SIMULATION OF A CORPORATE CASH BUDGET: APPLICATION AND VALIDATION

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ABSTRACT

The importance of cash budgeting has been thoroughly discussed in the financial literature, but the application of this technique to actual problems has not been fully developed. The failure to consider the random nature of certain critical financial variables such as sales and purchases is the concern of this paper. Frequently, it is suggested that the financial manager make point estimates of relevant variables such as gross sales and based upon these estimates calculate the resulting net cash flows. The authors of this paper question the validity of point estimates for use in short-term financial decision making.

This paper describes the results of an actual simulation of a firm's cash budget. The mathematical model utilized was developed previously by the authors. The modification and application of the model to a specific firm is described along with a step by step description of the actual cash budget simulation. The results of this application are presented along with a discussion of the model validation and a comparison of simulated versus actual end-of-month cash flows. The importance of this research lies in the resulting ability to provide the financial decision maker with a probability distribution of cash flows. Utilizing the probability distribution, the financial manager can select a short-term financial strategy consistent with his attitude toward risk.

INTRODUCTION

Academicians in the field of Management Science have been devoting a great deal of attention toward the creation of sophisticated mathematical models, which in theory appear quite realistic; these academicians have quite often suggested the "universal" or "general" applicability of their works to "real-world" problems and then, feeling as if their work is done, moved on to other projects--never testing or supporting the alleged universality or generality of these models.

This paper presents a description of the actual implementation of the authors' own generalized model, which the authors suggested in an earlier work [2]. The authors' implementation of this model resulted not from the pecuniary rewards expected from a consulting project or a strong commitment to support the contentions of their previous works, but rather, from a sense of responsibility and questioning on their own part as to "Does it really work?" and "How tough is it to really implement this thing?" The findings of the authors in their search for the usefulness of their model building efforts follow.

The remainder of this paper is presented in eight major sections. The first section is devoted to a discussion of the traditional use of cash budgeting--pointing out some of the weaknesses therein. The second section briefly describes the original mathematical model upon which this application is based. The third, fourth, fifth and sixth sections are devoted to description of the firm to which the model was applied, fitting the model to the firm, describing the simulation model and estimating the required variables needed, respectively. The seventh section discusses the results along with model validation. The final section summarizes the significance of the research and suggests certain guidelines for further applications.

TRADITIONAL CASH BUDGETING TECHNIQUES

Cash budgeting is an important tool used in making short-term financing decisions. Most corporate finance texts devote their attention to the development of procedures for budgeting cash [8, 9]. Typically the approach suggested is to calculate monthly cash inflows and outflows based upon monthly sales forecasts. The net difference between the cash inflow and cash outflow for corresponding months is the net cash flow. The value of the net cash flow for each month is then compared to the desired cash balance and, if the net cash flow minus the desired cash balance is negative, the financial decision maker attempts to arrange for the necessary financing. If the difference between the net cash flow and the desired cash flow is positive, the financial decision maker may plan strategies in order to reduce the cash balance--i.e., dividend payment, purchase of marketable securities or stock repurchase.

The textbook approach described above does not successfully account for the uncertainty of the forecasts. Under the present system the financial decision maker is given a series of monthly cash flow estimates. The uncertainty of these estimates must be considered in order to provide the decision maker with a more accurate description of his decision environment.

In order to overcome these drawbacks, some authors [7, 8] have suggested calculating the cash budget based upon a number of forecasts, i.e., optimistic, most likely and pessimistic forecasts. This approach certainly represents an improvement over the use of a point estimate. Other authors [1, 3, 4, 6] have suggested the use of simulation in order to develop a range of possible outcomes and associated probabilities. Eugene M. Lerner, in a 1968 article [3], illustrated the application of simulation to cash budgeting utilizing a simple example. Lerner emphasized the use of simulation to develop expected monthly values of a firm's cash balances. The standard deviations of the cash balances were also provided. The main shortcoming of Lerner's approach was the failure to utilize fully the information provided.

A more recent study by Laurence J. Moore and David F. Scott, Jr., [4], emphasized the value of providing the decision maker with more than merely the parameters (mean, standard deviation) of the distribution; but rather, providing the decision maker with information allowing him to answer questions such as "What will the minimum cash flow be in month x with 90% confidence?" This approach is superior to the use of point estimates or parameters of a simulated distribution. The major shortcoming seen in Moore and Scott's work lies in the mathematical model and the variables simulated. Their approach involved simulating a firm's gross cash inflows and gross cash outflows and then netting these figures to obtain net cash flow. These authors believe it would be more reasonable to develop a mathematical model to explain the resulting gross cash flows and then to generate the key stochastic variables and measure their effects on the net cash flows.

GENERAL MATHEMATICAL MODEL

The generalized cash budgeting model which was developed and described in detail in an earlier paper [2] is summarized in Equations (1) through (7) below.

$$I_t = \sum_{n=1}^{m} [j_{n,t}\,(c\cdot(1-e_o)) + \sum_{i=1}^{p} [f_i(1-(g_i+e_i)) \cdot j_{n,t-i}] - d\cdot k\cdot j_{n,t-1}] + a_t \qquad (1)$$

Where: I_t = Cash Inflow in period t

c = % of sales that were for cash
e_i = % of returns of merchandise for cash which resulted from a sale with time lag of i periods, i = o,p.
p = the number of period time lags considered.
f_i = % of sales which are made for credit whose payments lag i months, i = o,p.
g_i = % of returns of merchandise for credit with a time lag of i months.
$j_{n,t-i}$ = actual dollar sales of product n in time period t.
d = % of sales for which the cash discount is taken.
k = % cash discount given for early payment.
a_t = other cash income in time period t.
m = number of product lines.

$$\lambda_{n,t} = b_{n,t+\omega_n} + (j_{n,t-1} - b_{n,t-1}) \qquad (2)$$

Where: $\lambda_{n,t}$ = amount of each product n to be produced in time period t.
ω_n = production lead time in months for product n.
$b_{n,t+\omega_n}$ = monthly sales forecast in dollars for product n in time period $t+\omega_n$.

$$P_t = \sum_{n=1}^{m} \alpha_n \lambda_{n,t} \qquad (3)$$

Where: P_t = purchases in time period t.

α_n = % of expected sales represented by purchases for product n.

$$\pi_t = \sum_{n=1}^{m} w_n \lambda_{n,t} \quad (4)$$

Where π_t = wages in time period t.

w_n = % of total production cost represented by wages.

$$O_t = \ell \cdot P_t(1-s_o) + \sum_{i=1}^{p} [u_i(1-(s_i+v_i)) \cdot P_{t-i}] - q \cdot r \cdot P_{t-1} + \pi_t + X_t + Y_t + Z_t + \beta_t + \delta_t + \theta_t + D_t \quad (5)$$

Where: O_t = Outflow in time period t.

ℓ = % of purchases made for cash
s_i = % of purchases made which are returned with a time lag of i periods, i = 1,p.
u_i = % of purchases made on credit whose payments lag i months, i = 1,4.
v_i = % of purchases made in month i which are returned for credit in time period t where i = t-j; j = 1,p.
q = % of purchases on which the cash discount is taken
r = % cash discount on purchases
X_t = salary payments in time period t
Y_t = interest payments in time period t
Z_t = principal payment in time period t
β_t = lease payment in time period t.
δ_t = capital expenditure in time period t
θ_t = research and development expenditure in time period t
D_t = cash dividend paid in time period t

$$\Delta_t = I_t - O_t + \Gamma_t + T_t + \Phi_t \quad (6)$$

Where: Δ_t = Net Cash Flow in period t.

Γ_t = new bond issues (+) or retirements (-) in time period t.
T_t = new equity issues (+) or retirements (-) in time period t.
Φ_t = new term loans (+) or retirements (-) in time period t.

$$\nu_t = \Delta_t + \psi_t \quad (7)$$

Where: ν_t = ending cash in period t.

$\psi_t = \nu_{t-1}$ = beginning cash in period, which is actually the previous period's ending cash.

The generalized mathematical model was constructed in order to account for an n product firm, with specified lags in receipt of payment on accounts receivable and payment of accounts payable. As can be seen, other relevant variables including cash discounts, returns, financing capital expenditures, dividends, etc, have been included.

The only variable initially generated and assumed stochastic when applied to hypothetical data was the firm's sales. The generation of this variable was approached in a rather unique manner which involved generating a percent forecast error from an assumed a priori distribution and adjusting forecast sales by the resulting error to get actual sales. Utilizing hypothetical data the initial testing of the model was quite simple, but in actual application a number of variables had to be eliminated and certain additional variables--initially assumed deterministic had to be treated as stochastic.

SELECTION OF A FIRM FOR APPLICATION OF THE MODEL

A number of business firms were contacted and the possibility of application of the cash budgeting simulation model discussed. Some of the firms were not interested, others considered the information requested to be proprietary and another group indicated that they did not have sufficient time to devote to supplying the data required to implement such a model. The interest of the firms in this project seemed to be closely related to the disposition of the contacted manager toward the use of quantitative approaches for solving business problems.

Of the three firms willing to cooperate, an office equipment merchandising concern selling three basic product lines was selected; the application of the model to this firm appeared to be the most straight forward and the necessary data was available.

This office equipment firm (heretofore referred to as "the firm") had annual sales of approximately $17,000,000 divided between its three product lines--1) office furniture, 2) office supplies, and 3) other. Office furniture consisted of desks, chairs, and file cabinets; while office supplies consisted of staplers, paper forms, file folders, etc. The "other" classification consisted of stock forms such as accounting pads and legal pads. Since the firm accounted for each of these product lines separately, the type of information required was readily available.

FITTING THE MODEL TO AVAILABLE DATA

Upon careful analysis of the available data and product line configuration, a number of changes required for the initial model were determined. Alteration of various stages of the model in response to these required changes were made; each of the resulting changes and the justification thereof are discussed separately below.

A. CASH INFLOW: As a result of meetings with the company officials, the following facts pertaining to cash inflows were revealed:

1) Cash discounts are not extended to their customers.

2) Both cash sales and other income were so small as to be insignificant.

3) Any sales returns and allowances are netted into the monthly sales figures provided by the company.

4) The three basic product lines were office furniture, office supplies and other items.

5) The monthly percentages of lagged credit collections were 60%, 30%, 5% and 5%; this indicates that 60% of accounts receivable are collected with a one month lag, 30% with a two month lag, 5% with a three month lag and 5% with a four month lag.

6) Bad debt expenditures were considered insignificant.

With the above factors in mind, the new expression for cash inflow in time period t, I_t was developed and is presented below:

$$I_t = \sum_{n=1}^{3} \sum_{i=1}^{4} f_i(j_{n,t-i}) \qquad (8)$$

Where: f_i = percent of sales whose payments lag i months.

$j_{n,t-i}$ = actual dollar sales of product line n in time period t-i.

Actual sales, $j_{n,t-i}$, is a stochastic variable which is generated while the percentages of lagged credit collections are treated deterministically.

B. CASH OUTFLOW: The following facts concerning cash outflow were determined through discussions with the firm's management.

1) Cash outflows can be exhaustively classified as purchases, commissions, overhead and capital expenditures.

2) All purchases are made on a credit basis, i.e., no cash purchases exist.

3) Payments for purchases lag one month behind the actual purchase.

4) Purchase discounts and returns are netted into the purchase figures provided.

5) Sales Commissions are paid in the month following their incurrence.

Since commissions are paid on a sliding scale to each salesman, a direct rate applicable to total sales was not available.

Utilizing the facts presented above, the expression for cash outflow was developed and is given in Equation (9).

$$O_t = P_{t-1} + C_{t-1} + OV_t + S_t \tag{9}$$

Where: P_{t-1} = Purchases occurring in time period t-1 which are paid for in time period t.

C_{t-1} = Commissions earned in time period t-1 which are disbursed in time period t.

OV_t = Overhead incurred and paid for in time period t.

S_t = Capital expenditures made in period t.

In the simulation model purchases, commissions and overhead are treated as stochastic variables; while capital expenditures are deterministic.

C. NET CASH FLOW: The net cash flow is calculated by adjusting the difference between inflows and outflows in each period for the net changes in liquidity not accounted for in the mathematical model. Equation (10) illustrates this relationship.

$$\Delta_t = I_t - O_t + \Delta L_t \tag{10}$$

Where: ΔL_t = Net change in the firm's cash position not accounted for in the mathematical model, but occurring in time period t.

In the simulation model, ΔL_t is treated as a deterministic variable the values of which were provided by the firm's management.

D. ENDING CASH BALANCE: The ending cash balance is determined by adding the preceding periods ending cash to the net cash flow. See Equation (11).

$$\nu_t = \Delta_t + \nu_{t-1} \tag{11}$$

Where: ν_t = the ending cash balance in period t.

Equation (12) combines the preceding functional relationships into one mathematical expression.

$$\begin{aligned}
\nu_t = & \sum_{n=1}^{3} \sum_{i=1}^{4} [f_i(b_{n,t-i} + a_n(b_{n,t-i}))] && \text{Inflows} \\
& -[d(j_{1,t-i}) + \sum_{n=2}^{3} e_n(b_{n,t-i})] && \text{Purchase Outflows} \\
& - [\sum_{n=1}^{3} h(j_{n,t-i})] && \text{Commission Outflows} \\
& - [BOV_t + k(BOV_t)] && \text{Overhead Outflows} \\
& \pm \Delta L_t && \text{Change in Liquidity} \\
& - \delta_t && \text{Capital Expenditures} \\
& + \nu_{t-1} && \text{Beginning Cash}
\end{aligned} \tag{12}$$

Where: f_i; $j_{n,t-i}$, ΔL_t and δ_t as previously defined.

n=1 for office furniture

n=2 for office supplies

n=3 for other

$b_{n,t}$ = sales forecast of product n in time period t (provided by company management)

BOV_t = budgeted overhead in time period t (provided by company management)

a_n = stochastic variable representing the sales forecasting error of management stated as a percentage for each product line n

d = stochastic variable representing the value of office furniture purchases as a percentage of actual office furniture sales.

e_n = a stochastic variable representing the value of purchases as a percentage of forecast sales for product line n, where n equals 2 or 3.

h = a stochastic variable representing the values of sales commissions stated as a percentage of actual sales

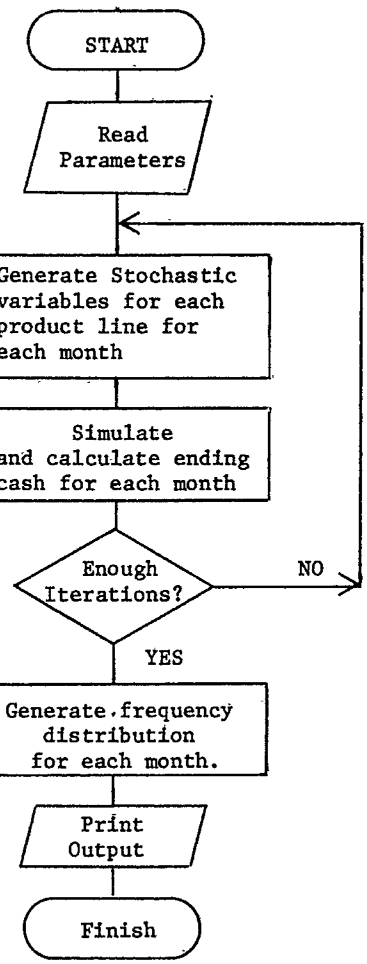
k = a stochastic variable representing the forecast budget error stated as a percentage.

THE SIMULATION MODEL

As indicated in Illustration 1, the logic of the simulator is rather simplistic. Initially, all input parameters are read by the model. These parameters include such variables as monthly sales forecast for each product line, probability distribution parameters for purchases, commissions, etc. Next the various stochastic variables are generated and input to the mathematical model to calculate net cash flow and ending cash balance. The process is repeated for each time period (each month). When enough iterations have been run, a frequency distribution of net cash flows and ending cash balances is determined and printed.

ILLUSTRATION 1

Flowchart of Cash Flow Simulator



ESTIMATION OF STOCHASTIC VARIABLE DISTRIBUTIONS AND ASSOCIATED PARAMETERS

In order to determine the parameters and types of distributions of each of the stochastic variables--a_n, d, e_n, h, and k--twenty-four monthly values for: actual and forecast office furniture sales; actual and forecast office supply sales; actual and forecast other supplies sales; actual office furniture purchases; actual office supply purchases; actual other supplies purchases; actual and budgeted overhead; and actual commissions were utilized.

By using curve fitting routines and testing the empirical distribution against known distributions using the Kolmogorov-Smirnov goodness of fit test the parameters and types of distributions for each stochastic variable were determined. These results are presented in Table 1.

TABLE 1

Distributions and Parameters for Stochastic Variables

Variable	Description	Type of Distribution	Mean	Standard Deviation	Max $\lvert F_x(x) - S_t(x)\rvert$*
a_1	Forecast Error Furniture	Normal	-.002	.322	.0520
a_2	Forecast Error Supplies	Uniform	-.150	.025	.0500
a_3	Forecast Error Other	Uniform	.055	.253	.0556
d	Office Furniture Purchases	Normal	.789	.158	.0830
e_2	Office Supplies Purchases	Uniform	.596	.109	.083
e_3	Other Purchases	Exponential	.172	.178	.0656
h	Sales Commisions	Normal	.0766	.0119	.0636
k	Budget Overhead Error	Uniform	.001	.064	.2000

*The critical value of $\lvert F_x(x) - S_t(x)\rvert$ with an α of .2 was .295 where:

$F_x(x)$ = cumulative frequency distribution of the theoretical distribution

$S_t(x)$ = cumulative frequency distribution of the observed distribution

MODEL VALIDATION AND VERIFICATION

Simulation model verification and validation is perhaps the most frequently overlooked task in designing and applying a digital simulator. The reason verification is neglected "is that the problem of verifying simulation models remain today perhaps the most elusive of all the unresolved problems associated with computer simulation techniques." [5, p. 310]. Unless a researcher has some degree of confidence that his model accurately represents the "real-world" system, he cannot advocate its use in making decisions concerning the actual system.

In order to attain a sufficient degree of confidence in the model, four steps have been taken. Firstly, the logic of the driver program and all the subroutines have been thoroughly tested to insure that the model is performing as designed with the absence of coding errors as well as logic errors. Secondly, each process generator used to generate the various stochastic variables was tested using Kolmogorov-Smirnov non-parametric goodness of fit test. All generating functions easily passed the test. The third step in the verification procedure has required the replication of the experiments and one way analysis of variance. The simulation model has been run ten times with all controllable variables, except the random number seed, held constant. Before each run, the random number seed has been replaced by another random number. Table 2 reflects the results of these ten verification runs. The null hypothesis is that all ten runs produces expected ending cash balances which were equal. As depicted in Illustration 2, the null hypothesis cannot be rejected. The fourth step taken can be classified as historical verification. Basically, the mean of ending cash balances from the simulation run were compared to actual ending cash balances of historical data on which the model parameters are based. The rationale behind this comparison was not to prove statistically that the residual errors were small enough to be treated as merely sampling errors; but rather, to examine the error to determine whether they were within the tolerance limits of a financial decision maker. Table 3 presents the actual and simulated cash flows, along with the standard deviation of the simulated cash balances. Also included are the associated absolute and percent errors for the ten months simulated. The actual and simulated values are plotted in Illustration 3. These results indicate that if management can live with 5-10% tolerance, this model should be quite useful.

CONCLUSIONS

The use of simulation in cash budgeting allows the financial decision maker much more flexibility in making short-term financing decisions. Instead of merely a few point estimates of ending cash flow expected for each month, the model described in this paper provides the decision maker with a broad range of ending cash flow values along with the probability of occurrence of the respective values.

Both the mathematical model and the simulation scheme presented have illustrated the applicability of simulation to the cash budgeting problem. By applying the simulation model to an actual firm, the authors have demonstrated the relative ease with which the model can be molded to a particular company and utilized in a very meaningful and practical manner.

TABLE 2

Results of Validation Replication Runs

Expected Values of Ending Cash Balance (in Thousands of Dollars)

RUN NO.	JAN.	FEB.	MAR.	APR.	MAY	JUNE	JULY	AUG.	SEP.	OCT.	NOV.	DEC.
1	175.65	238.98	226.17	221.80	240.05	212.99	35.24	94.64	74.92	57.93	69.85	13.01
2	175.71	238.95	226.14	222.14	239.99	212.79	35.43	94.90	75.07	58.17	70.08	13.19
3	175.60	238.66	225.90	221.91	239.82	212.69	35.07	94.77	75.17	58.10	70.08	13.20
4	175.51	238.45	225.51	221.32	239.25	212.20	34.77	94.33	74.92	57.81	69.75	12.90
5	175.49	238.43	225.56	221.35	239.27	212.17	34.69	94.14	74.51	57.55	69.48	12.63
6	175.34	238.24	225.63	221.26	239.38	212.52	34.75	94.25	74.68	57.63	69.58	12.74
7	175.47	238.61	226.07	221.57	239.55	212.91	35.11	94.40	74.84	57.82	69.73	12.92
8	175.25	238.02	225.56	221.54	239.56	212.45	34.92	94.10	74.46	57.51	69.43	12.62
9	175.50	238.65	226.17	222.14	240.19	213.10	35.44	94.51	74.71	57.83	69.73	12.92
10	175.51	238.70	226.19	221.99	239.90	212.58	34.80	94.27	74.70	57.68	69.66	12.83

TABLE 3

Comparison of Actual and Simulated Month-End Cash Balances

Month	End of Month Simulated Mean Cash Balance	Standard Deviation of Simulated Cash Balances	Actual Cash Balance	Absolute Error	Percent Error
Jan.	-4525.95	121.503	-4324.75	201.200	4.65229
Feb.	-4593.35	170.970	-4354.11	239.240	5.49458
Mar.	-4752.29	240.017	-4751.28	1.010	0.02126
Apr.	-4772.82	181.830	-5096.90	324.080	6.35837
May	-4997.83	176.294	-5124.79	126.960	2.47737
June	-5195.70	179.203	-5159.53	36.170	0.70103
July	-5104.83	191.315	-5491.21	386.380	7.03634
Aug.	-5594.88	193.823	-5701.96	107.080	1.87795
Sept.	-5782.78	205.829	-5788.01	5.230	0.09359
Oct.	-5745.59	210.685	-5598.99	146.600	2.61833

ILLUSTRATION 2

One Way Analysis of Variance Mean Cash Balance for March

Null Hypothesis $H_0 := \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 = \mu_6 = \mu_7 = \mu_8 = \mu_9 = \mu_{10}$

Alternative Hypothesis H_1: All means are not equal

$\alpha = .05$ = Level of significance

$n = 1000$ = Total number of observations

$k = 10$ = The number of simulation replications

$df_1 = k-1 = 9$ = The degrees of freedom dependent on the number of replications

$df_2 = n-k = 991$ = The degrees of freedom dependent on the total number of observations

$F\alpha = 1.88$ = Critical Value of F (Table lookup - F Distribution)

$$F = \frac{\hat{\sigma}_c^2}{\hat{\sigma}_\varepsilon^2} = \frac{\text{Variance between replications}}{\text{Variance within each replication}}$$

$$\hat{\sigma}_c^2 = .077432$$

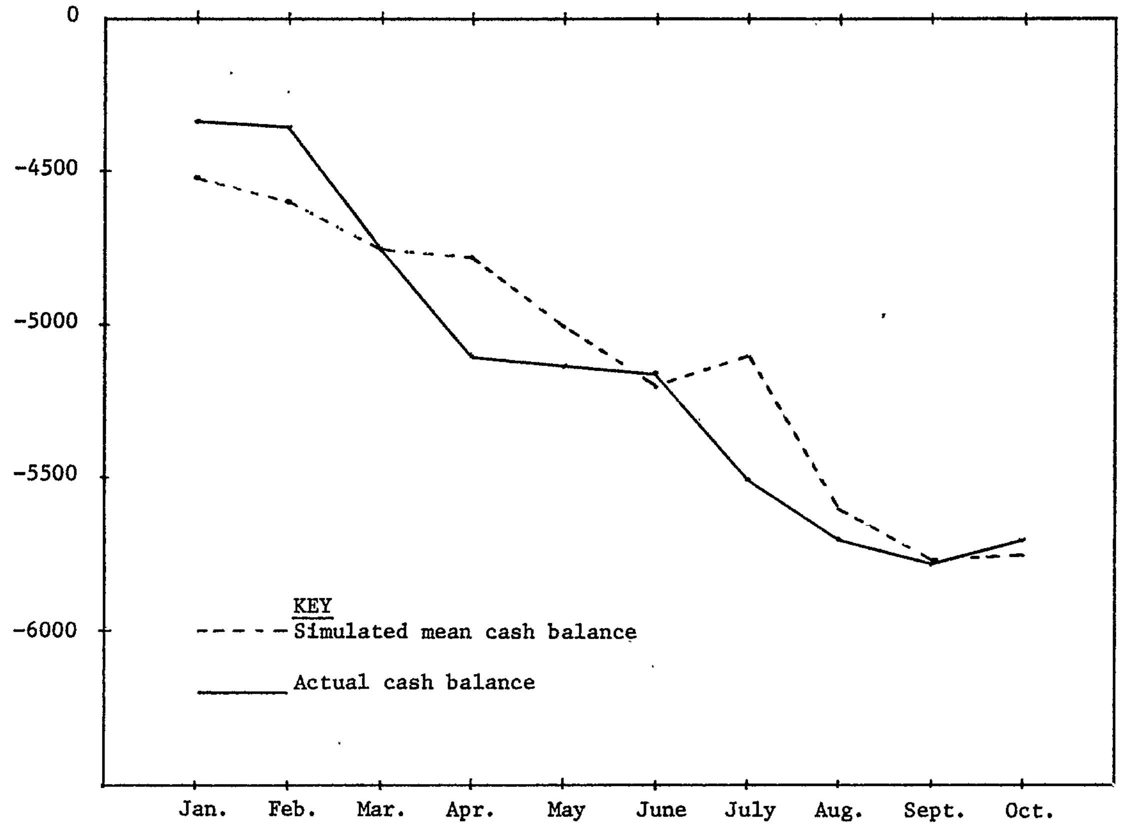
$$\hat{\sigma}_\varepsilon^2 \doteq 146.2334$$

$$F = \frac{.077432}{146.2334} = .00052$$

Because $F < F_\alpha$ the null hypothesis is accepted.

ILLUSTRATION 3

Graphical Comparison of Actual and Simulated
Month-End Cash Balances



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