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SIMPLE MULTIVARIATE TIME SERIES FOR SIMULATIONS OF COMPLEX SYSTEMS

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ABSTRACT

Recent work has made the generation of univariate time series for inputs to stochastic systems quite simple. The time series are all random linear combinations of i.i.d. random variables with Exponential, Gamma and hyperexponential marginal distributions. The simplicity of structure of these time series models makes it practical to combine them to model multivariate situations. Thus one can model, for example, alternating sequences of response and think times at a terminal in which response and think times are not only autocorrelated, but also crosscorrelated.

Recent work by Gaver and Lewis (1980), Jacobs and Lewis (1977), Lawrance and Lewis (1977, 1980, 1981a, 1981b, 1981c) and others has made the generation of univariate time series for inputs to stochastic systems quite simple. These time series are all random linear combinations of i.i.d. random variables with Exponential, Gamma and hyperexponential marginal distributions. Thus the range of marginal distributions and correlation structures which can be modelled is very broad.

The original formulations of these models was as first-order autoregressive processes. They can, however, be extended to moving average and mixed autoregressive-moving average structures. The key to this extension is that for each marginal distribution $F(x)$, a way has been found to obtain a random linear combination of independent random variables with distribution $F(x)$ so as to give a new random variable with the distribution $F(x)$. This is, of course, well known for normal random variables since linear, constant coefficient combinations of normal random variables are normal; this result is the basis for the widely used ARMA(p,q) models. The results are new for exponential, Gamma and hyperexponential distributions.

We describe here only the Gamma process, since it is new. Thus let $G_n(k,\mu)$, $n=0,1,2,\ldots$ be an i.i.d. sequence of Gamma variates with shape parameter $k(>0)$ and mean μ. Let $B_n(q,k-q)$ and $C_n(k-q,q)$, $n=1,2,\ldots$ be mutually independent sequences of i.i.d. Beta variables $(q \leq k)$, independent of $\{G_n(k,\mu)\}$. Set $X_0 = G_0(k,\mu)$. Then

$$X_n = B_n(q,k-q)X_{n-1} + C_n(k-q,q)G_n(k,\mu) \qquad n=1,2,\ldots$$

is a one parameter, autoregressive Markovian process with Gamma(k,μ) marginal distributions for the X_n's. It is more regular than the original, defective Gamma process given by Gaver and Lewis (1980). It is a special case, after reformulation, of the Gamma GBAR(1) process introduced by Lawrance and Lewis (1981b) and Fishman (private communication). However the broader GBAR(1) process is not as useful since it cannot always be expressed as a random linear combination of independent Gamma variables.

The corresponding first-order moving average Gamma process is

$$X_n = B_n(q,k-q)G_n(k,\mu) + C_n(k-q,q)G_{n+1}(k,\mu) \qquad n=1,2,\ldots$$

and combined autoregressive-moving average processes are easily constructed.

The simplicity of structure of these univariate series models makes it practical to combine them to model multivariate situations. This has been described for computer system modelling by Lewis and Shedler (1977) and for some simple queues by Jacobs (1978, 1980). As another example one can model alternating sequences of response and think times at a terminal in which response and think times are not only autocorrelated but crosscorrelated.

A fairly broad scheme for generating these multivariate time series is given, and some of its properties are described.

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