ECONOMIC CIRCUIT ANALYSIS

Walter L. Whipple
The University of Michigan, Ann Arbor, Michigan

INTRODUCTION

This analysis is an outgrowth of a previous study aimed at optimizing the profit of a commercial computer center (7). That study was restricted to a single kind of commodity. While it is consistent with the results of this paper, its limitations required the preparation of a theory based on utility and production functions.

In order to give an economic framework for the theory, the order of a microeconomic theory text was followed (2). The treatment was shortened by ignoring storage elements and their contribution to market equilibrium and transient behavior. The paper considers only simple multi-product consumers and firms under the assumption that once a consistent framework for static elements is provided, the extensions to dynamic elements will follow.

A consumer of multiple products is first considered in the network formulation. In order to combine the flows, it is necessary to transform each commodity to utility via the marginal utility of the commodity. Simple equivalent networks are used for converting circuit topologies into forms suitable for discussion. Substitution and complementation are briefly discussed. Then a firm is modeled in a similar fashion using the implicit form of the production function. Multiple products are briefly considered and then the problems of optimization are treated in some detail. The equivalence between matching impedances and equating marginal cost to price is stated. It is then observed that economic circuit analysis for static or cyclic problems is equivalent to the electric utility load flow problem. Fortunately, practical solution methods exist for large scale problems of this type.

The fact that consumption and production are usually irreversible in the short run, especially for consumers, is ignored in this paper. The inclusion of unidirectional flow elements analogous to electrical diodes is straight forward albeit messy to analyze. If computer simulation is to be used, the methods must permit such constraints along with the other difficult non-linearities which will be found in economic problems.

THEORY OF THE CONSUMER

A consumer's satisfaction from the purchase of a variety of goods and services may be termed the utility he receives from their consumption. For this study it is assumed that utility is a cardinal quantity which may be computed from knowledge of a consumer's purchases. Further, utility will be assumed to be an additive function.

A consumer is assumed to optimize his utility by purchasing all components of utility at the same price per unit. In other words, the price of a good divided by its marginal utility does not vary across the collection of goods and services purchased.

In utility price and utility per unit time we have the basic potential and flow variables of a network problem for a consumer. The product of these variables is a flow of money per unit time and the time integral of their product is an amount of money. These two derived measures are analogous to power and energy in physical systems. The ratio of differential utility and price is an elasticity akin to admittance whose reciprocal is impedance. Table 1 summarizes these qualities.

A graphical representation of a consumer with income I who purchases n commodities is shown in Figure 1. If the n^{th} commodity is of special interest, Figure 1 may be redrawn with the first n - 1 impedances lumped together as shown in Figure 2.

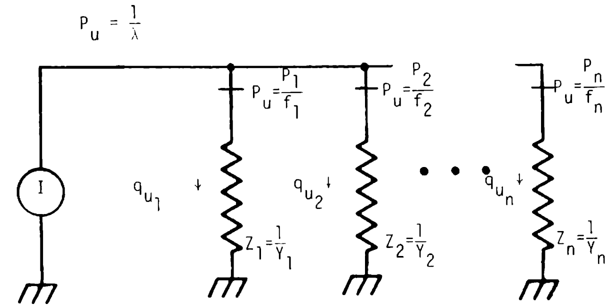
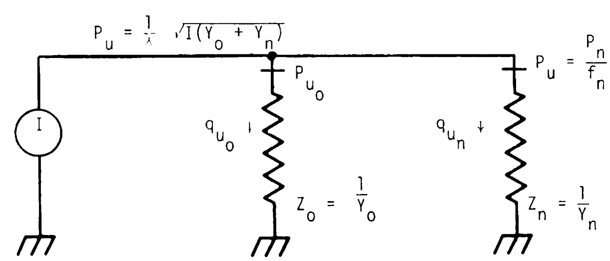


Figure 1: n commodity consumer



$$P_{u_0} = P_{ui}, \qquad i = 1, n - 1$$

$$q_{u_0} = \sum_{i=1}^{n-1} q_{u_i} = \sum_{i=1}^{n-1} f_i q_i$$

$$Y_0 = \sum_{i=1}^{n-1} Y_i$$

$$Z_0 = \frac{1}{\sum_{i=1}^{n-1} \frac{1}{Z_i}}$$

Figure 2: Two-commodity equivalent of n commodity consumer

THÉVENIN AND NORTON EQUIVALENTS (6)

In some cases it may be more desirable to represent the two-commodity equivalent in a different form in which the income source is a price source in series with an impedance. If the price and utility with respect to commodity n are to remain invariant with variations in Z_n, then the circuit shown in Figure 3 results.

The Thévenin equivalent circuit is derived by separately equating utilities and prices with respect to the n^{th} commodity. The equivalent cirucit is valid only with respect to that commodity. The remainder of the circuit may have quite different characteristics. The advantage of the Thévenin equivalent is that the transormation is independent of the value or form of Z_n. Other

TABLE 1 NETWORK VARIABLES AND RELATIONSHIPS

Quantity	Economic Interpretation	Physical Analogy
$p_u = \frac{1}{\lambda}$	Price of utility	Potential
$p_{u_i} = \frac{p_i}{f_i}$	Price of utility through purchase of commodity i	Potential
$q_{u_i} = q_i f_i$	Flow of utility through purchase of commodity i	Flow
$I_i = p_{u_i} q_{u_i} = p_i q_i$	Amount of money per unit time spent on commodity i	Power
$A_i = \int p_{u_i} q_{u_i} dt = \int {}_i q_i dt$	Total amount of money spent on commodity i	Energy
$Z_i = \frac{\partial p_{u_i}}{\partial q_{u_i}} = \frac{\partial p_i}{\partial q_i} \cdot \frac{1}{f_i^2}$	Impedance of utility through consumption of commodity i	Impedance
$Y_i = \frac{\partial q_{u_i}}{\partial p_{u_i}} = \frac{\partial q_i}{\partial p_i} f_i^2$	Price admittance of utility through consumption of commodity i	Admittance
$\varepsilon_i = \frac{p_{u_i}}{q_{u_i}} \cdot \frac{\partial q_{u_i}}{\partial p_{u_i}} = \frac{p_i}{q_i} \cdot \frac{\partial q_i}{\partial p_i}$	Price elasticity of commodity i. This is normalized price admittance	Per unit admittance
$\eta_i = \frac{q_{u_i}}{p_{u_i}} \cdot \frac{\partial p_{u_i}}{\partial q_{u_i}} = \frac{q_i}{p_i} \cdot \frac{\partial p_i}{\partial q_i}$	Stiffness of commodity i. This is normalized impedance	Per unit impedance

For a consumer at equilibrium:

$$\frac{1}{\lambda} = p_u = \frac{p_i}{f_i} \quad \text{for all } i \le n \qquad q_u = \sum_{i=1}^{n} q_{u_i} = \sum_{i=1}^{n} q_i f_i$$

$$\text{Income} = \sum_{i=1}^{n} p_u q_{u_i} = \sum_{i=1}^{n} p_i q_i$$

equivalents may be derived which will not provide invariance with respect to Z_n but preserve some other characteristic. For instance a power equivalent exists which keeps the dissipation of money invariant in the two impedances.

For a Thévenin equivalent circuit, it may be shown that the money dissipation is given by:

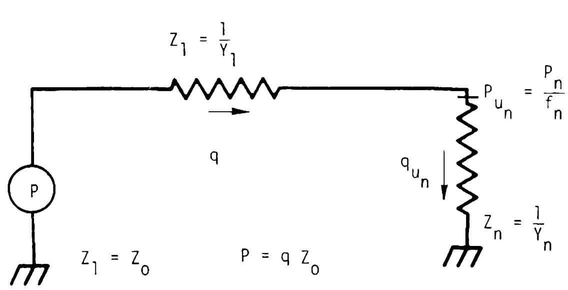


Figure 3: Thévenin equivalent circuit

$$I_{Z_1} = \frac{Z_o^2}{Z_n^2} I_{Z_o}$$

The quantity in the source impedance is given by:

$$q_{Z_1} = \frac{Z_o}{Z_n q_{Z_o}}$$

The price difference is:

$$p_{Z_1} = \frac{Z_o}{Z_n} p_{Z_o}$$

These equations may be used to interpret the values associated with the equivalent commodity zero.

If on the other hand, one desires to obtain an equivalent for the network of Figure 3 in the form of Figure 2, then the dual problem solution is a Norton equivalent obtained in an analogous fashion.

INDIFFERENCE CURVES

A network for a constant income consumer of two commodities is shown in Figure 4.

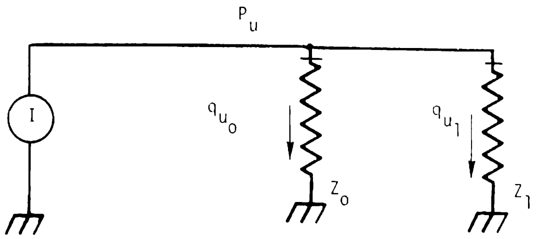


Figure 4: Two-commodity consumer

Assuming all of the consumer's income is spent, one would like to know what amounts of the two commodities are considered equivalent by the consumer. The relationship is:

$$\frac{I}{P_u} = q_{u_0} + q_{u_1} = q_0 f_0 + q_1 f_1$$

If the marginal utilities of the two commodities are constant, then the straight line of Figure 5 results.

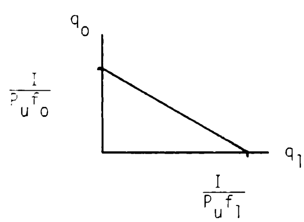


Figure 5: Indifference map for two commodities with constant marginal utilities

Suppose that the two commodities interact in such a way that the consumer's utility from consumption of one commodity increases in proportion to his consumption of the other commodity. Then $f_1 = c_1 q_0$ and $f_0 = c_0 f_1$ so that

$$\frac{I}{P_u} = (c_1 + c_0) q_1 q_0$$

and the line of Figure 6 results.

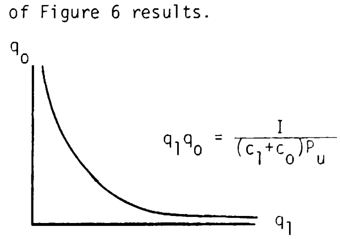


Figure 6: Hyperbolic indifference curve

In fact, as any of the quantities c_1, c_0, p_u, or I changes, a family of curves results. These indifference curves will have shapes entirely determined by the values assumed for f_1 and f_0, the marginal utility with respect to q_1 and q_0.

It is a consequence of the assumption of consumer reationality that the normalized prices he pays for goods are equal to the price of utility. If it is assumed that prices are fixed by the market place, then the f_i are not truly the consumer's marginal utilities. By altering the f_i in such a way that the income equation still holds, the solution can be moved to another indifference curve which will be optimum for the assumed f_i's but not for the true marginal utilities.

Independent of the marginal utilities, but linked through quantities purchased, is the income dissipation in the impedances Z_1 and Z_0. This dissipation is given by:

$$I = p_1 q_1 + p_0 q_0$$

Figure 7 shows the budget constraints implied by the above equation assuming constant prices with the indifference curve of Figure 6 superimposed upon it.

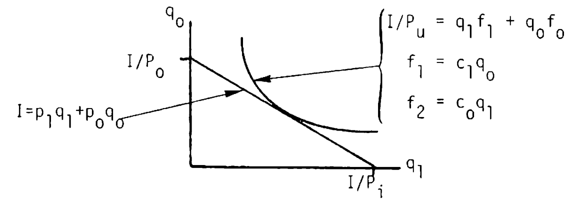


Figure 7: Budget constraint and optimal indifference curve

Where the two curves intersect, it must be that:

$$p_1 = f_1 p_u = c_1 p_u q_0$$

$$p_0 = f_0 p_u = c_0 p_u q_1$$

which is the required condition of optimality and tangency of the intersection. Any other values of f_i may intersect with the budget line, but will not be tangent. In this case their utility values will be lower.

Ordinary Demand Function

The ordinary demand function for a commodity is the quantity he will buy at each price and income level. Suppose the consumer buys n commodities using all of his income in a rational manner. Figure 1 illustrates his network equating normalized prices for all commodities. Figure 2 lumps all but the n^{th} commodity (which is of interest) into one equivalent category termed "all other goods." Interpretation of Figure 2 is somewhat complicated by the fact that an increase in consumption of commodity n may be accompanied by a compensating change in the marginal utilities, a change in prices or income. The parallel nature of Z_0 and Z_1 is the difficulty. Figure 3, the Thévenin equivalent circuit, is more suitable since the price quantity relationship is apparent in the configuration, especially for a linear case. Figure 8 shows the ordinary demand curve for commodity n with constant marginal utilities and constant income.

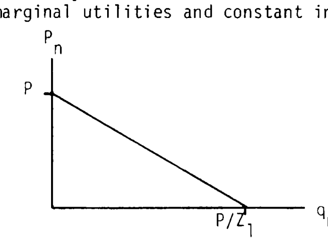


Figure 8: Ordinary demand function for constant income and marginal utilities

Clearly this model, even for the usual nonlinear utilities, will satisfy the properties of demand functions:

1. Demand is a single valued function of prices and income.

2. Demand is homogeneous of degree zero in income and prices unless a specific dependence on price

or income appears in marginal utilities or impedances.

<u>Compensated Demand Function</u>

A compensated demand function is easily constructed by changing the income source to a utility source. Whenever the term I/p_u appears in the above equations, the term q_u is substituted. In the former case, the amount of utility actually consumed is given by the ratio I/p_u and the value p_u may vary with q_u to maintain constant I. With compensated demand circuits the value of q_u is fixed and variations in p_u imply corresponding variations in I according to $I = p_u q_u$. Since income is conserved (more accurately, money), a variation in income dictates other variations to maintain:

$$I = p_u q_u = \sum_{i=1}^{n} p_i q_i$$

<u>Price and Income Elasticities of Demand</u>

The price elasticity of demand is related to the commodity impedance and admittance as follows:

$$\varepsilon_{nn} = Y_n \frac{p_n}{q_n} = \frac{1}{Z_n} \frac{p_n}{q_n}$$

Price elasticity of demand is essentially normalized load admittance for each commodity. It is usually not as informative as the unnormalized admittance since it is rarely even approximately constant. An important special case of a normalized impedance appears in the analysis of electrical power networks where per unit admittance is defined (using economic quantities) as:

$$Yp_u = \frac{\partial q}{\partial p} \frac{P_o}{Q_o}$$

The values P_0 and Q_0 are constant in that context and allow one to ignore voltage level transformations in power distribution systems. The transformer turns-ratios which determine voltage levels are analogous to the marginal utility ratios in economics. See the end of the paper for further comments in this regard.

Cross price elasticities may be similarly defined as:

$$\varepsilon_{ij} = Y_{ij} \frac{p_i}{q_j} \qquad Y_{ij} = \frac{\partial q_j}{\partial p_i}$$

However, the principal use of such cross elasticities is in linear or quasi-linear networks. These are primarily useful as an aid in writing network equations from node prices.

The income elasticity of demand for commodity n is given by:

$$\varepsilon_n = \frac{I}{q_2} Y_{2i} = \frac{\sqrt{I}\, f_2}{2 q_2 f_1} \left(\frac{1}{Z_1} + \frac{1}{Z_2} \right)^{1/2}$$

In complicated networks it will be necessary to compute cross and income elasticities and even self elasticities by incremental changes on a computer model as is done for many electrical networks as a byproduct of the numerical solutions.

Similarly the terms of the Slutsky equation can be determined from the incremental demands with income and utility constant. These are just the price sensitivities of the solution quantities. The substitution effect may be analyzed from Figure 9.

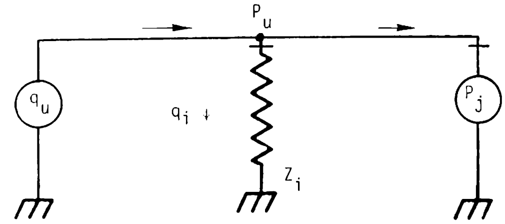


Figure 9: Circuit for analysis of substitution

It may be easily shown that:

$$q_i = \frac{f_i}{f_j} \frac{p_j}{Z_i}$$

and, if f_i, f_j, and Z_i are independent of p_j:

$$\left(\frac{q_i}{p_j}\right)_{U = \text{constnat}} = \frac{1}{Z_i} \frac{f_i}{f_j}$$

If all quantities are positive,

$$\left(\frac{\partial q_i}{\partial p_j}\right)_{U = \text{constant}} > 0$$

and commodities i and j are substitutes. If $f_i f_j < 0$, then

$$\left(\frac{\partial q_i}{\partial p_i}\right)_{U = \text{constant}} < 0$$

and commodities i and j are complements. This statement seems to refute the statement that all commodities cannot be complements for each other. However, only very unusual interrelationships will satisfy the requirements for complementarity. If by commodity, one means a flow of positive marginal utility, then no problem exists. Only in the case for which the marginal utilities are not all of the same sign can all commodities exhibit complementarity. All sorts of tied-in or barter sales exhibit such behavior. For instance the owner of a large commercial truck may be required to purchase a computer-controlled, anti-skid braking system with his truck. Such a system is currently viewed as having dubious to negative utility by the industry while the utility of a truck is strongly positive. Therefore, the two commodities may be viewed as complements even if no other purchases are made.

Additional possibilities for the complementarity of commodities i and j involve dependencies of f_i, f_j, or Z_i on p_j. Assume, for instance, that commodity j is one whose demand is dominated by the Veblen effect (4). In this case the demand and presumably the marginal utility of the commodity increase with the price. For instance the marginal utility may be given by:

$$f_j = a + bp_j^2$$

Then

$$\left(\frac{\partial q_i}{\partial p_j}\right)_{U = \text{constant}} = \frac{a - bp_j^2}{(a + bp_j^2)^2}$$

If $p_j > \sqrt{a/b}$ then

$$\left(\frac{\partial q_i}{\partial p}\right)_{U = \text{constant}} < 0$$

and the commodities are complements.

With multiple products there exist less contrived mechanisms for complementary behavior between products.

THEORY OF THE FIRM

Just as the theory of the consumer is based on a utility function, the theory of the firm is based on a production function. For this analysis some of the differences are ignored. For instance, utility is assumed to possess an objective and unambiguous cardinal measure or at least one that will work sufficiently well. Additional optimizations for the firm will be imposed on the network. Multiple outputs for the firm will be handled as well. The firm will be seen to be easily incorporated into a network approach with quite natural interpretations.

The Production Function

Rather than provide a duplicate development for single and multiple producers, this paper will skip immediately to the multiple joint product case. The implicit form of the production function is:

$$F(q_1, q_2, \ldots, q_{s+n}) = 0$$

where s of the commodities are outputs and n are inputs. This paper will assume that the implicit production function may be stated in terms of functions g_i as follows:

$$F(q_1, \ldots, q_{s+n}) = \sum_{i=1}^{s+n} g_i q_i$$

Furthermore, for simple analyses it will be assumed that all inputs and outputs are reversible so that no constraints exist on the signs of the q_i. This will be recognized as an economic form of Kirchoff's current law: the sum of all flows at a node of a network is zero. The functions g_i are marginal products for inputs. In the case of a multiple product firm, the product is some common denominator of the actual products. The g_i for outputs (negative q_i) are marginal actual products with respect to this common product. Each connection to a common node thus represents a flow of inputs or outputs scaled to the common product. In the case of a dual product operation such as sheep-raising, the common product is sheep. The sheep may be bought or sold while no amount of wool will produce another sheep and shearing does not destroy the sheep. Therefore, such a joint product is more complicated and actually requires storage elements which are avoided in this paper.

Productivity Curves and Isoquants

Total productivity curves are traces of the values of an explicit production function in much the same manner as the characteristic curves of a vacuum tube or transistor. A two-input, single-output production function is shown by assigning discrete values to one input and tracing the partial function which remains. Average and marginal productivity curves may also be generated in the usual fashion. These curves are the basic data required for a network analysis in addition to the structure and elasticities of the firm. On the other hand, they may be degermined for composite firms by simulation of simple firms interconnected in a network.

Similarly, isoquant lines may be generated by computing $(dx_i)/(dx_j)$ from simulation sensitivities and connecting points by the method of isoclines used in control theory. Elasticity of substitution is computed in the same way.

Elasticities of the Firm

Each of the parameters of the implicit production function has associated with it an elasticity. The inputs have the normal demand elasticities associated with those commodities. The outputs are connected by elasticities which account for the difference between cost for the common product and the market price for the actual product. It also accounts for any added expenses peculiar to the individual product. In the case of a single product firm, the output elasticity represents just profit. Figure 10 represents a simple circuit diagram of a firm such as a proposed offshore power plant which would produce gaseous hydrogen and oxygen from the electrolysis of water.

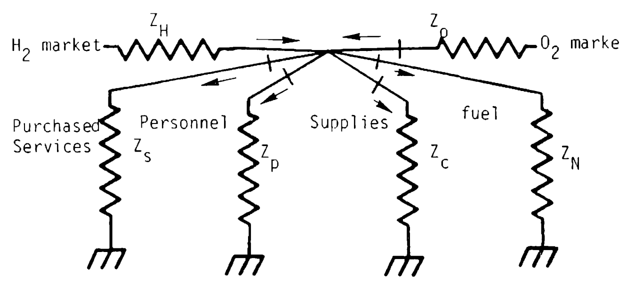


Figure 10: Circuit diagram of water electrolysis plant

The essential feature is a common product, electrolyzed water in the form of gaseous hydrogen and oxygen, which is then sold in the respective markets. All the factors may be expanded in terms of their markets and ultimately the firms of which they are composed. All flows are normalized in terms of their marginal products with respect to the common product.

The situation in Figure 10 should be compared with that of Figure 11 which shows a plant which produces gases by liquefaction from the atmosphere. Since each gas liquefies at its own characteristic temperature after all gases of higher boiling point have been liquefied, the common product becomes a series of common products. For simplicity, only energy is shown as an input.

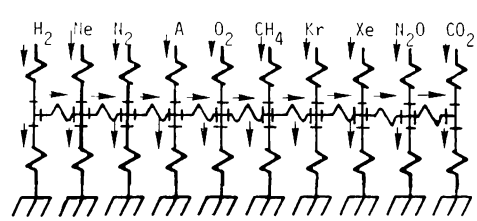


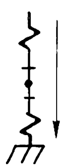
Figure 11: Liquified gas plant network

The order of gases is determined by the boiling points and any gas for which no economic market exists may be simply vented. If no gas to the left of a useable gas is desired, then the remaining gas is simply vented without further liquefaction. For simplicity, marginal product functions are not shown and elasticities are not labeled. Note that for a single plant the connecting impedances become shorts since only energy is expended for each additional gas. The mixtures of partially liquified air and energy are regulated by the production functions as are the quantities of the outputs.

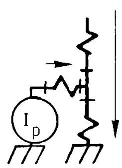
Both Figures 10 and 11 contain short bars interrupting flow paths or branches of the network. At each of these bars the price and quantity are scaled by the functions g_i such that the production function holds. In this way the diverse types of products involved in an enterprise may be combined to form new products without adding dissimilar quantities. Note that it is the production function which determines the roles of the products and any constraints between them. It is assumed here that all products are completely sold on some market and that wasted, non-polluting byproducts are not shown.

A polluting product which is not corrected by the producer may be viewed as having been sold at some price to those who will bear the price of either a lower quality of life or equipment to remove the pollution. Figure 12a represents a power generation plant which discharges heat effluent into a river. In Figure 12b, the firm passes the heated effluent through a cooling tower provided at public expense to secure a cool effluent. Figure 12c is a network for a plant which pays for its own cooling tower and essentially has to include the cooling tower operating expense in the cost of generation.

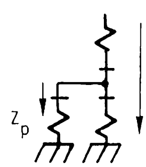
Electricity
Market



12a. Free pollution discharge



12b. Polution burden borne by separate entity



12c. Polution burden borne by firm

Figure 12

<u>Optimizing Behavior</u>

In the network formulation, the implicit production function, $F(q_1, q_2, \ldots, q_n) = 0$ is replaced by an economic analog of the Kirchoff current law at the node representing the firm:

$$\sum_{i=1}^{n} f_i q_i = 0$$

Since the amount of money flowing in any branch is invariant across the transformation from output proportion to input factor, it must be that:

$$\frac{1}{\mu} = \frac{p_1}{f_1} = \ldots \frac{p_n}{f_n}$$

But this is precisely the first order condition for maximum output for fixed cost or minimum cost for fixed output. Therefore, the network formulation assumes that such an optimization is achieved. Deviations from this optimality will require additional elements or a different configuration to account for the differences.

In all of the circuit diagrams for the firm there has appeared an impedance connecting the product to the consumer or the product marketplace. The price differential across the impedance is the marginal gross profit retained by the seller. When multiplied by the product flow, the resulting number is total gross profit. Figure 13 illustrates a Thévenin equivalent of a market from the point of view of one supplier.

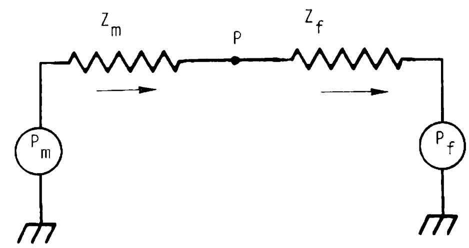


Figure 13: Thévenin market seen by single firm

In this case, the production costs and market source are represented as constant prices in dollars per product sold. Similarly the impedances are assumed to be constant. The question is posed: What value of Z_f will maximize the income dissipated in Z_f?

$$I_{Z_f} = (P_m - P_f)^2 \frac{Z_f}{(Z_m + Z_f)^2}$$

$$\frac{\partial I_{Z_f}}{\partial Z_f} = (P_m - P_f)^2 \frac{(Z_m - Z_f)}{(Z_m + Z_f)^3}$$

$$\frac{\partial^2 I_{Z_f}}{\partial Z_f^2} = -2(P_m - P_f)^2 \frac{2Z_m - Z_f}{(Z_m + Z_f)^4}$$

The first and second order conditions for a maximum are met by $Z_m = Z_f$. This condition, known in electrical engineering as the maximum power transfer theorem, could well be called by economists the principle of maximum money transfer (6). Matching impedances (or their complex conjugates for alternating flow analysis) is necessary for maximization of profit. The same result applies of course if the point of view shifts to the consumer.

This profit optimization applies to cases in which the producer can affect the market price and therefore applies to situations in which some form of monopoly exists. The monopoly situation may be due to spatial, temporal, or product differentiation as well as to control of entry and other covert or overt monopolistic practices.

In the absence of monopolistic factors Z_m is zero. No optimum then exists with p_m and p_f constant; however, increasingly large values of profit occur as Z_f approaches zero. If P_f is increasing with production, then the cost includes an impedance and the optimum value of Z_f occurs when Z_f is equal to the impedance. Similarly, if P_m decreases with production, the source is equivalent to another one with an impedance and Z_f must be equal to the impedance.

In the case of negative Z_m, zero Z_m and decreasing P_f, or zero Z_m and increasing P_m, matching impedances minimizes profit. The effect of equalizing impedances in this case is equivalent to directly connecting P_m and P_f and leaving the price indeterminate. The minimum is realized with $q_f = p_f = 0$ and no income is generated or dissipated. However, the solution is highly unstable in that a small displacement will result in a large quantity flow and price differential. The profit function for negative Z_m is sketched in Figure 14.

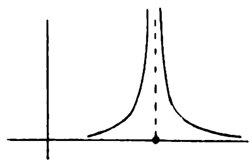


Figure 14: Profit with negative source impedance.

One may easily show that these optimizations are equivalent to equating marginal cost and price as is done in the usual economic analysis.

METHOD OF SOLUTION

The preceding sections have outlined the considerations required for modeling consumers and firms. Discussion has been made of income, price and commodity sources. Impedances, admittances, or elasticities have been used to represent sloping sources or loads. It would not be difficult to introduce storage elements of the form:

$$\Delta p = L\frac{dq}{dt} \quad \text{or} \quad q = C\frac{d\Delta p}{dt}$$

These would permit analyzing cyclical or transient phenomena and the usual long-run/short-run effects.

The one feature which has complicated all diagrams and analysis to this point is the representation of marginal utilities and production functions which has been shown by a small line interrupting a circuit branch. This detail is an impedance transformation implemented by multiplying and dividing the quantities and prices by the square root of the impedance scale factor. It is sufficient for each branch and node to specify a reference price or quantity flow since that will enable determination of the marginal functions at each transformation.

Several computer methods exist for solution of large networks such as an economic analysis would be likely to produce. Completely nonlinear programs up to 600 consumers and firms and 600 interconnections are commercially available (for instance, IBM's ECAPII, the Air Force Weapons Laboratory's SCEPTRE, or TRW's TESS). Such programs now permit static, cyclic, and general transient analysis. However, they can be expected to exhibit some numerical or topological difficulties with one-way flows and discontinuous controls. Special versions of these circuit analysis programs permit various special problems to be solved (North American Rockwell's SYSCAP has permitted solution of networks containing up to 70 exponentially nonlinear sources; the Air Force's TRAFFIC program solves cyclic problems for very large numbers of elements). The closest type of analysis to the economic circuits formulated in this paper is the Bonneville Power Administration's LOAD FLOW program. This program would permit efficient solution of networks containing 2000 consumer and firm sources with up to 4000 interconnecting impedances. Interconnections, loads, and sources may have real or complex values so that either static or cyclic analyses may be performed. Included in the Newton-Rapheson iteration algorithm is a provision for a variety of optimizations. As is customary for electric power load flow programs, the marginal utilities and products may be indirectly entered via nominal price levels. Non-optimal marginal utilities may be specified along with a limited number of variable ratios. It would appear that solution of an electric power network and an economic circuit are very closely related and that a modified version of one of the Newton-Rapheson load flow programs could provide a significant simulation capability for economic problems.*

*Information on these programs is generally available in the various user's manuals which are usually distributed by the vendors or the computer firms which offer the use of the programs. Rather analytical material is contained in references 1, 3, and 5.

REFERENCES

(1) Bowers, James C. and Stephen R. Sedore. SCEPTRE, A Computer Program for Circuit and Systems Analysis. Englewood Cliffs, New Jersey: Prentice-Hall, 1971.

(2) Henderson, James M. and Richard H. Quandt. Microeconomic Theory, A Mathematical Approach. 2nd ed. New York: McGraw Hill, 1972, chapters 2 and 3.

(3) Jensen, Randall W. and Mark D. Lieberman. IBM Electronic Circuit Analysis Program. Englewood Cliffs, New Jersey: Prentice-Hall, 1968.

(4) Leibenstein, Harvey. "Bandwagon, Snob, and Vevlen Effects in the Theory of Consumers' Demand" in Micro-Economics, Selected Readings. Edwin Mansfield, ed., 2nd ed. New York: W.W. Norton & Company, 1975.

(5) Stagg, Glen W. and Ahmed H. El-Abiad. Computer Methods in Power System Analysis. New York: McGraw-Hill Book Company, 1968, chapter 8.

(6) Walsh, John B. and Kenneth S. Miller. Introductory Electric Circuits. New York: McGraw-Hill, 1960, chapter 1.

(7) Whipple, Walter Leighton. "Optimal Profit Scheduling." M.S.E. thesis in Computer, Information and Control Engineering, The University of Michigan, Ann Arbor, 1974.