A STATISTICAL RESPONSE SURFACE GENERATOR

James Roger Corliss

Farmers Bank of the State of Delaware

ABSTRACT

Small scale special purpose analog simulators are finding many applications as teaching aids in the classroom. One type of simulator design is currently being used in graduate level statistics courses at the University of Delaware. It was developed primarily for teaching Statistical Response Surface Technology.

Statistical Response Surface Technology (SRST) is an effective way of optimizing variables in a system or process. Samples at various operating parameters are taken and analyzed according to one of many Response Surface Design techniques. This statistical sampling process can produce an accurate estimate of a maximum or minimum point, or can give the optimum coefficients of a quadratic model.

The Response Surface Generator (RSG) described in this paper incorporates multivariable function simulators and a random error generator to produce a variety of response surface designs. In general, the student is given a problem based on some process or model which is simulated in the RSG. He can manipulate any of the six input variables and then read the output response. The student may be asked to find the minimum or maximum operating point, the minimum or maximum cost, or the quadratic model for the process using SRST techniques.

I. SIMULATOR SPECIFICATIONS

In 1971, specifications for a Response Surface Generator were developed by the Statistics and Computer Science Department of the University of Delaware. The design was to consist of a multifunctional response system with six variable inputs and one variable output. The functions to be generated were to include linear and nonlinear functions with linear-linear, linear-nonlinear, or nonlinear-nonlinear interactions. The linear functions were to have various negative and positive slopes. The nonlinear functions were to consist of several different positive and negative peaking functions plus an increasing and decreasing exponential function.

A random error generator was to be incorporated. It was to have a Gaussian response and would be used to simulate sampling error. Switches were to be used to vary the amount of random error and to alter the functions controlled by each of the inputs. The front panel controls were to be coded so that the student would not know what functional relationships were being used.

II. SIMULATOR DESIGN

The RSG design incorporates function generators which produce eight different response types. These responses are shown in Illustration 1. They include an increasing and a decreasing linear function, a peak and a valley function, an increasing and a decreasing exponential function and their inverse functions. The generated functions are combined with various switchable values of resistance to produce a total of twenty six different responses. These responses can be rearranged or modified by an interior patchboard.

Up to six responses are connected to the six independent variable input potentiometers located on the front panel. A six position panel mounted response selector switch is used to activate the desired internally patched six-dimensional response surface model.

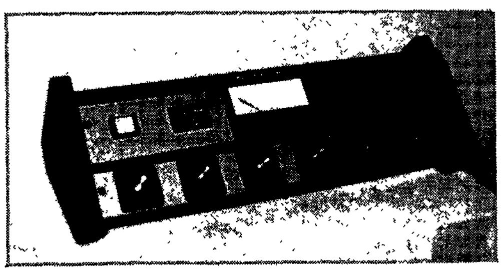


ILLUSTRATION 1

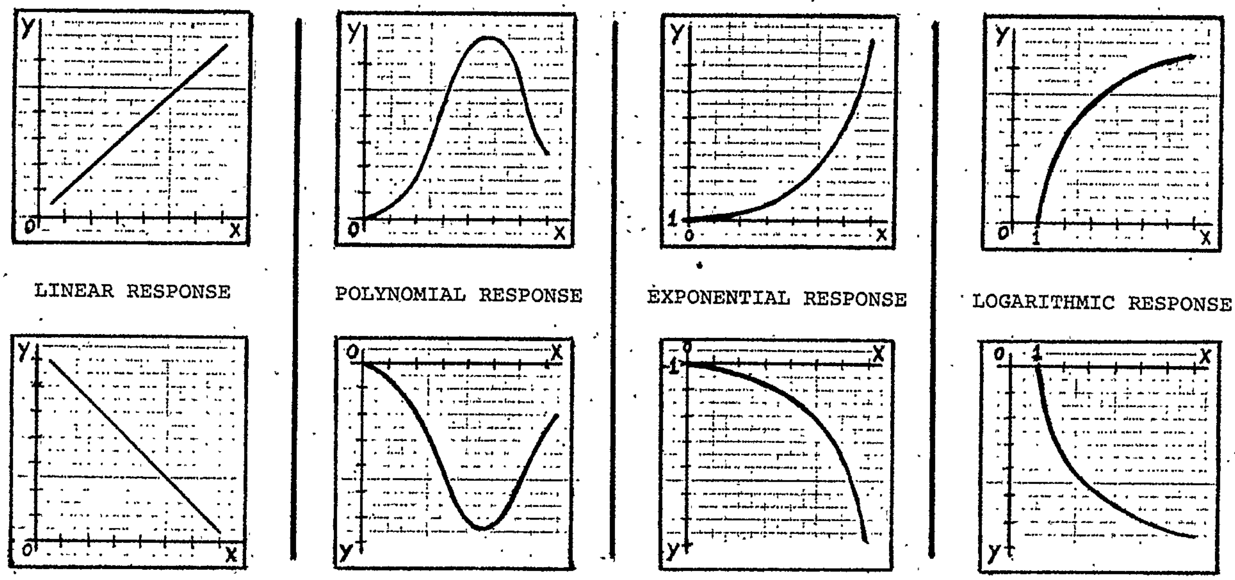


Illustration 2 presents the individual responses for the inputs A, B, C, D, E and F when the Selector Switch is in position 1. Inputs A and B together, produce a linear interaction. Inputs D and E both give a linear response.

The linear interaction is produced by two operational amplifiers and a zener diode. The nonlinear responses are developed over diode-resistor operational amplifier circuits. The nonlinear interactions come from the summation and interrelation of various nonlinear responses.

ILLUSTRATION 2

Individual Responses For
Selector Switch Position 1

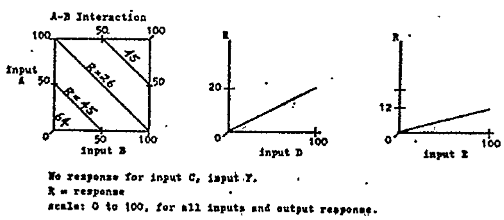
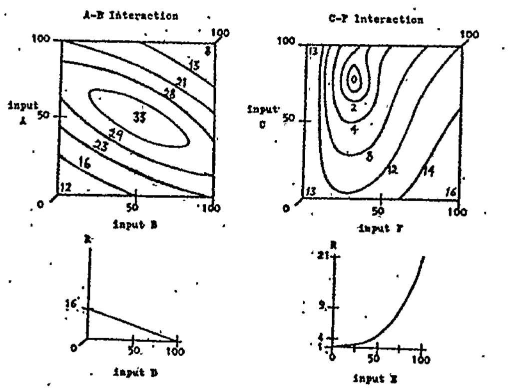


Illustration 3 shows the responses used when the Selector Switch is in position 6. Two nonlinear interactions are produced in conjunction with inputs A and B and inputs C and F. Input D has a decreasing linear response. Input E gives an exponential response.

ILLUSTRATION 3

Individual Responses For
Selector Switch Position 6



The random error generator incorporates a unique design. A chatter-jammer type circuit was modified to produce random variations. Certain transistors, operated in the avalanche condition, give a pink noise response. Pink noise, a special class of white noise, consists of all frequencies in the audio spectrum and is considered to be a pure Gaussian random signal. An operational amplifier was used to match the generator's output impedance and to amplify the signal.

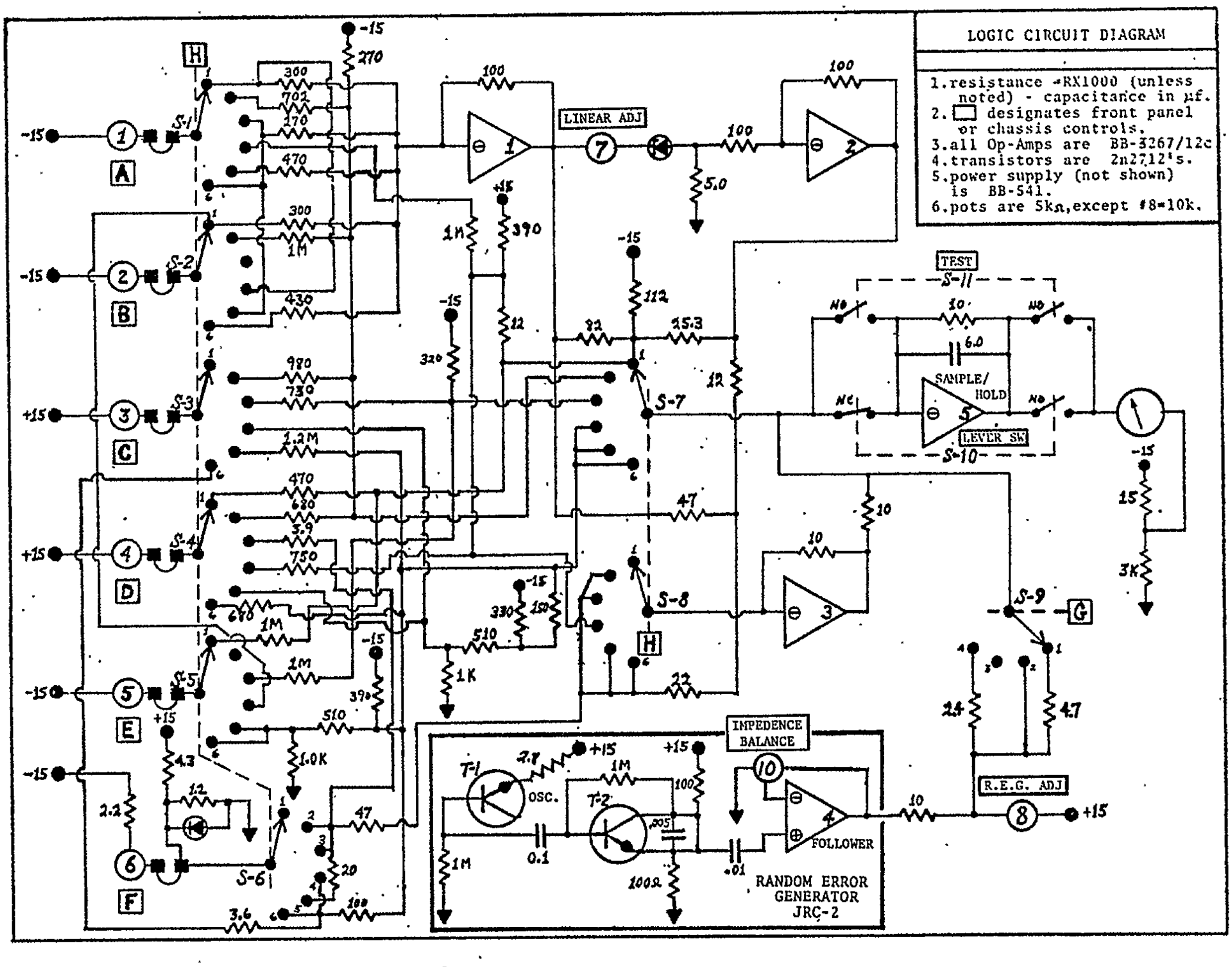
All active generator outputs, along with the random error signal, are fed to a final operational amplifier which acts as both a summer and a track/hold unit. The track/hold circuit was incorporated to prevent the student from reading the response directly from the output meter when the inputs were being set.

Four operational amplifiers produce the eight basic functions. OA1 and OA2, in the circuit diagram of Illustration 4, generate the linear interaction and the peaking function. OA1 and OA2 are used with other circuitry to produce some of the nonlinear interactions. OA3 is both an inverter and a generator for the exponential response. It is also used in the generation of nonlinear interactions. OA5 adds together all the generated functions, outputs the final response surface, and acts as a sample/hold circuit.

A lever switch, located on the front panel, operates the sample/hold circuit. When depressed, it samples and holds the current response surface at the output of OA5, moves the response to the output meter and disconnects all inputs to OA5.

ILLUSTRATION 4

Logic Circuit Diagram



III. SIMULATOR OPERATION

To operate the Black Box the student would set the inputs, following one of the response surface designs pertinent to the particular problem being solved, press the lever switch to connect the sampled output value to the meter, and record the reading. He would then release the lever switch and set the inputs for the next sampling run.

Additional circuitry includes a test switch which allows the instructor to bypass the sample/hold circuit and to observe all responses and random error signal fluctuations on the output meter. Trim pots are used for calibration and adjustment of the random error generator and the functional interactions.

Long-term reliability and stability tests were run. Randomness of the error generator was measured using Spectral Analysis and Auto Correlation Analysis in conjunction with sampled output data. The data consisted of a continuous sampling of up to 5,000 data values. Test results verified circuit reliability and a near perfect randomness for the Random Error Generator.

IV. CONCLUSIONS

The RSG has proved to be an effective learning tool. It illustrates the effectiveness of systematic experimentation in finding the optimum conditions of a process. It demonstrates that experimental design is better than the Edisonian Approach, the hunt-and-peck method. During the RSG's short period of classroom use, the instructor reports that the students have gained a better understanding of Response Surface Design Technology and its operation. The change of pace from the normal class lecture and the greater class involvement and participation required by its use seems to have increased the enthusiasm and learning potential of the student.

ACKNOWLEDGEMENTS

Professor Arthur Hoerl
Statistics and Computer Science Dept.
University of Delaware