Applications of Simulation in Project Management

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Abstract

Van Slyke introduced simulation as a method for the analysis of project networks in 1963. Since that time many new simulation procedures have been developed and/or adapted for use in project management. The purpose of this paper is to review the traditional simulation procedures that have been applied to project management and to present and discuss new developments regarding the application of simulation to project management. Specifically, this paper will include coverage of simulation as it relates to PERT networks, GERT project networks, and project control systems, i.e., the simulation of updated networks. The intent is to provide project managers with a basic understanding of the role of simulation in project network analysis and to improve both the efficiency and effectiveness of using simulation as a tool for decision analysis in project management.

INTRODUCTION

Project Management encompasses the planning, scheduling, and control of both human and non-human resources, which are acquired and/or assembled in a "temporary" organizational unit, for the purpose of achieving a specific objective. There are many facets to project management, and a number of "tools" have been developed and/or adapted to assist project managers in dealing with the complexities inherent in the performance of their duties.[1]

One of the better known and most commonly utilized project management tools involves the modelling of a project as a network. The result is referred to as a "project network model". The principal focus of this paper will be on the utilization of simulation in the analysis of project network models.

Regarding the use of simulation in the analysis of project network models there are two main areas of application. The first application area is critical path analysis. In conjunction with critical path analysis, simulation can be be very useful in estimating the value of certain time-related variables such as activity completion times, event realization times, and project duration, as well as criticality indices. The second area of application occurs when resource considerations become a factor. This application area is referred to as resource requirements/allocation analysis, which deals with resource loading, resource leveling, and/or time/cost tradeoff decisions.

Both PERT and CPM are network-based techniques, which originally incorporated analytical procedures for conducting critical path analysis. In 1963, Van Slyke [10] reported on the usefulness of utilizing simulation in the analysis of project network models and introduced the concept of the activity criticality index. In PERT, there is only one aspect of uncertainty - the activity duration. In 1966, Pritsker [7, 8] introduced GERT, which is a generalized technique involving network modelling and simulation analysis. Since its introduction, GERT has gone through several evolutionary changes,[2] and was refined for project management purposes by Hebert [3]. GERT incorporates a second aspect of uncertainty into project modelling - the structure of the network itself can be stochastic.

In the next section we will briefly review the fundamentals of traditional project network models and analysis. In subsequent sections we will direct our attention to the applications of simulation in support of critical path analysis and resource requirements analysis with regard to both the traditional PERT/CPM networks and the more contemporary GERT project models.

TRADITIONAL FUNDAMENTALS

Both PERT and CPM are well-known and well-documented project planning techniques.[3] The fundamental steps comprising these closely related procedures can be summarized as follows:

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1. Partition the referrent project into a mutually exclusive and exhaustive set of activities. (This is often referred to as the Work Breakdown Step.)

2. Determine and/or estimate the following attributes for each activity:

   a. precedence relationships

   b. duration

   c. resource requirements

3. Construct a network model of the project using the set of activities and the corresponding precedence relationships.

4. Analyze the resulting project network model. Specifically, perform:

   a. a critical path analysis, and

   b. a resource requirement/allocation analysis.

Traditionally, the analysis phase of the PERT/CPM procedure was performed analytically using a single-valued estimate for each activity duration. This type of critical path analysis resulted in dterministic time estimates and a crude (0,1) probability estimate of activity criticality.

We will now turn our attention to the improvements in project network analysis resulting from the incorporation of stochastic dimensions into the models and utilizing simulation as an integral part of the analysis procedure.

TRADITIONAL SIMULATION PROCEDURES

In 1963, Van Slyke [10] introduced the idea of utilizing simulation techniques in the project management environment. His method is summarized as follows: (a) utilize the Monte Carlo Technique to randomly generate a sample duration for each activity from its estimated duration distribution, which need not be BETA; (b) perform a critical path analysis on the network model using these sample durations, and record the results; (c) repeat this procedure until acceptable estimates for the time-related variables of interest are obtained.

Classical applications of simulation in the project management environment are typically devoted to generating estimates of activity times (early and late schedules), criticality indices, and/or resource requirement profiles. In general, the rationale for simulating the realization of a project network model is to obtain this information for planning and/or control purposes.

In the vast majority of these applications the referrent project is modeled as a PERT/CPM network. An interesting, but often overlooked, point regarding the simulation of PERT/CPM-type networks is that these simulations are not, in general, conducted in a dynamic mode [4]. More specifically, the progressive realization of these PERT/CPM project models is not actually simulated when "simulation" is used to generate estimates for the project variables of interest. It is not necessary to track and record the simulated activity completions and event realizations as they occur in an artifical time frame in order to generate this information. This point is illustrated in FIGURE 1.

In using simulation to obtain estimates of desired information when a project is modeled as a PERT/CPM network, it is only necessary to generate a sample duration for each activity, and then process this data in a sequence which will not violate any of the project-network precedence constraints. It is possible to omit the step involving the simulation of network realizations when using PERT/CPM models because of the deterministic structure of these networks.

Suppose the activities of a network are designated a(i,k), where a(i,K) is the unique activity emanating from event i (start node) and incident to event k (end node). The structure of PERT/CPM networks enable the events to be numbered so that for every activity a(i,k), $i<k$. Once the event numbers are established (generally a non-unique assignment), the activities can be ordered in an ascending sequence based on their start nodes. A critical path analysis can then be conducted by processing the randomly generated activity duration data according to this ordered list of activities. The event numbering scheme and the ordering of the activity list insure that no precedence constraints will be violated during the analysis.

Given an ordered list of activities, each with three attributes; (1) start node, (2) end node, and (3) duration, a procedure for performing an event-oriented critical path analysis is detailed in FIGURE 2. All of the information needed by the analyst can then be generated from the early and late event realization times and the activity durations. This type of simulation analysis is typically conducted exclusively during the planning stage. While useful for planning purposes, the resulting information quickly becomes obsolete once the project itself gets underway and proceeds toward completion.

FIGURE 1

PROJECT NETWORK SIMULATION PROCEDURES

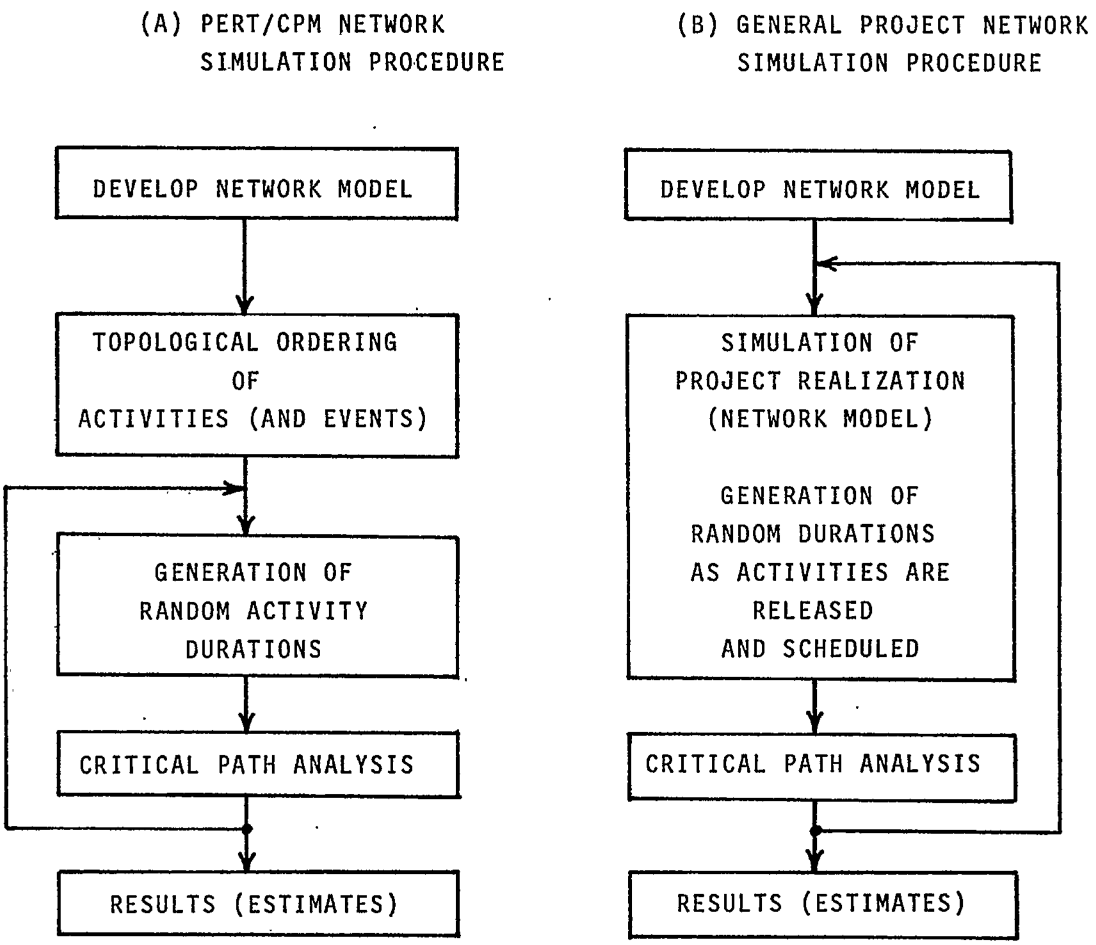

FIGURE 2

DETAILED EVENT ORIENTED CRITICAL PATH ANALYSIS FOR PERT/CPM NETWORKS

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    FORWARD PASS

    DO 10 N = 1,NE
 10 TER(N) = 0.0
    DO 20 J = 1,NA
    I = A(1,J)
    K = A(2,J)
    D = A(3,J)
    X = TER(I) + D
    IF(X.GT.TER(K))  TER(K)=X
 20 CONTINUE
    MPD = TER(NE)

    BACKWARD PASS

    DO 30 N = 1,NE
 30 TLR(N) = MPD
    DO 40 JJ = 1,NA
    J = NA - JJ + 1
    I = A(1,J)
    K = A(2,J)
    D = A(3,J)
    X = TLR(K) - D
    IF(X.LT.TLR(I))  TLR(I)=X
 40 CONTINUE
```

WHERE:

NE = the number of events
NA = the number of activities
TER(J) = the early realization time of event (J)
TLR(J) = the late realization time of event (J)
MPD = the minimum project duration

SIMULATION FOR PROJECT CONTROL

Revised estimates of pertinent project information can be obtained by re-simulating a network model once an update procedure has recorded the activity starts and completions occurring before a specified point in time. The first approach employed in resolving this problem was to simply change the estimated duration distribution of a completed activity to a constant, and then repeat the simulation. Currently, if project progress is updated by recording the start (AST) and finish (AFT) times of activities, (as attributes (4) and (5), respectively), then the critical path analysis in the UPDATE or CONTROL MODE is performed as detailed in FIGURE 3.

GERT PROJECT MODELS

There are several modeling alternatives, other than PERT/CPM, available to the project analyst. One of these alternatives is the Graphical Evaluation and Review Technique, which is more commonly referred to by its acronym GERT.[2]

The structure of GERT networks is stochastic, which generally prohibits the use of the short-cut method commonly employed for computing estimates of project variables in deterministic models. The progressive realization of stochastic (GERT-type) networks must actually be simulated by tracking activity completions and event realizations over time using a next event approach.[3] This procedure id detailed in FIGURE 4. Because the realization of the project has been simulated in a dynamic mode, it is not necessary to include the forward pass in the critical path computations. The early realization time (TER) of the events are simply recorded during the simulation, and only the backward pass (calculation of TLR's) is necessary.

The stochastic nature of PERT project networks requires that deterministic critical path concepts be extended in order to accommodate new dimensions of un-ertainty (specifically multiple activity realizations). Simulation analysis of stochastic project networks leads to the generation of new information useful to the project manager, specifically conditional criticality indices and related probabilistic estimates (indices) involving multiple activity realizations.

In PERT/CPM network analysis, the computation of criticality indices is simply a matter of noting the ratio of the number of times a given activity was critical to the total number of (simulated) network realizations. However, the flexibility of GERT permits situations such as probabilistic activity releases and cancellation of in-progress activities to be represented in the project model.

FIGURE 3

CRITICAL PATH ANALYSIS FOR CONTROL
(FORWARD PASS PROCEDURE)

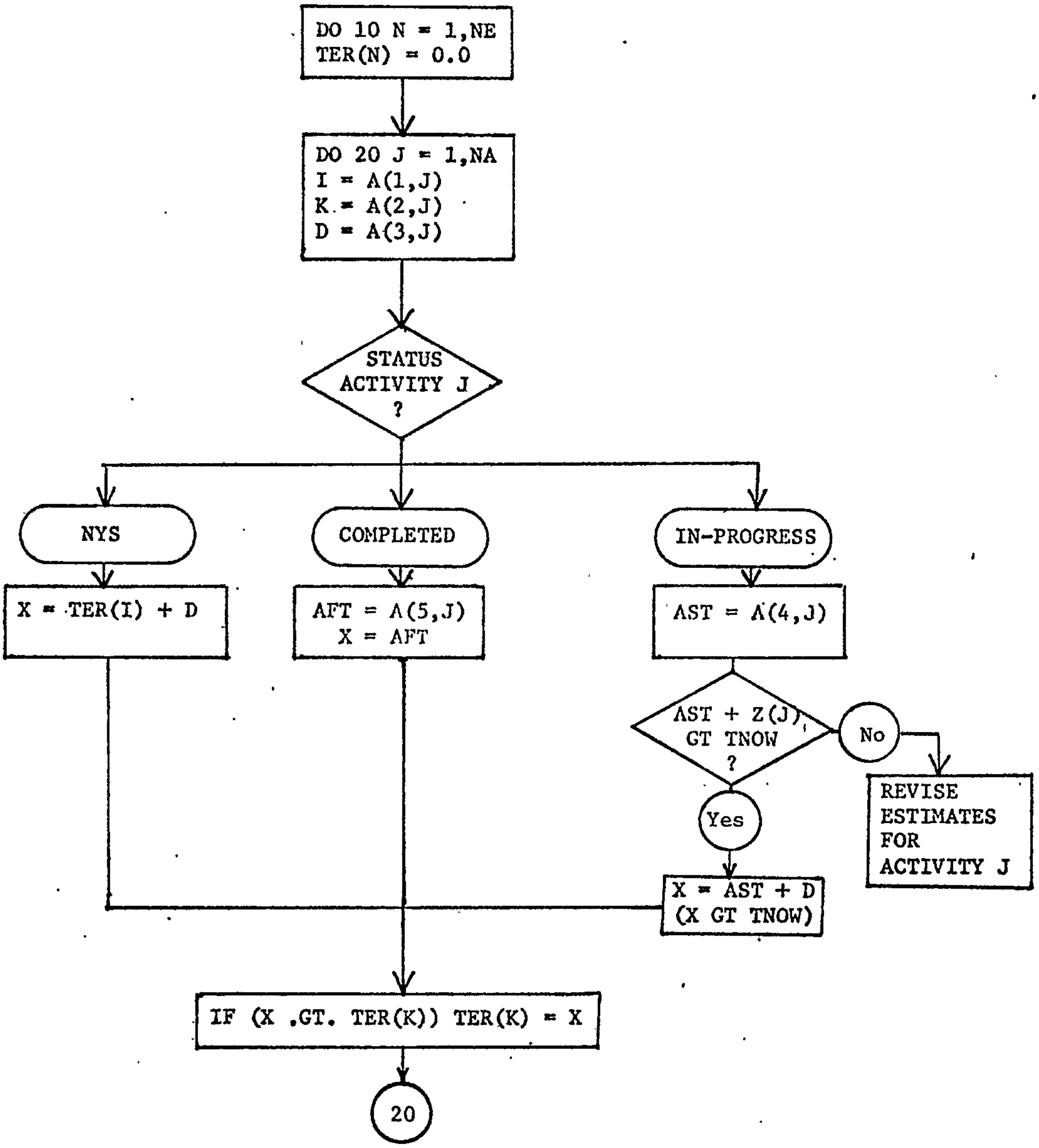
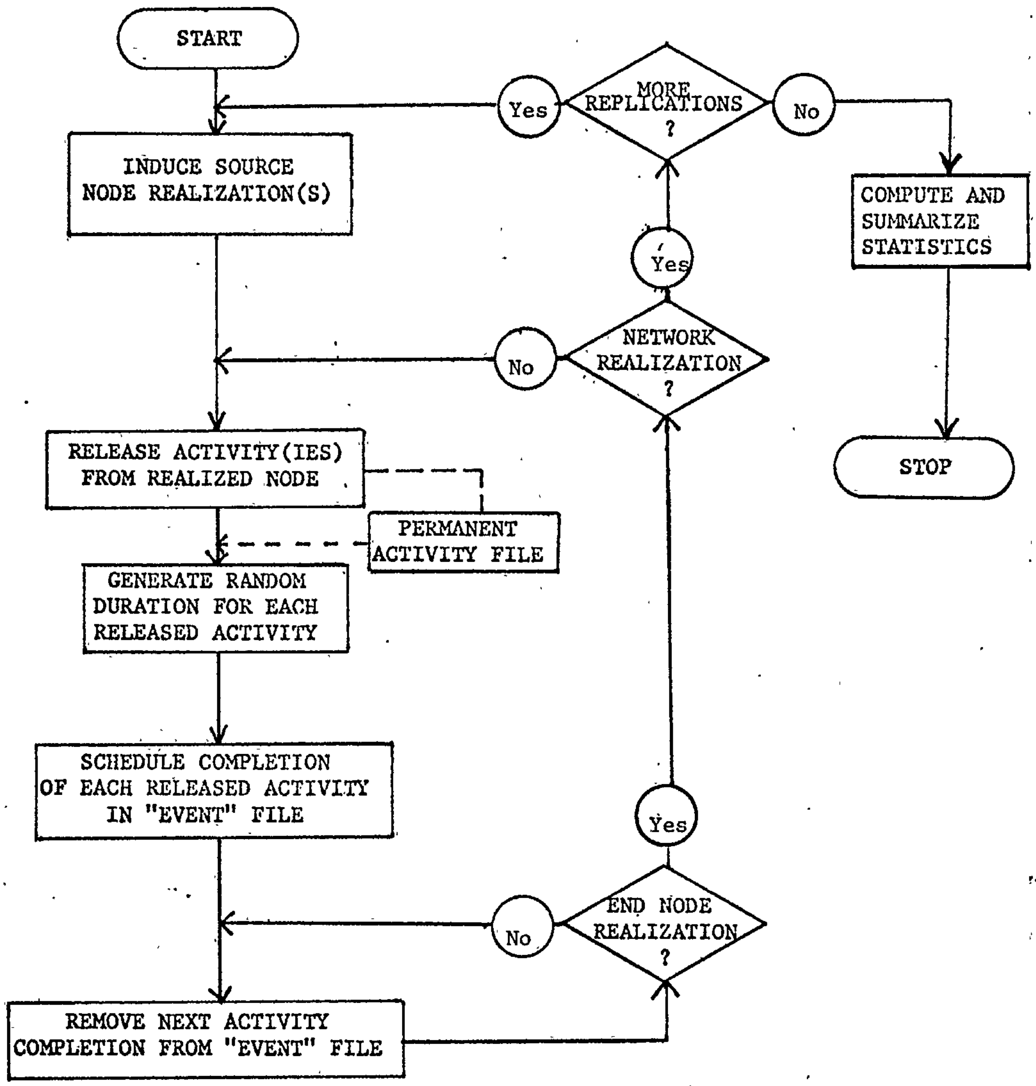


FIGURE 4

NEXT EVENT PROCEDURE FOR PROJECT SIMULATION



The conditions complicate the concept of criticality relative to GERT project models and necessitated the development of new concepts which included conditional criticality indices. These new criticality concepts are summarized in FIGURE 5.

FIGURE 5

CRITICALITY CONCEPTS

Let N = the total number of simulated network realizations.

a.) Assume a PERT/CPM network structure:

$$w_n(x) = \begin{cases} 1, & \text{if activity x is critical during simulation experiment n} \\ 0, & \text{otherwise} \end{cases}$$

$$W(x) = \sum_{n=1}^{N} w_n(x)$$

$C(x) = W(x)/N$ = Van Slyke's Criticality Index

b.) Assume a GERT network structure: (0,1) Realization Case w/o Abandonment

$$t_n(x) = \begin{cases} 1, & \text{if activity x is realized during simulation experiment n} \\ 0, & \text{otherwise} \end{cases}$$

$$T(x) = \sum_{n=1}^{N} t_n(x)$$

$P_r(x) = T(x)/N$ = REALIZATION INDEX - an estimate of the probability that activity x will be realized

c.) Assume a GERT network structure: (0,1) Realization Case w/Abandonment

$$s_n(x) = \begin{cases} 1, & \text{if activity x is started during simulation experiment n} \\ 0, & \text{otherwise} \end{cases}$$

$$s(n) = \sum_{n=1}^{N} s_n(x)$$

$P_s(x) = S(x)/N$ = START INDEX - an estimate of the probability that activity x will be started

$C_r(x) = W(x)/T(x)$ = CONDITIONAL (r) CRITICALITY INDEX

$C_s(x) = W(x)/S(x)$ = CONDITIONAL (s) CRITICALITY INDEX

$P_{r/s}(x) = T(x)/S(x) = P_r(x)/P_s(x)$ = an estimate of the conditional probability of realization given activity was started

d.) Assume a GERT network structure: Multiple Realization Case

In this situation, distinct realizations of an activity must be monitored during each simulation experiment. Following the reasoning of the previous sections, the unconditional criticality index would be defined and computed as follows:

$C(x,y) = W(x,y)/N$ = the unconditional probability that the y-th realization of activity x will be critical

The conditional indices follow easily as logical extensions of the (0,1) Realization Case.

RESOURCE REQUIREMENTS/ALLOCATION ANALYSIS

The procedures for resource requirements/allocation analysis are essentially the same for PERT/CPM and GERT. An activity is released for scheduling when its precedence constraints are satisfied. However, the scheduling of the activity (actually the activity start) is conditional on the availability or resources. Resource usage or consumption is monitored by accumulating the simultaneous requirements of a given resource in time slots. Representation of simultaneous resource requirements over time is referred to as a resource profile, which may be either numerical or graphical. In situations where the activity duration estimates and/or the network structure (realization sets) is stochastic, there will actually be a distribution of resource usage in each time slot.

In the case of resource loading, resource constraints are not considered. Activities are scheduled under the assumption that adequate resources are available, and as the network realization is simulated, resource requirements are accumulated. The resource profile in this case becomes a forecast of resource demand.

In the case of resource leveling, a specific amount of a resource is available. This will usually cause a conflict in the scheduling of activities and delay the completion of the project. Heuristic rules are used to determine activity scheduling priorities when insufficient resources are available. Multiple network realizations are simulated using each of the heuristic rules of interest. The results of these simulations are subjected to statistical analysis and inferences can be drawn regarding the anticipated performance of the heuristic scheduling rules under actual operating conditions.[6] Problems involving variables resource constraints would be analyzed using the same basic approach.

Deterministic time/cost tradeoff problems have been the focus of considerable research.[7] However, relatively little has been done in this area when the activity duration estimates and/or the network structure is stochastic. On an aggregate level there are two basic decisions to be made. How much to spend on a periodic basis, and what combination of resources to purchase given a specific spending decision. Simulation can be used in the analysis of this type of problem using an approach similar to that used in the resource leveling problem.[8]

The author is unfamiliar with simulation-based research conducted in a stochastic environment which addresses the time/cost tradeoff problem when each activity possesses a specific tradeoff function. From a practical standpoint, it seems unlikely that the effort required to gather the data and/or estimates needed to specify stochastic tradeoff functions for each activity in a large project could be justified on the basis of anticipated improvement in performance.

CONCLUSION

Our discussion has briefly reviewed some of the major applications of simulation in the area of project management - in particular, project network analysis. From a practical viewpoint, simulation may be invaluable in the analysis of stochastic project network models - because there are few, if any, alternatives currently available.

FOOTNOTES

1. See Cleland and King [1] for details.
2. See Pritsker [6], and the preface and bibliograpy in Pritsker [9, 1st edition] for details.
3. See bibliographies in Davis [2] and Weist and Levy [11].
4. See [5], [6], [7], and [8] for details.
5. See Chapter 5 in Hebert [3] for details.
6. See Chapter 6 in Hebert [3] for details.
7. See bibliography in Weist and Levy [11] for references.
8. See Chapter 6 in Hebert [3] for details.

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