INDUSTRIAL DYNAMICS SIMULATION OF THE EFFECTS OF ADVERTISING ON PRODUCTION--DISTRIBUTION SYSTEM

C. L. Hwang, Reza Shojalashkari

and L. T. Fan

Kansas State University

ABSTRACT

Industrial dynamics simulation is employed to investigate the effects of some advertising practices on the behavior of production-distribution system. An advertising model is reformulated in the framework of industrial dynamics and is incorporated into the system model. Three different variations of the advertising model are considered: protracted, short-time intensive, and impulse type. The results of the system simulation, obtained on a DYNAMO compiler, are compared on the basis of such criteria as the total cost of production and inventory, fluctuations in system variables, and total sales generated. Protracted and impulse type campaigns seem to be the most feasible. The choice between the two depends on the product type as well as on the production and inventory policies.

I. INTRODUCTION

The so-called industrial dynamics has found applications in industrial systems analysis over the last decade. It traces the behavior of systems through the feedback relationships that exist among the components of each system. As such, it is a study of the interactions among the "feedback loops" of a system; such loops exist in economic, social, industrial, and biological systems. A feedback loop exists whenever a decision results in actions which change the state or condition of a variable in a system, and, therefore, change the information about that variable; the new information is utilized to guide the future decision. See Illustration 1. If the action generates still greater action, a growth process is resulted, and the feedback loop is a "positive" feedback loop. On the other hand, if the results of the action are in a direction so as to eliminate the discrepancy between the present state and the desired state of the system, the loop is called a "negative" feedback loop.

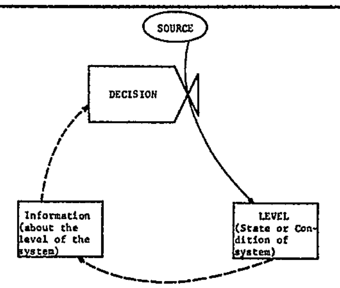


Illustration 1. Structure of a Feedback Loop

This study deals with the application of industrial dynamics to investigate the effects of some advertising practices on the behavior of an industrial system whose dynamic model is proposed by Forrester (3).

A mathematical model of sales response to advertising, suggested by Vidale and Wolfe (9, 10), is used in this work to describe the "effectiveness" of advertising campaigns. However, the original model explores only the changes in retail sales as the result of advertising. The model is modified for the purpose of developing its industrial dynamics representations for three different conditions, namely, a) A protracted advertising campaign; b) An intensive, short-time campaign; c) An impulse type campaign. They are added to the industrial system model to investigate the effect of such campaigns on the production and inventory practices of production-distribution systems.

II. INDUSTRIAL SYSTEMS AS VIEWED BY INDUSTRIAL DYNAMICS

An industrial system, hereafter referred to as "system", in its simplest form consists of production, distribution, and retail sectors. There are, of course, subdivisions within each sector, and also interactions between the system and market, but the core of such a system is essentially made of a production sector, a distribution sector, and a retail sector. The production sector is the highest, and the retail sector, the lowest sector in the system. Illustration 2 shows the simplified organization of a typical industrial system. Materials flow downward from each sector to the next, and orders flow upward between successive sectors. The flow of materials and orders are the "rate" variables, and would necessarily pass through "levels", which are the inventories and order backlogs at each sector.

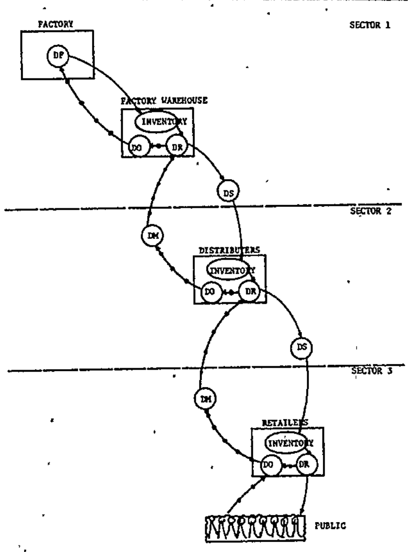


Illustration 2. Organization of a Production-Distribution System

In an industrial system, there are ordinarily six flow channels; they are: 1) order flow; 2) material flow; 3) information flow; 4) personnel flow; 5) equipment flow; and 6) capital flow. Illustration 3 shows the industrial dynamics representation of these flow channels. Only the first three flow channels are considered in this work; the others are assumed not to affect the system behavior.

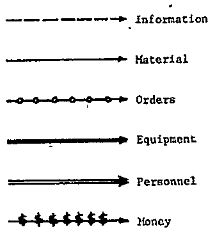


Illustration 3. Flow Symbols

Within the organization of an industrial system, the level and rate variables as well as the time delays are to be identified. The most important time delays in the system are: a) Factory lead-time delay, DF; b) Shipping delays between sectors, DS; c) Order-filling delays at all three sectors, DR; d) Delays in making decisions about the orders to the next higher sector, DO; and/ e) Mailing delays of orders between any sector and the next higher one, DM.

Since the three sectors are similar in function, as far as the modeling is concerned, the level and rate variables associated with these sectors will also be similar. Considering the three flow channels at work in this system, the level and rate variables can be enumerated as follows:

LEVEL VARIABLES

The level variables at each system sector are a) Backlog of unfilled orders received; b) Inventory of items in stock; c) Average sales level, which is the basis for the ordering policy from one sector to another.

RATE VARIABLES.

At each system sector, the rate variables are a) Rate of outgoing orders to the next sector; b) Rate of incoming materials from the next higher sector (shipping rate); c) Rate of outgoing materials to the next lower sector (shipping rate); d) Rate of incoming orders from the next lower sector.

The above time delays and variables are the essential factors in the feedback structure of the system, as represented in the flow diagram of Illustration 4 (3), which shows the very basic feedback structure of the system at the retail sector. Other sectors have similar structures. For a complete description of the diagram as well as the dynamic model of the system, written in DYNAMO, readers are referred to Forrester (2, 3).

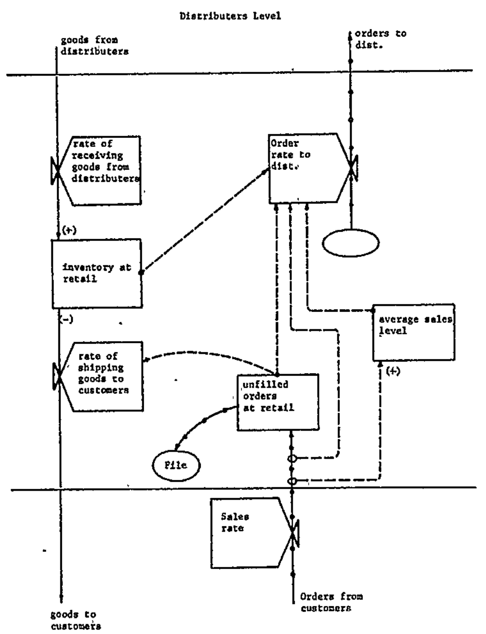


Illustration 4. Basic Feedback Structure of Retail Sector

III. ADVERTISING PRACTICES IN INDUSTRAL SYSTEMS

In modeling the organization of the industrial system, simplification was obtained by not considering the market sector initially. However, the behavior of the production-distribution system would not be meaningful unless the interactions between the system and market are considered. In the following study of the production-distribution system, it is assumed that the retail sales are greatly influenced by the promotional efforts, which are now part of any such system.

In recent years, operations research methodology has been extensively applied to industrial situations; however, it has not been widely used in the field of advertising due to the difficulties in hypothesizing and estimating the response of people to promotional efforts. Although it is not a promising field for OR studies, some quantitative aspects of advertising have drawn attention recently, such as evaluation of advertising effectiveness, allocation of advertising budgets among products and media, determining the advertising budget, and so on (1, 5, 7).

Measuring advertising effectiveness is the most difficult part of the model building in this case, since there are no clear-cut and agreed-upon definitions of effectiveness, and accordingly there are various methods for measuring the effectiveness of advertising campaigns (6). In general, effectiveness "Is used to denote the degree to which advertising can change people's external or internal behavior with respect to an item-product, service, or idea-advertised, and in the direction desired by advertiser." (8)

A mathematical model of sales response to advertising was proposed by Vidale and Wolfe (9, 10), based on their extensive experimental study over large portions of U.S. market.

DESCRIPTION OF THE MODEL

The model is based on three parameters affecting the sales response to advertising, a) the Sales Decay Constant, b) the Saturation Level, and c) the Response Constant.

The Sales Decay Constant

This determines how the sales rate decreases in the absence of advertising. The rate of decline (sales decay constant) has been found to be generally constant for a product, meaning that a fixed portion of the sales is lost each year, provided that market conditions are relatively constant, and that allowances are made for seasonal changes and similar random factors.

The sales decay rate is proportional to the sales rate and, therefore, if $S(t)$ = rate of sales at time t; λ = sales decay constant; then, the change in the sales rate is:

$$\frac{dS(t)}{dt} = -\lambda S(t) \qquad (1)$$

The solution to this differential equation is:

$$S(t) = S(0) \cdot e^{-\lambda t} \qquad (2)$$

where $S(0)$ is the initial sales rate.

The Saturation Level This is the practical upper limit of sales that an item may acquire regardless of what is spent on advertising thereafter. Its value depends on the product itself, as well as on the advertising medium used. When the sales rate; $S(t)$, equals to the saturation level, SL, the change in sales rate is zero; i.e.,

$$\frac{dS(t)}{dt} = 0.$$

The Response Constant This parameter, the response constant or more accurately zero-sales response constant, r, is the sales generated per dollar of advertising when the initial sales are zero. As the sales reach the saturation level, the effect of advertising decreases because the number of potential customers decreases. It has been assumed a linear decrease in advertising effectiveness, i.e., a constant value for response constant. Therefore, if $S(t) = 0$, then $\frac{dS(t)}{dt} = r$, and if $S(t) = SL$, then $\frac{dS(t)}{dt} = 0$.

In many cases sales are not zero when the advertising campaign begins; therefore, the campaign is directed at that portion of the market that has not been affected by it. If the sales level prior to advertising is S_0, and the saturation level is SL, the unaffected part is $(1 - \frac{S_0}{SL})$. In this case, the sales generated per advertising dollar are given by: $r[1 - \frac{S_0}{SL}]$.

Mathematical Representation of Model. The mathematical model of the sales response to advertising states, in effect, that the change in the sales rate equals the sales generated in the "unaffected portion" of the market, minus the sales lost due to sales decay function; in mathematical notations:

$$\frac{dS(t)}{dt} = r\,A(t)\,[1 - \frac{S_0}{SL}] - \lambda\,S(t) \qquad (3)$$

in which: $\frac{dS(t)}{dt}$ = Change in sales rate at time t, (\$/time)/time; r = Response constant, 1/time; $A(t)$ = Advertising rate, \$/time; S_0 = Initial rate of sales prior to advertising, \$/time; SL = Saturation level, \$/time; λ = Sales decay constant, 1/time; and $S(t)$ = Sales rate at time t; \$/time.

SYSTEM MODEL WITH ADVERTISING To add the model of the market interaction to the system model, it is assumed that the production level at the factory constitutes the information on which the advertising budget is based. This information is not readily available because of the time delays necessary to smooth the average rate of manufacturing.

Decisions regarding the advertising expenditures are usually made after a time delay, and advertising agencies and media would cause additional delays before the actual initiation of the advertisement. The effect of advertising is not immediate; there is evidence that awareness about the product being advertised builds up gradually (3, 9, 10). This gives rise to an additional delay.

The system model, in this form, is basically a positive feedback loop; as the retail sales increase, the production level at the factory will eventually increase; a higher production level means a greater advertising budget and eventually a higher level of awareness among the public. If advertising awareness has any effect, it would make the prospective customers purchase sooner than they would otherwise; this increases the retail sales. The growth within the positive loop continues, but there are controlling factors which affect the growth process, as is the case with most positive feedback loops. These factors are the declining advertising awareness, the sales decay function, and the saturation level, which altogether contribute to the declining number of customers. The structure of the system with advertising is depicted in Illustration 5.

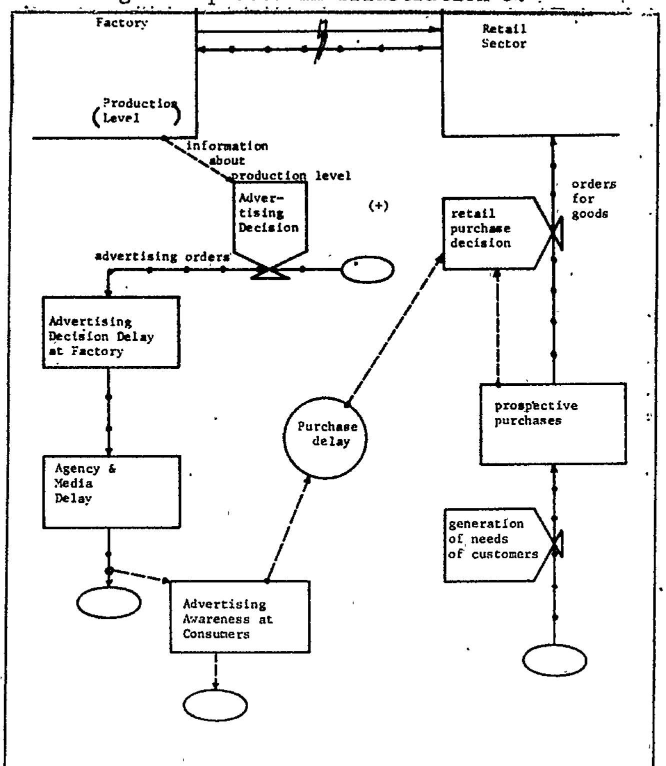


Illustration 5. Flow Diagram of the Advertising Sector

Different advertising campaigns, based on the mathematical model of equation (3), along with their industrial dynamics representations, are discussed in the next section, and the results of the simulation of the system behavior are interpreted.

In all the examples, the values of the parameters as listed in Table 1, which represent a particular product, are used in the system model.

IV. INDUSTRIAL DYNAMICS SIMULATION OF THE SYSTEM UNDER DIFFERENT ADVERTISING PRACTICES

CONSTANT ADVERTISING OVER A PERIOD

In some campaigns, the amount spent on advertising is constant for a period of time T and becomes zero thereafter. The analytical solution to equation (3) for a constant rate of advertising over a time period T is (9):

$$S(t) = \left(\frac{SL}{1 + \frac{\lambda SL}{rA}}\right)\left(1 - e^{-(\frac{rA}{SL} + \lambda)t}\right) + S_0\, e^{-(\frac{rA}{SL} + \lambda)t} \qquad (t \le T) \qquad (4)$$

$$S(t) = S(T)\, e^{-\lambda(t-T)} \qquad (t > T) \qquad (5)$$

To simulate the behavior of the system under constant advertising for a period of time, it is assumed in this example that a budget equal to 10% of the initial sales volume is allocated to advertising. The duration of the campaign is assumed to be 25 weeks or approximately 6 months. After a time delay representing the decision process at the factory, and another delay at advertising agencies and media, the campaign is actually presented to the public.

This scheme is programmed for DYNAMO, and then added to the production-distribution system model with slight modifications to fit the situation.

Illustration 6 shows the simulated behavior of the system for 2 years. The initial response to the advertising campaign is slow; as the advertising "awareness" or "effectiveness" gradually builds up, the sales rate accelerates, and the inventory at retail starts to fall because there has not been any preparations in advance and also because the orders for goods placed at a higher sector would encounter various delays as explained earlier and would not arrive immediately. It can be observed that the orders received at the factory are constant for about 2 months (8 weeks) after the advertising starts. As a result, the inventory at the factory warehouse remains constant for more than two months.

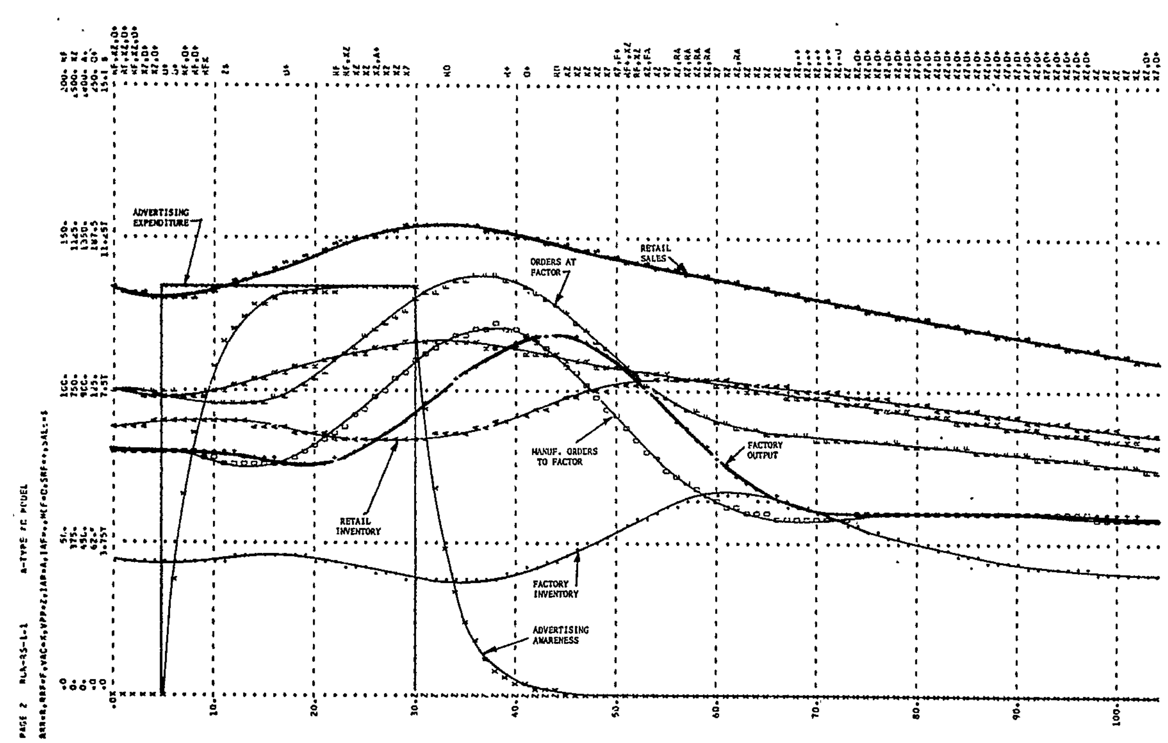
Retail sales continue to rise as the awareness increases; they reach a peak (at the 36th week) and thereafter decline exponentially according to the sales decay function.

Because of the time delays in the system structure, the rising rate of retail sales is not immediately reflected at the factory, and orders to the factory warehouse start rising only 7 weeks after the retail sales have started to increase at the 6th week. However, once started, the orders received at the factory warehouse will increase faster than the sales rate (amplification), and as a result, they reach a maximum almost at the same time as the retail sales rate.

Manufacturing orders to the factory from the factory warehouse follow a similar pattern, and 6 weeks after they are at the maximum, the production rate at the factory reaches its maximum (44th week). The 6-week delay corresponds to the factory lead-time.

Inventories at both the retail and factory warehouse behave very similarly. In the beginning, they are almost unchanged, but as soon as the

Illustration 6. Constant Advertising for a Period of Time



advertising campaign starts, and as soon as the retail sales go up, both inventories start to fall; however, the inventory at the retail sector reaches a minimum sooner than it does at the factory warehouse because of the delays in the flow channels of the system. The retail inventory is minimum at the 25th week, while the factory warehouse inventory reaches its lowest level at the 30th week.

At the 30th week, advertising stops and loses its effectiveness gradually. At a point where its effectiveness is not great enough to encourage additional sales, retail purchases start to fall at an exponential rate. With retail sales going down, manufacturing orders at the factory will gradually decrease, and so will the factory output. Because of the amplifications in the system, it can be observed that although retail sales fall at a small rate (SDC = .005 or .5% per week), the rate of change is amplified through the system sectors, and will result in a greater rate of change in the factory output (2.8% per week). It takes a relatively long time (more than 20 weeks) before orders placed at the factory and consequently the factory output start to decrease.

Although advertising stops at the 30th week and retail sales start to fall at the 36th week, inventories at the retail and factory warehouse continue to increase. This is because the steady increase in the retail sales from the 6th week to the 36th week can be easily interpreted as the new sales trend, and therefore, orders are placed accordingly to increase the inventory and pipeline contents. It takes more than 5 months before the falling retail sales affect the inventories; they only start decreasing after the 60th week.

In this example, it was assumed that the factory production can be expanded to meet the requirements. In a more realistic situation, there might be a limitation on the factory production expansion.

SHORT-TIME, INTENSIVE ADVERTISING

Advertising campaigns are not always protracted over time; some advertisers prefer to spend the whole advertising budget on a single, and short-time advertisement. This can be called a "single-pulse campaign."

In equation (3), if time T becomes negligible, the solution to the equation is (9):

$$S(t) = SL\ e^{-\lambda t} - (SL - S_0)\ e^{-(\frac{ra}{SL} + \lambda)t} \qquad (6)$$

where a is the total money spent on advertising.

For the dynamic model of this advertising campaign, it is assumed that the total advertising budget, which in the previous example was spent over a time period of 25 weeks, is now being spent in a short time, say one week; the same considerations regarding the time delays at the factory and at the agencies and media apply.

The results of the system simulation for 2 years are shown in Illustration 7. Advertising campaign is shown as a single pulse occurring at the end of the 6th week. Sales that have been declining previous to the advertising campaign pick

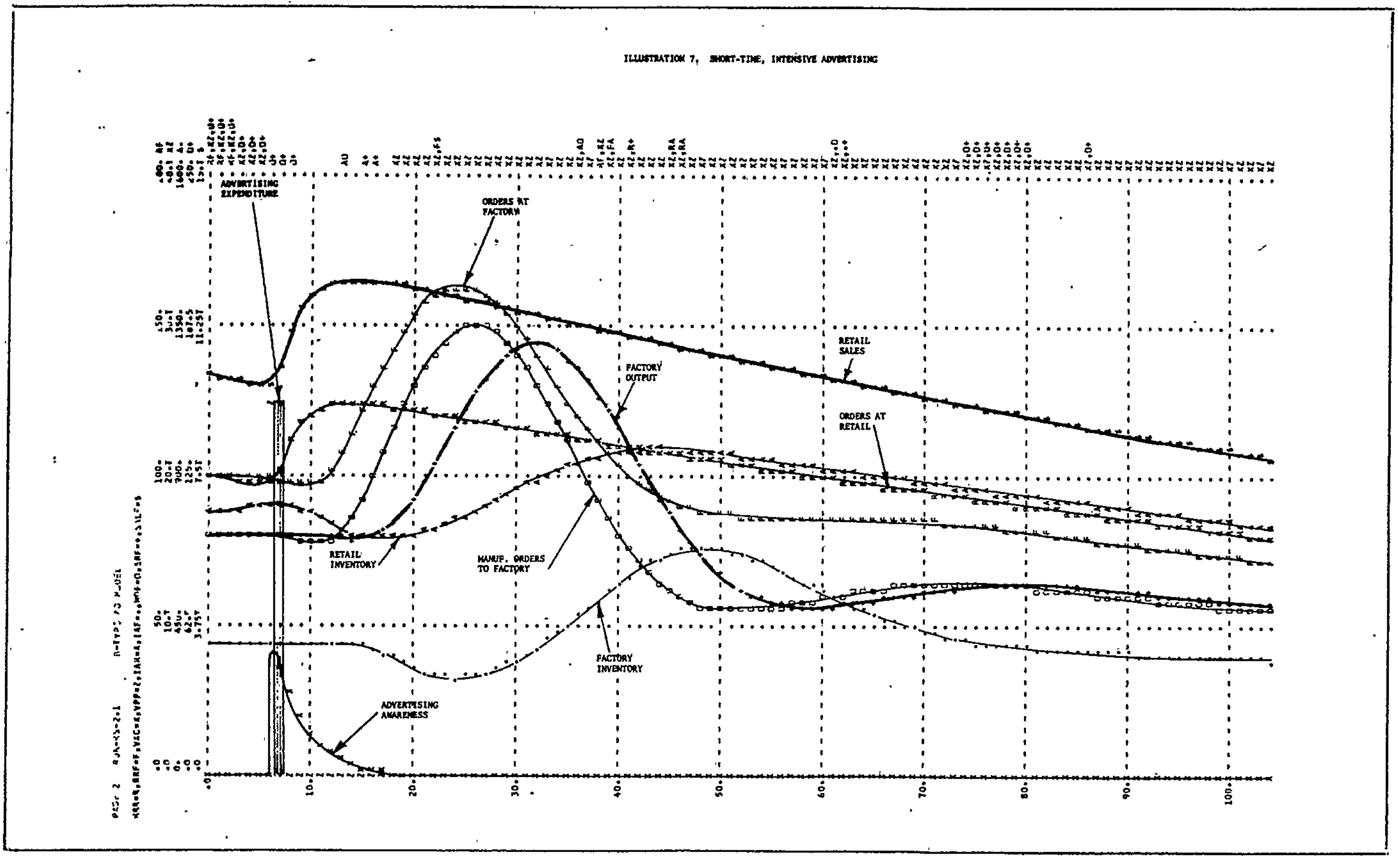


TABLE 1
Parameter Values for System Model

Parameter	Value
Sales Decay Constant	.005/week
Saturation Level	15000 $/week
Response Constant	0.5/week

up after a very short time and continue to rise up to the 15th week; this is mainly because of the sales response function which is at work throughout this period. After the sales rate reaches a maximum at the 15th week, the sales decay function takes effect, because the advertising effectiveness is now at such a low level that it cannot create any additional sales.

The orders received at the factory warehouse are almost constant in the beginning, and they start to increase at the 11th week; a week later, the manufacturing orders to the factory encounter a similar trend. The factory output has a 6-week time lag in all cases and, therefore, it reaches a maximum at the 32nd week, while manufacturing orders are at peak at the 26th week.

The behavior of inventories is similar to that of the previous example. Both retail and factory warehouse inventories decline in the beginning, and only after a long time delay (18 weeks at retail, and 24 weeks at factory warehouse) do they start to increase. Although sales at the retail are declining at this time, inventories keep rising, partly because of the need to replace the items sold and partly because the sales level is higher than it was in the beginning, and, therefore, the retail sector requires a higher level of inventory. However, because of delays and amplifications, the declining sales rate is reflected in the inventory levels after the 44th week at retail, and after the 48th week at the factory warehouse. From then on, they decrease in correspondence with the retail sales.

Unlike the previous case, the factory output in this example has two peaks, one at the 32nd week and about 80% above the initial level, and another at the 82nd week and about 18% below the initial level; the peaks are almost a year apart. However, the first peak is much more conspicuous than the second.

IMPULSE-TYPE ADVERTISING

In the previous example, the advertising campaign is carried out only once. It is observed that the sales rate started to rise following the campaign, but a decreasing trend started after a short time, as the advertising campaign lost its effectiveness. The problem of fast-decreasing effectiveness of any advertising campaign can be overcome by carrying out another one at a later time, when the effectiveness of the previous one becomes so low that it is no longer capable of increasing the sales. This type of advertising campaign can be called "impulse-type" advertising, and it basically consists of a series of "short-time intensive" advertisements, carried out at intervals of time. Neither the budget spent on advertising each time nor the interval between the subsequent "impulses" need be constant.

The analytical solution to equation (3) in the case of impulse-type advertising with constant interval and budget, would be a series of analytical solutions to the short-time, intensive kind of advertising, separated from each other by intervals equal to T, that is,

$$\left.\begin{aligned}
S_1(t) &= SL\, e^{-\lambda t} - (SL - S_0)\, e^{-\left(\frac{r\,a}{SL} + \lambda\right)t} && 0 < t \leq T \\
S_2(t) &= SL\, e^{-\lambda (t-T)} - (SL - S_1(T))\, e^{-\left(\frac{r\,a}{SL} + \lambda\right)(t-T)} && T < t \leq 2T \\
S_3(t) &= SL\, e^{-\lambda (t-2T)} - (SL - S_2(2T))\, e^{-\left(\frac{r\,a}{SL} + \lambda\right)(t-2T)} && 2T < t \leq 3T \\
&\vdots
\end{aligned}\right\} \tag{7}$$

where a is the amount spent each time, and A =n a, is the total advertising budget; and n is the number of campaigns.

In modeling this advertising scheme for DYNAMO, it is assumed that the total advertising budget of the first example, which was spent constantly over a time period of 25 weeks, is now being spent equally on 7 "impulse" campaigns, separated from each other by an interval of 16 weeks.

Illustration 8 shows the simulated behavior of the system under impulse-type advertising for a period of 2 years. The first campaign is carried out at the end of the 5th week, representing the delays at the factory, agencies, and media.

Before the first campaign takes place, sales have decreased slightly. A short time after the campaign, they start increasing at a small pace, and then remain constant for a period thereafter. They start decreasing again until the next "pulse" is applied when the same pattern, with slight variations, occurs. It can be observed that, in general, sales decrease but at a very small rate; at the end of the 2-year period the sales rate has decreased only about 5% from the initial level or 2.5% per year. Results of additional simulation runs have indicated that this is due to the insufficient advertising budget.

The factory output shows slight variations; it reaches a minimum at the 29th week, which is about 8% below the initial level. At the 45th week, it reaches a level which is about 6% below the initial level, and remains constant thereafter for a period of 32 weeks. It further decreases by 1% after the 77th week, and the new level holds up to the end of the simulation run.

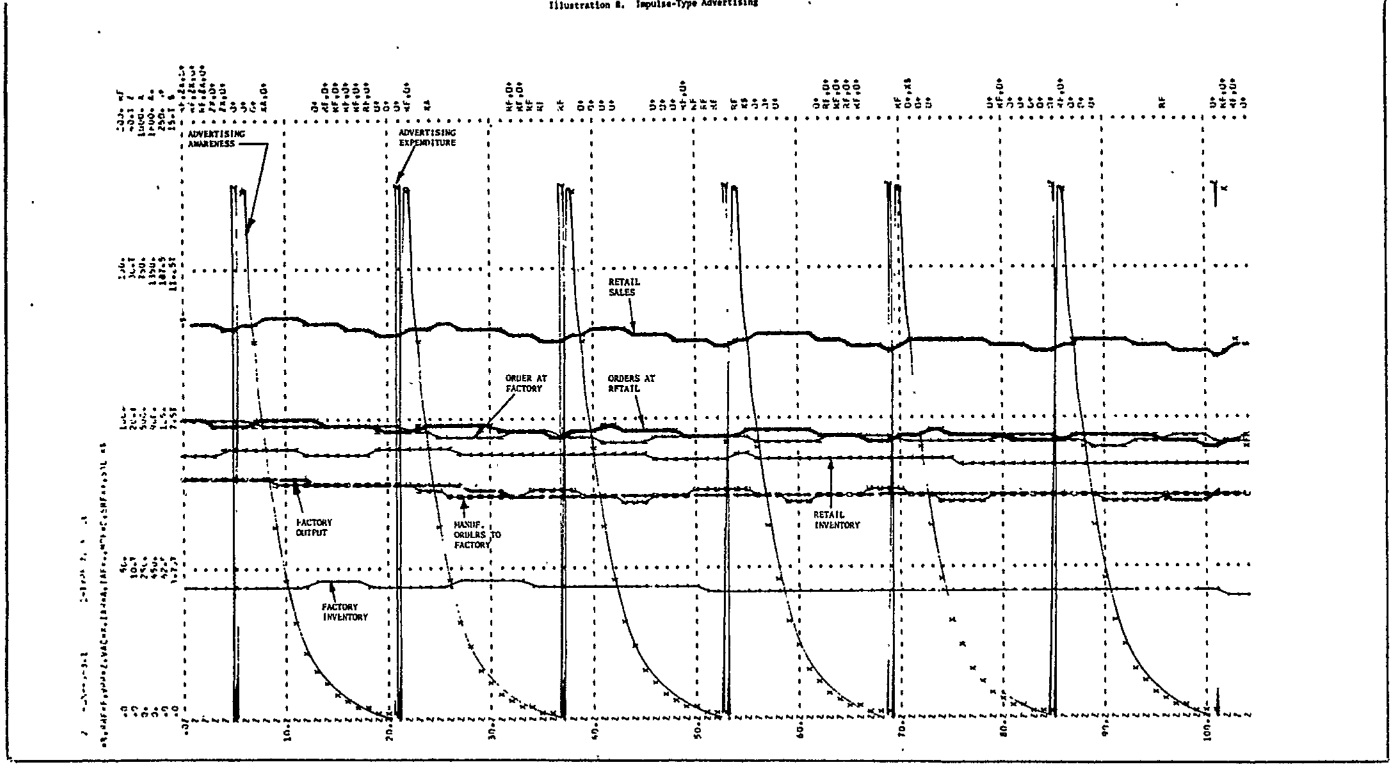
Inventories at the retail and factory warehouse behave similarly; their fluctuations are very slight. At the end of the simulation run, inventories are down about 4% at retail, and about 5% at the factory warehouse.

RANDOM FLUCTUATIONS IN RETAIL SALES

In considering the three models of sales response to advertising, it is assumed that the sales change over time in a deterministic way. However, there are many factors that affect a purchase in a particular situation which cannot be taken into account individually; instead, their effects can be approximated by introducing random variations into the sales rate at the retail.

The effect of random fluctuations on the retail sales in the case of constant advertising for

Illustration 8. Impulse-Type Advertising



a period, can be observed in Illustration 9 which corresponds to Illustration 7 for the deterministic sales rate. The random variations are sampled from a "random number generator" which generates normally distributed random numbers with a mean of 0.0 and standard deviation of .05. Similar results can be obtained for the other two cases of advertising (11).

V. DISCUSSION AND CONCLUSION

The results of the three different advertising practices* were examined in the last section. It was observed that:

1) For an A-type campaign, the retail sales rate grows in the beginning, reaches a maximum, and then declines exponentially.

2) For a B-type campaign, the sales rate immediately starts to rise, reaches a maximum value in a relatively short time, and then drops exponentially.

3) For a C-type campaign, the behavior described for the B-type campaign takes place within each interval, and on the whole, sales decline.

In addition to the moment-by-moment sales at retail, three other criteria may be employed to compare the three campaign practices. The first criterion is the total sales generated as a result of advertising. Table 2 shows the total sales for the three models from the beginning of the advertising campaign to the end of the simulation run. The total sales for the A-type and B-type advertising are practically equal. However, about $57,000 less in sales occurred in the C-type advertising.

The second criterion is the magnitude of fluctuations in the production rate at the factory and in inventories. There are costs associated with changes in the production rate and in the inventory level. Therefore, smooth rates of production and inventory levels should be considered, in some occasions, in addition to total sales. Table 3 summarizes the changes in the factory output and the factory warehouse inventory from their nominal values in response to different advertising campaigns.

The third criterion is the cost incurred when production rate and inventory level change to new values. The cost function may be represented by (Holt, et al., 4):

$$S = \sum_{n=1}^{N} [C(\theta_n-\theta_{n-1})^2 + D(E-I_n)^2] \qquad (9)$$

where S = Total cost of changing production rate and inventory level; n = Production rate during the n-th period; I_n = Inventory level at the end of the n-th period; E = Desired inventory level; C,D = Cost coefficients, constant; and N = Number of periods. Here, it is assumed that, for a duration of 2 years, 7 periods each of 15 weeks exist. Based on this assumption Table 4 shows the estimated costs S, according to Equation (9), for the three advertising campaigns, and for different

*For convenience of reference, the three advertising practices are coded as follow:

- Protracted advertising: A-Type
- Short-time, intensive advertising: B-Type
- Impulse-type advertising: C-Type

Illustration 9. Constant Advertising for a Period of Time; Random Fluctuations in Sales

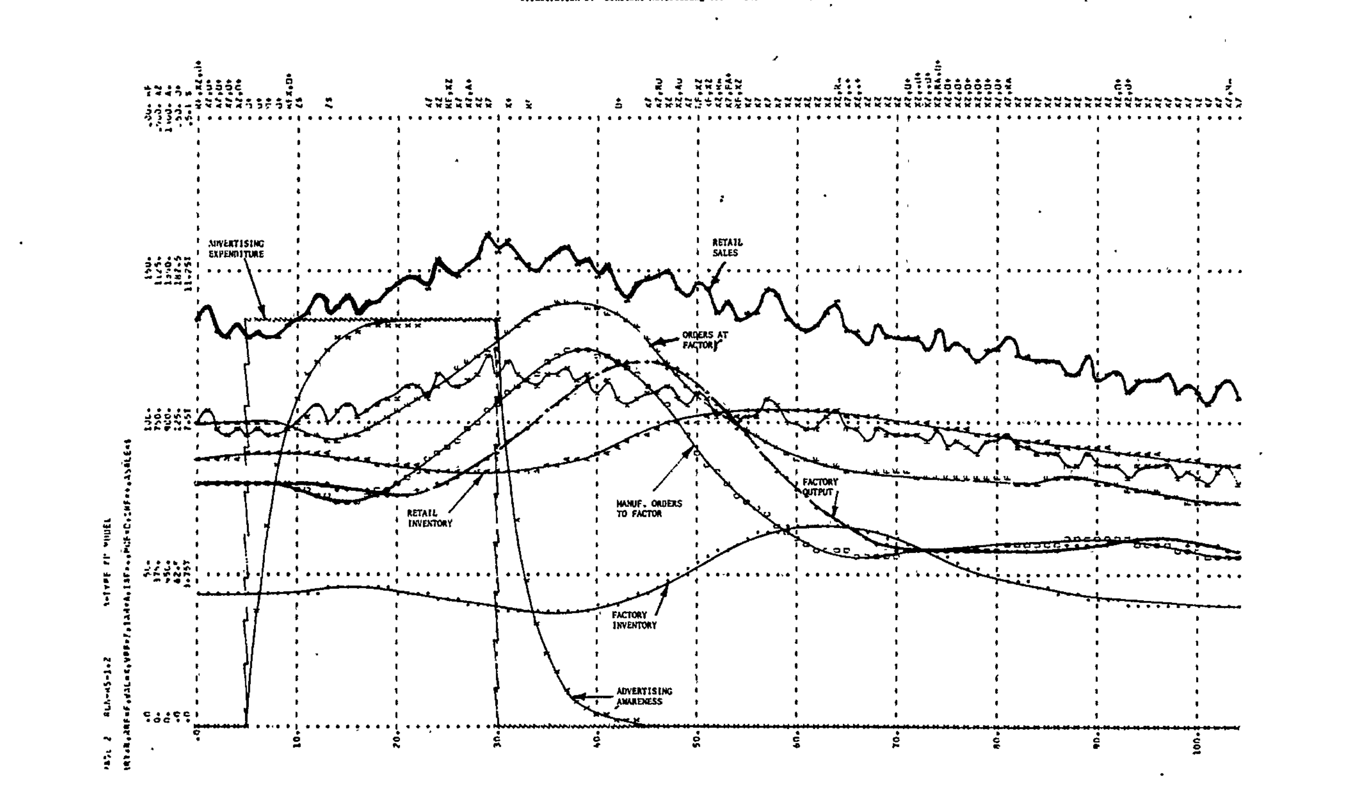


TABLE 2

Total Sales Generated After Advertising

Advertising Type	Total Sales Generated After Advertising, 1000$
A=Protracted	1000
B=Short-Time Intensive	1004
C=Impulse	943

TABLE 3

Variations of Factory Output and Inventory

Advertising Type	Factory Output (Units/Week)			Factory Warehouse Inventory (Units)		
	Initial Value	Max. Min.	% Variations	Initial Value	Max. Min.	% Variations
A	100	14 8 73	+ 48 - 27	400	590 350	+ 47.5 - 12.5
B	100	180 71	+ 80 - 29	400	680 283	+ 70 - 29.3
C	100	100 91	0 - 9	400	420 370	+ 5 - 7.5

TABLE 4

Total Cost of Changing Production Rate and Inventory Level

Parameter Values		Advertising Type: Protracted Type (A)	Short-Time, Intensive Type (B)	Impulse Type (c)
relatively large	C = 100 D = 20	$787,100	$1,891,000	$51,460
intermediate values	C = 10 D = 2	78,710	189,100	5,146
relatively small	C = 1 D = .20	7,871	18,910	514.6

TABLE 5

Values of the Objective Function

(1000 $)

C & D	A - Type	B - Type	C - Type
C = 100 D = 20	213	-887	891
C = 10 D = 2	921	815	938
C = 1 D = 0.2	992	985	943

values of constants C and D. The objective function can be defined as:

$$F = \begin{pmatrix}\text{Total Sales Generated} \\ \text{after Advertising}\end{pmatrix} - \begin{pmatrix}\text{Total Cost of Changing} \\ \text{Production and Inventory Level}\end{pmatrix}$$

The values of the objective function for the three advertising campaigns, and for different C and D values are shown in Table 5.

Since the values of C and D depend on the specifications of the product, production techniques, inventory practices, and so on, the decision belongs to the advertiser as to what kind of advertising would be feasible. If C and D are relatively large (as in the first row of Table 5), the C-type advertising seems to be feasible since it yields a higher objective function value. On the other hand, if C and D are very small (as in the third row of Table 5), the A-type advertising yields a higher value of the objective function. For intermediate values of C and D, the C-type advertising is justified, although there is no significant difference between the C- and A-type campaigns. The B-type campaign does not seem to be feasible in any case.

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