THE USE OF SIMULATION IN THE DESIGN OF AN INVENTORY CONTROL SYSTEM

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ABSTRACT

Although the literature abounds with inventory models developed for almost every conceivable set of circumstances, when it comes time to develop an inventory system for a particular application, none of the literature models seem to fit exactly. This requires the analyst to do one or both of the following:

a) Modify an existing model to fit the given situation.

b) Assume that even though not all of the assumptions in the existing model are satisfied, it will do an adequate job of controlling the inventory.

Frequently, simulation is used to evaluate the consequences of following approach b (see, for example references 1, 3, 5, and 8).

The purpose of this paper is to demonstrate the use of simulation in the design of a forecasting and inventory control system. The use of alternate approximate modelling strategies will be simulated using an analytic simulation model of the real system on a relatively large sample of parts. The design questions under study will include the forecasting model, the frequency of updating the forecasts, the order quantity model, and the reorder point model. In each case, a very simple and a more complex model will be used. The major thrust is to point out the use of simulation as an aid in evaluating design alternatives.

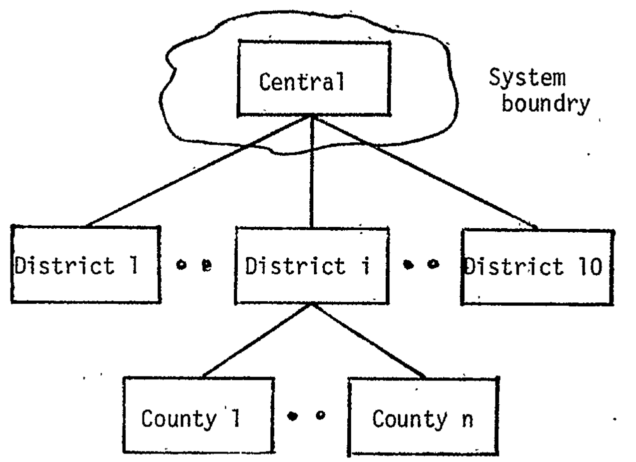
I. PROBLEM AND CONSTRAINTS

The specific problem is to be studied is a multi-level inventory system containing spare parts and equipment for a large fleet of maintenance vehicles as diagrammed in Illustration 1.

Each organization indicated in the diagram has a maintenance shop and carries an inventory of parts and equipment. Replenishments orders at lower level organizations are filled from higher level organizations. Only the central organization places replenishment orders outside.

ILLUSTRATION 1

Inventory System



Early in the study, it was determined that the major control effort would take place at the central facility and that the demands from districts and from the central shop would be considered exogeneous events. This decision was made for two reasons; a lack of information and a lack of skilled personnel at the lower levels. Thus, the system under study is a single location inventory system as outlined by the system boundary.

The inventory consists of approximately 10,000 identifiable parts, each to be reviewed on a continuous basis. The system is to be computerized and is composed of a forecasting model to estimate usage during the upcoming period and an inventory model to specify the order quantity and reorder point. The forecasting model and inventory control model will be the same for all parts and all parameters are to be updated on a periodic basis to adapt to changing conditions.

II. DESIGN ALTERNATIVES

The problem described in the previous section does not fit the assumptions of any standard inventory model. Demand for each item is stochastic in nature and occurs in "lumps" of various sizes from the ten districts and the central maintenance shop. In addition, the level of demand for some items varies over time as the age and composition of the fleet changes. A simulation model is used to evaluate how well some standard models will perform in controlling the inventory, even though the assumptions upon which the model is based are not valid.

For purposes of this paper, four types of design questions will be considered with two alternatives each. This is not intended to be an exhaustive list of alternatives but rather to represent the kinds of alternatives that may be evaluated. The first question concerns what type of forecasting model to use. Two models will be evaluated; a simple exponential smoothing model in which demand is assumed constant (6, p. 48) and a double exponential smoothing model which allows for a trend (6, p. 56). In both cases, certain seasonal items will use separate models for the summer and winter seasons.

The second design question concerns the frequency of revising the forecasts and likewise, the inventory control parameters. Again, two designs will be evaluated; one in which forecasts are updated quarterly and the other in which forecasts are updated annually.

The third design question concerns the quantity of items to order. One design approach is to use the standard EOQ formula (4, p. 29) derived for inventory systems with deterministic demand. The other is to use Hadley and Whitin's (4, p. 162) approximate order quantity formula (HW) based on stochastic demand. Demand is assumed to follow either a Normal distribution (for high demand items) or a Poisson distribution (for low demand items.[1] Actually, this formula will be used in the form shown by Nahmias (7) for service level type policies.

The final design question concerns the selection of a reorder point or equivalently a safety stock level. It will be assumed that a shortage cost is not available and thus, a service level must be specified. The service level policy (called a β policy by Nahmias) indicates the fraction of time a given demand will not be filled from stock. Obviously, the selection of β represents a continuum of design alternatives. It will be demonstrated how simulation may be used to set β as low as possible without increasing costs over those in the current system.

[1]Kolmogorov-Smirnov goodness of fit tests were performed on the monthly demand for all parts studied. The hypotheses that demand was Normally distributed or that demand was Poisson distributed were rejected (each at α=10%) for only five of the forty-four parts. Available data ranged from 15 to 36 months.

III. SIMULATION MODEL AND DATA

The objective of this paper is to demonstrate how simulation may be used to evaluate the design questions posed above on a relatively large sample of parts. To accomplish this, an analytic simulation model is used to describe the results of applying a given design to the system. The simulation is performed over a one year period on forty-four parts that are representative of the entire inventory system.

Prior to the beginning of the simulation, forecasts of average monthly demand are made for the upcoming period. Approximately three years of historical data is available for each part. The first two years of data are used to initialize the exponential smoothing models. Preliminary runs indicated that smoothing constants of .14 and .06 would produce the minimum squared error for the simple and double smoothing models, respectively. Forecasts for the first and subsequent periods will vary according to the forecasting model used and the updating period under consideration.

Based on these forecasts, reorder quantities and reorder points are established for each part in the inventory system. These will also vary according to the forecast received, the order quantity model used, and the reorder point model used. These design parameters are then used to control the inventory during a one year period divided into four quarters. Demand during a given quarter is assumed to occur at a constant rate with a mean estimated from actual usage during the quarter.

Given the simplifying assumptions described above, a Monte Carlo simulation will not be necessary. The traditional measures of effectiveness, including number of orders placed, average investment in inventory, and number of shortages may all be computed analytically as indicated below.

$$OC = A*\lambda/Q$$

$$CC = (Q/2+R-\mu)*I*C \quad \text{if } R\geq\mu$$
$$CC = (Q-\mu+R)^2*I*C/2Q \quad \text{if } R<\mu \text{ and } Q\geq(\mu-R)$$
$$CC = 0 \quad \text{if } R<\mu \text{ and } Q<(\mu-R)$$

$$NS = 0 \quad \text{if } R\geq\mu$$
$$NS = (\mu-R)*\lambda/Q \quad \text{if } R<\mu \text{ and } Q\geq(\mu-R)$$
$$NS = \lambda \quad \text{if } R<\mu \text{ and } Q<(\mu-R)$$

where
- OC = ordering cost per quarter
- CC = carrying cost per quarter
- NS = number of shortages per quarter
- A = cost per order ($3.50)
- I = carrying cost per dollar invested (10% per year)
- λ = actual demand during quarter
- μ = actual lead time demand during quarter
- C = unit cost of item
- Q = order quantity based on forecast demand
- R = reorder point based on forecast demand

These measures of effectiveness may then be aggrated for all four quarters and for all forty-four parts to evaluate a given design alternative.

IV. ADVANTAGES OF PROPOSED MODEL

It should be noted that the simulation model could have been designed to simulate the design alternatives using the actual demand data observed on a day by day basis rather than totalling the demand for the entire quarter and assuming that it is used at a constant rate. Obviously, this approach is much more realistic. However, the simulation approach described in the previous section offers three important advantages over the empirical approach described above.

First, using the proposed model there are no problems with initial conditions; steady state results are obtained directly from the analytic equations. Using the empirical approach, the initialization of inventory levels can significantly bias the results.

Second, periods of stockout in the actual system may cause demand, if measured by shipments as in this case, to follow unusual patterns. For example, there may appear to be no demand during the stockout and then an unusually high demand when the replenishment order finally comes in. The proposed approach tends to average out these unrealistic variations while the empirical approach would exaggerate their impact. One could simply not consider any items with prolonged stockout for evaluating alternative designs but eliminating these potentially hard to control items can also bias the results.

Third, it is easier to handle a large sample of items, since data analysis only involves obtaining the total quarterly usage for each item. Thus, even though realism may be lost when modelling each individual item, the realism of the entire system should be improved by the larger sample of parts.

V. RESULTS

The results of the simulation are tabulated in Table 1. The left hand column indicates six design alternatives, representing various combinations of forecasting models, updating procedures, and order quantity formulas all designed to meet five service levels (β=.004,.008,.012,.016, and .020). Three measures of effectiveness are recorded for each alternative; ordering cost, carrying cost, and stockout percentage. These measures were computed by summing over all forty-four items studied for all four quarters of the study year. Design 7 provides the simulated results obtained using the current reorder points and reorder quantities with no updates. This serves as a basis for comparing the design alternatives.

The ordering cost for the current system is well below that of any of the design alternatives studied. This highlights the fact that they currently tend to order in larger quantities than would be suggested by either the EOQ or HW. Likewise, as expected, the HW order quantities are larger than the EOQ order quantities. The ordering costs are roughly in a ratio of 1:2:3; thus, the HW order quantities are roughly 1/2 of the current order quantities and the EOQ order quantities are roughly 1/3 of the current order quantities.

The carrying cost of each design alternative varies with the specified service level; the greater the service provided, the greater the carrying cost. The service levels were chosen such that the carrying costs would come out above and below those of the current system. In general, the carrying cost does not vary much with the other design factors such as the forecasting model, the updating period, or the order quantity formula.

The stockout percentage of each design alternative varies directly with the specified service level but at a much higher level. Theoretically, one would expect the service level and the stockout percentage to be the same. However, due to the fact that the forecasting model is unable to predict exactly the actual demand that occurs, the stockout percentage is much greater than the service level specified. Also, as the forecasting model is improved and the updating procedure becomes more frequent, the forecasts are better and thus, the stockout percentage is reduced. Note also that the larger order quantities specified by HW provide greater protection against inaccurate forecasts and thus, give smaller stockout percentages.

Illustration 2 displays the results presented in Table 1 in graphical form. The Y axis is the stockout percentage and the X axis is the sum of the ordering cost and the carrying cost. Each curve represents one of the six alternative designs with the large dots representing the five service levels used. The special dot, labelled 7, represents the simulated results using the current inventory control parameters.

These curves are a useful way to summarize the results because they allow one to look at the differences in stockout percentage of alternate designs for a fixed level of ordering and carrying costs. The largest reduction in stockout percentage may be obtained by using HW instead of the standard EOQ. To see this reduction, compare curves 1 and 2, 3 and 4, and 5 and 6. Smaller reductions are possible by using quarterly updating (compare curves 1 and 3 and curves 2 and 4) and double smoothing (compare curves 3 and 5 and curves 4 and 6).

These curves may also be used to select a design service level such that system costs will not be larger than those currently used. For example, suppose design 6 is selected and it is desired to determined β such that system costs equal \$4, 547 (ordering cost plus carrying cost of current system). Drawing a vertical line up from the X axis at this point and observing where it crosses 6 indicates that the service level is about half way between .012 and .016. Thus, β=.014 would be an appropriate design parameter.

TABLE 1

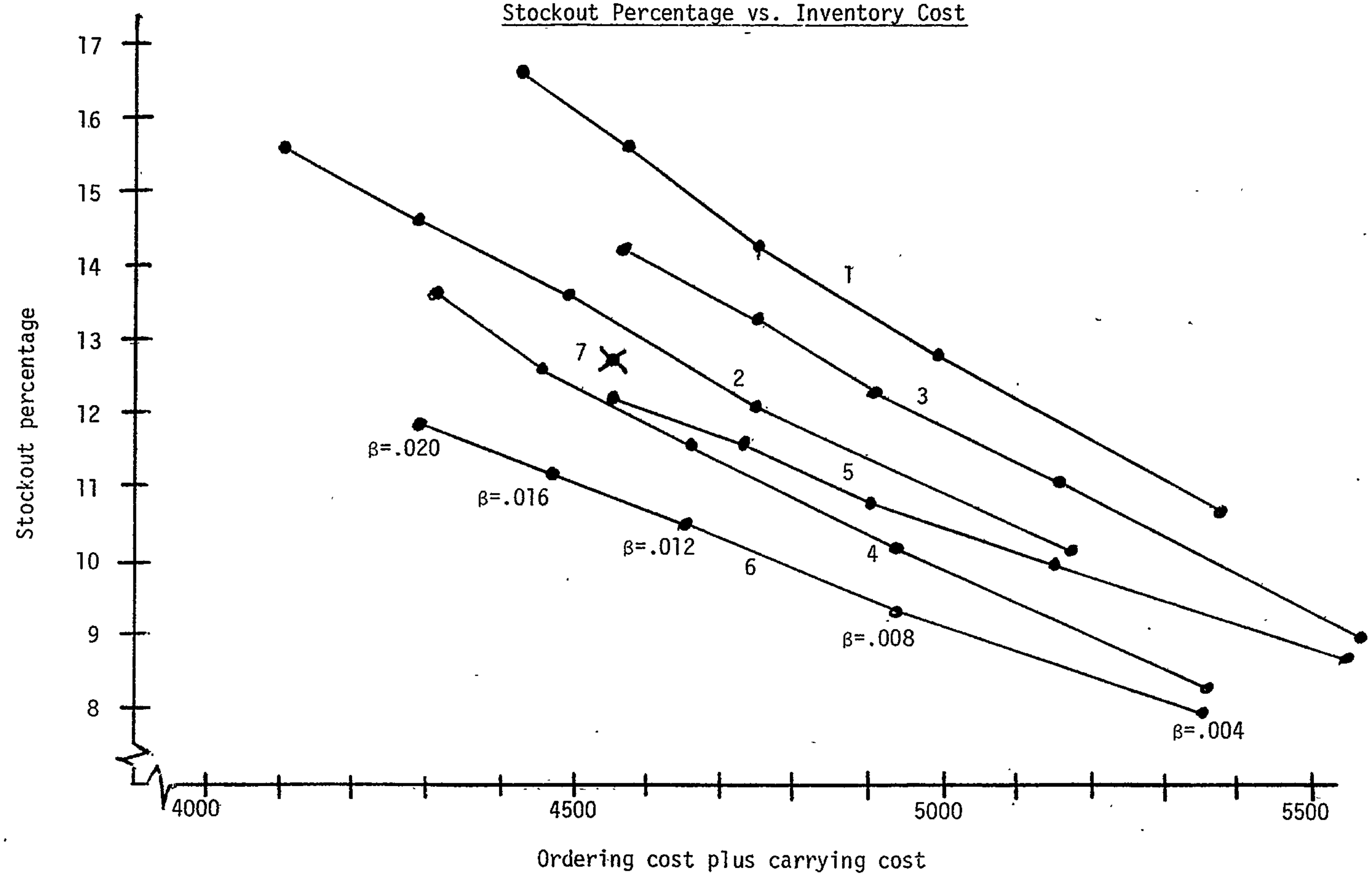
Results of Simulation

Design alternatives	Service level (β)	Ordering cost	Carrying cost	Stockout percentage*
1. Simple smoothing, annual update, EOQ	.004	896	4478	10.7
	.008	896	4088	12.8
	.012	896	3842	14.3
	.016	896	3673	15.6
	.020	896	3525	16.6
2. Simple smoothing, annual update, HW	.004	610	4555	10.2
	.008	596	4145	12.1
	.012	591	3899	13.6
	.016	582	3704	14.6
	.020	578	3524	15.6
3. Simple smoothing, quarterly update, EOQ	.004	865	4700	9.0
	.008	865	4290	11.1
	.012	865	4044	12.4
	.016	865	3883	13.4
	.020	365	3701	14.2
4. Simple smoothing, quarterly update, HW	.004	597	4762	8.4
	.008	585	4349	10.2
	.012	576	4085	11.6
	.016	568	3886	12.7
	.020	563	3744	13.6
5. Double smoothing, quarterly update, EOQ	.004	842	4700	8.8
	.008	842	4298	10.0
	.012	842	4056	10.8
	.016	842	3881	11.6
	.020	842	3706	12.2
6. Double smoothing, quarterly update, HW	.004	589	4761	8.0
	.008	577	4353	9.4
	.012	568	4082	10.5
	.016	560	3901	11.3
	.020	556	3727	11.9
7. Current reorder points and reorder quantities, no update		276	4271	12.7

*Stockout percentage = 100 * Total number of shortages/Total annual usage

ILLUSTRATION 2

Stockout Percentage vs. Inventory Cost



The most surprising result to this author was that the current system performs reasonably well and in fact better than designs 1, 2, and 3 for the given level of system costs. This is surprising since the current reorder points and reorder quantities are established in such a subjective way. However, a little reflection reveals the reason for this. It was learned earlier that the inventory manager tended to order in relatively large quantities. In fact, he orders inexpensive items in even larger quantities and expensive items in relatively smaller quantities. This type of logic is particularly well suited to the measures of effectiveness established for the system. That is, to minimize the number of stockouts at a given level of system costs, it is advantageous to keep larger supplies of the inexpensive items.

The logic described above suggests another design alternative which may be evaluated using the simulation model. In this approach, design 6 will be used but the service level will vary according to the cost of the item.

β	Cost
.006	$C < \$5$
.012	$\$5 < C \leq \$50$
.024	$C > \$50$

This case was simulated and produced an ordering cost of $569, a carrying cost of $3,886, and a stockout percentage of 9.1%.

VI. SUMMARY

Many authors have suggested using simulation to evaluate approximate inventory modelling strategies. This paper provides another example of this approach and includes several new ideas in the design of the simulation model.

1. The forecasting model was evaluated along with the inventory control model and found to have a large effect on the results.

2. A relatively large sample of parts was simulated to evaluate the alternate designs and aggregate measures were used to compare alternatives.

3. Illustration 2 was found to be a useful way of representing the results for service level type policies.

VII. REFERENCES

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