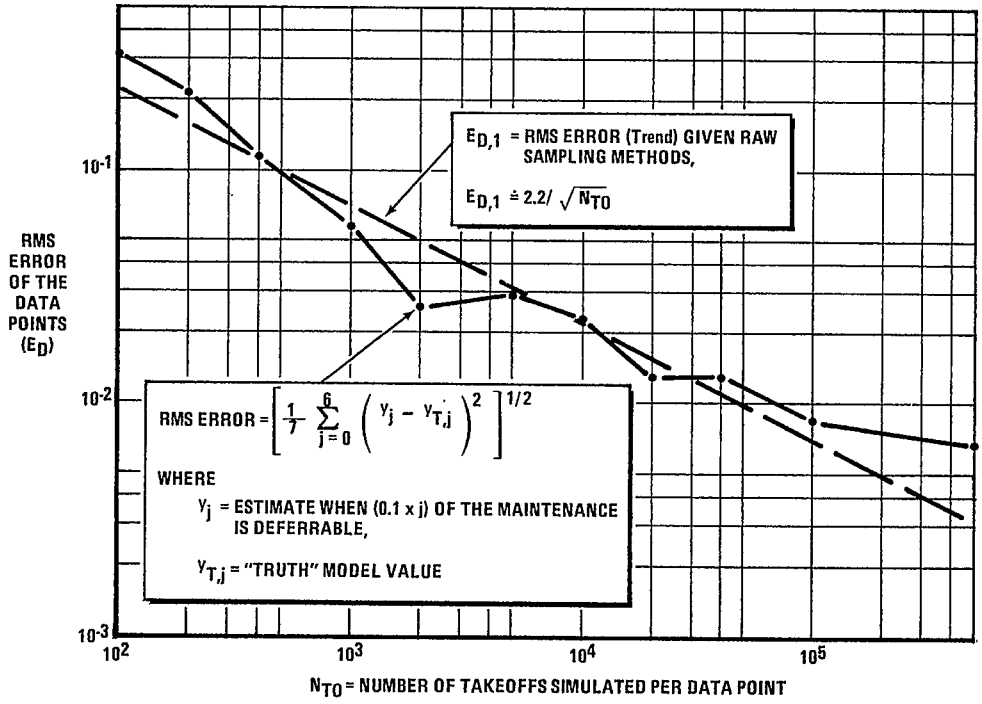


CONVERGENCE, RANK PRESERVATION, NOT EASILY ACHIEVED

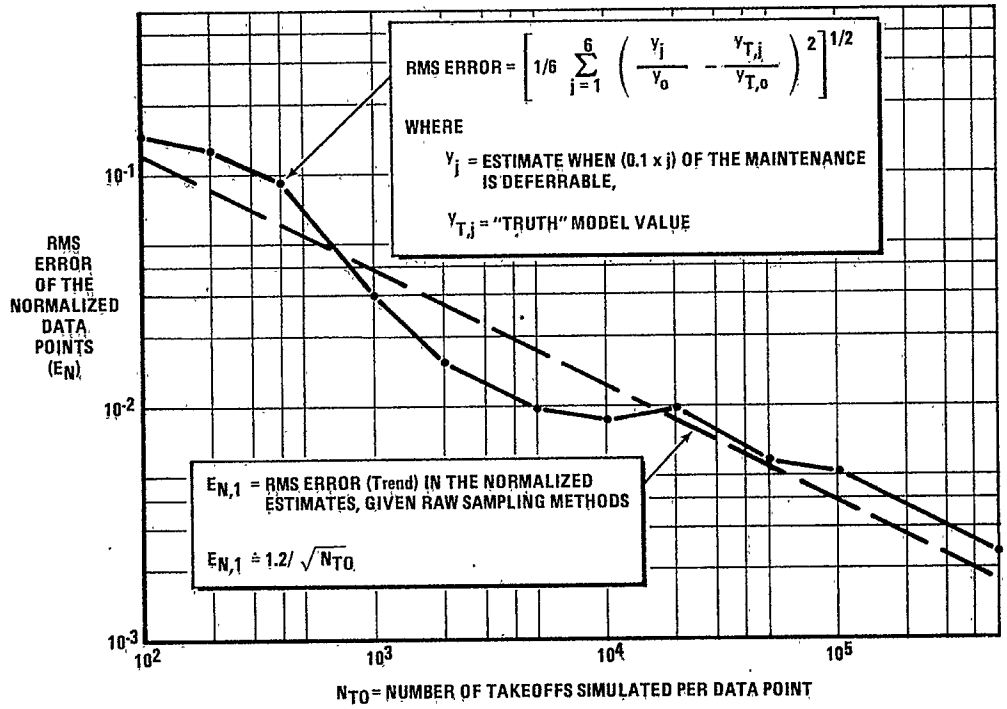
• RAW SAMPLING METHODS



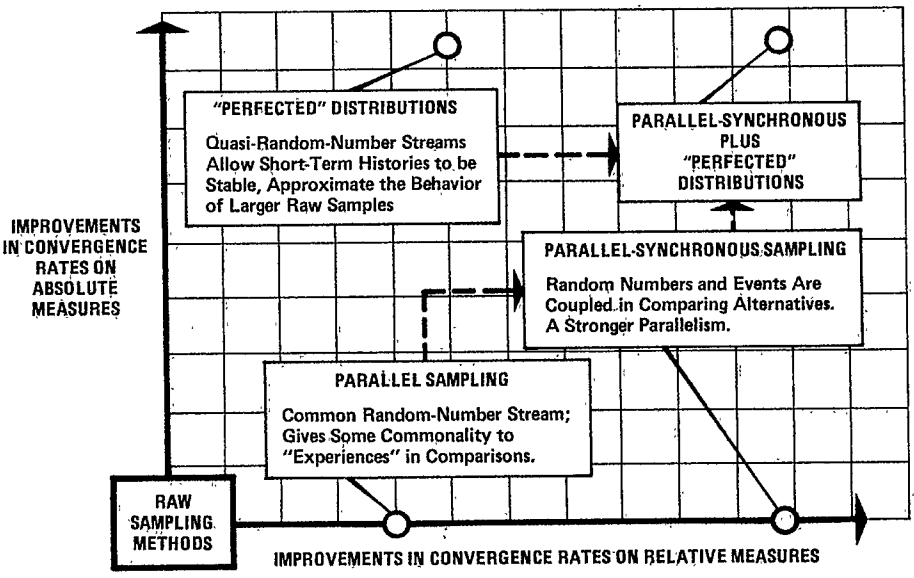
RAW SAMPLING METHODS PRODUCE CONVERGENCE AT STANDARD RATE



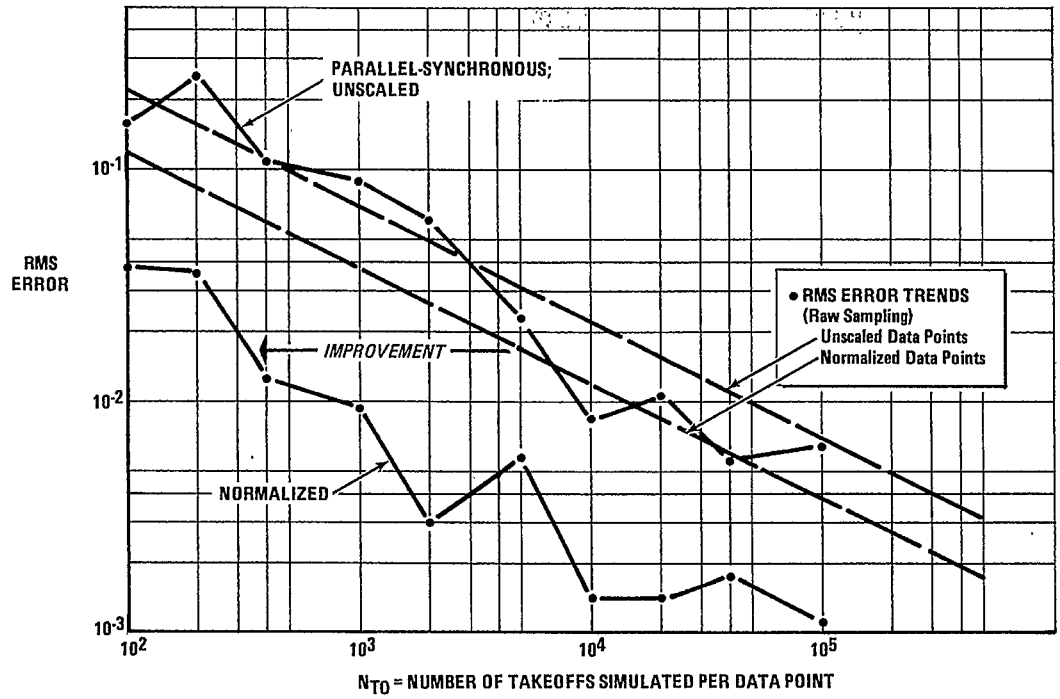
RAW SAMPLING: CONVERGENCE TREND ON RELATIVE MEASURES



VARIANCE REDUCTION TECHNIQUES – ANTICIPATED PAYOFFS



PARALLEL-SYNCHRONOUS SAMPLING: USEFUL ON RELATIVE MEASURES



PERFECTING THE DISTRIBUTION OF THE RANDOM NUMBERS

COMPUTE MIDPOINTS OF n EQUI-SIZED INCREMENTS SPANNING [0,1]

$$y_j = \frac{j - \frac{1}{2}}{n}, \quad j = 1,n$$

DERIVE A SET OF "PERFECTLY DISTRIBUTED" RANDOM NUMBERS BY INVERSE USE OF THE CUMULATIVE DISTRIBUTION FUNCTION, F.

$$x_j = F^{-1}(y_j), \quad j = 1,n$$

GENERATE A SET OF n RANDOM NUMBERS

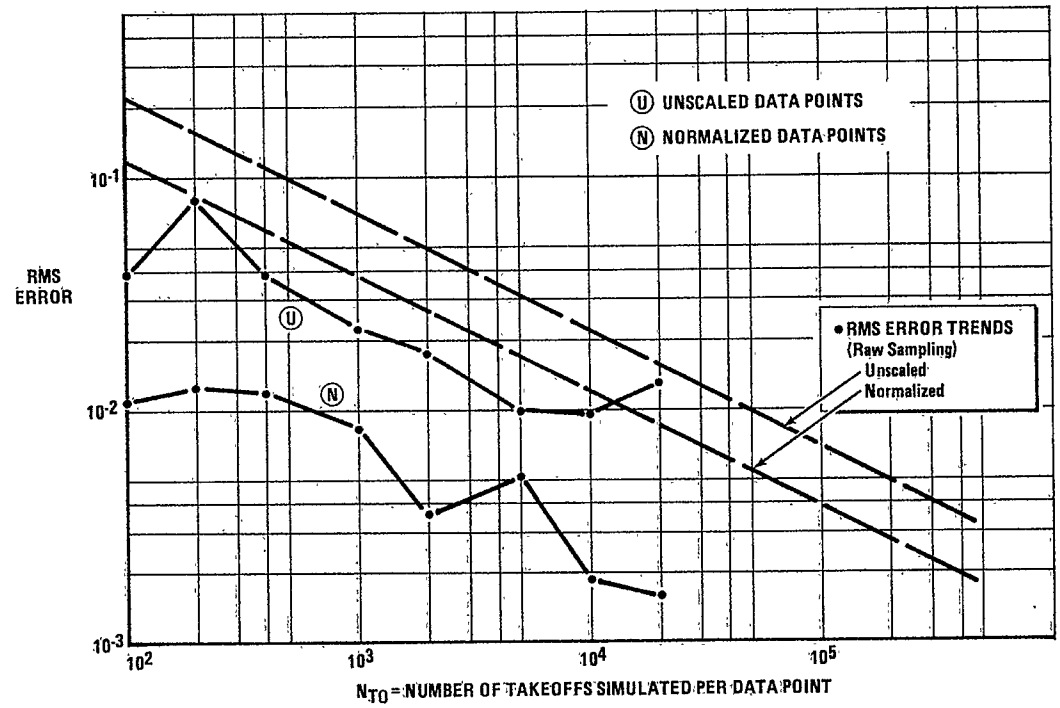
r_1	r_2	r_3	....	r_{n-1}	r_n

USE THE RANKED $\{r\}$ TO SELECT THE USAGE-ORDER OF $\{x\}$

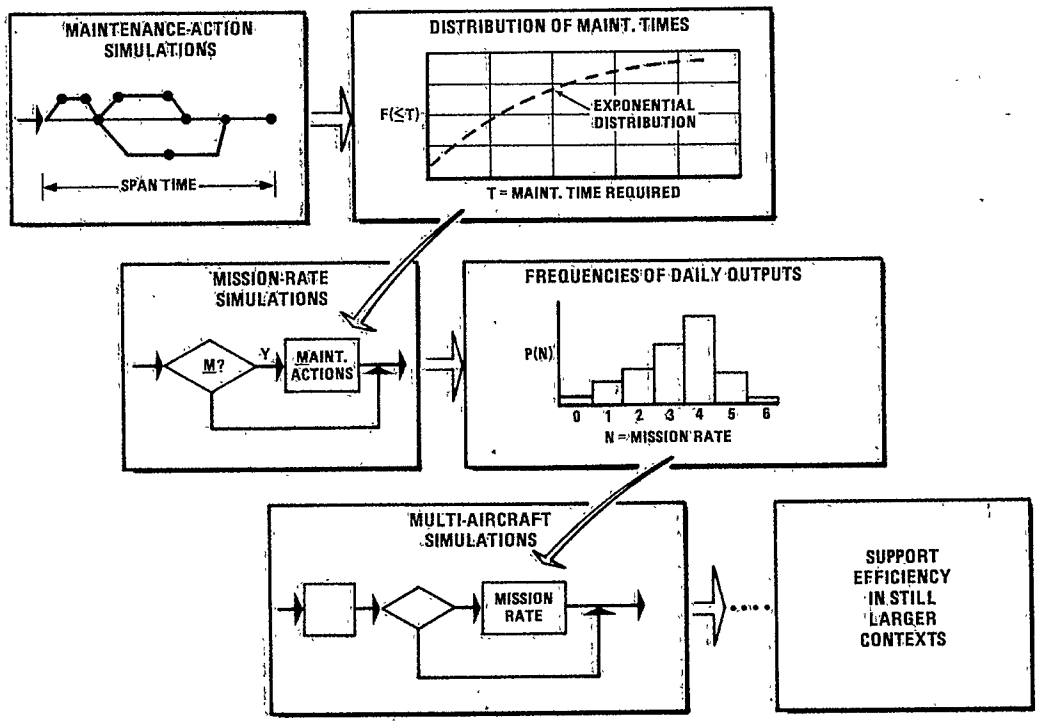
x_{72}	x_{14}	x_{95}	....	x_{17}	x_{54}

THAT IS, THE SMALLEST r WAS r_{72}; THE SECOND SMALLEST WAS r_{14}; ETC.

PARALLEL-SYNCHRONOUS SAMPLING OF "PERFECT" DISTRIBUTIONS: HELPS ON BOTH ABSOLUTE & RELATIVE MEASURES

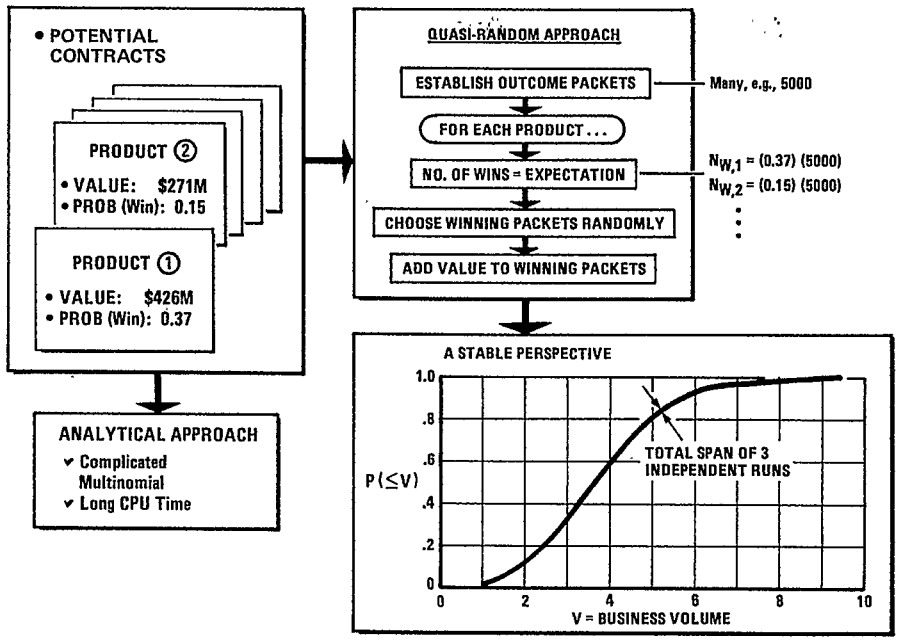


CAPITALIZE ON INSIGHTS: USE NESTED SIMPLIFICATIONS



USING QUASI-RANDOM METHODS TO FIND A DISTRIBUTION

• EXAMPLE: ANTICIPATED BUSINESS VOLUME



CHOOSING MONTE CARLO SIMULATION?

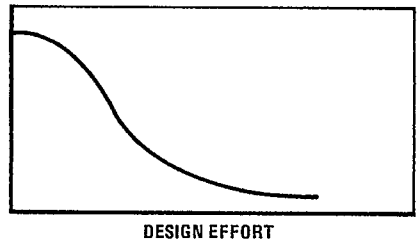
LOOK BEFORE YOU LEAP . . .

- SCOPE THE OBJECTIVES
- DESIGN OBJECTIVE-ORIENTED SAMPLING TECHNIQUES
  - ✔ Focus on Study-Relevant Variance
  - ✔ Constrain Variance Irrelevant to the Focus

. . . BECAUSE . . .

DESIGN EFFORT CAN HAVE A HIGH PAYOFF

- ✔ Lower Costs
- ✔ Consistent Trends
- ✔ Improved Insight



. . . AND . . .

UNLIKE LIFE, IN SIMULATION WE CAN CONTROL THE STRUCTURES OF THE SAMPLES

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Kahn H, Marshall AW (1953), Methods of reducing sample size in Monte Carlo computations, Journal of the Operations Research Society of America, Vol. 1, No. 5, pp. 263-278.

Kleijnen JPC (1974), Statistical Techniques in Simulation, Part I, Dekker, New York.