"MONTE CARLO SIMULATION OF CROSSTALK IN COMMUNICATION CABLES"

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Abstract

One way of developing a communication cable of new design is to make many experimental cables and to study their path-to-path crosstalk properties. This method is both time-consuming and expensive. In this paper, use is made of evaluating "synthetic" cables. First, we fit statistical models, which are functions of design parameters, to measurements of crosstalk in one or more experimental cable(s). Then we use these models and estimates of associated components of statistical variability in Monte Carlo simulation of crosstalk thereby enabling the exploration of cable designs different from those of the experimental cables.

1. Introduction

Far-end crosstalk is one of the major sources of interference in communication paths. Cable designers would like to develop new communication cables whose path-to-path crosstalk* is at a desired level. One way of developing a communication cable of a new design is to make many experimental cables and to study their path-to-path crosstalk properties. This method is both time-consuming and expensive. Here we describe another approach which develops and evaluates "synthetic" cables.

First we analyze path-to-path crosstalk measurements for experimental cable(s) to estimate components of statistical variability and to fit statistical models, which are functions of design parameters, to these measurements. Then these models are used in Monte Carlo simulation of path-to-path crosstalk for cables with designs different from that (those) for the

* From now on, far-end crosstalk is abbreviated as crosstalk.

experimental cable(s). The individual crosstalk into each path (pair) from the remaining paths are "added" to get overall indices of performance called power sums. The power sum distributions of "synthetic" cables are used to evaluate their designs. This evaluation leads to further exploration of new cable designs.

2. Data Collection

Statistical analysis of crosstalk measurements from one or more experimental cables is the backbone of the approach described here. An experimental low capacitance cable was designed and manufactured during the course of recent development of a new transmission system. We will use this cable to illustrate the approach. A cross-section of this cable is shown in Figure 1. This cable has 102 pairs and it is made up of six layers. In each layer there is a certain number of distinct twist lengths which are repeated a certain number of times (Table 1). The six layers are stranded (i.e., the pairs in each layer go around the center) alternately in the right and left directions.

The crosstalk measurement equipment was limited physically to measure not more than 50 pairs in a setup and it was feasible to measure only two 50-pair sets in total. The two 50-pair sets chosen for measurement (yielding $2450=2\binom{50}{2}$) pair-to-pair crosstalk measurements) are indicated in Figure 2. These two sets were selected to provide sufficient data for estimating the performance of the current cable design and for exploring other twist length selections in future cable designs.

3. Model Development and Simulation of Crosstalk

3.1 Model Development

Table 2 gives a breakdown of the 2450 measurements into three groups: (i) within layers (i.e., when both the disturbed pair i and the disturbing pair j belong to the same layer), (ii) between layers when $D_{ij} \leq 3$, where

TABLE 1

Twist Length Assignment (inches)

Layer No.	Total No. of Pairs	No. of Distinct Twists	Distinct Twist No. 1	2	3	4	5	6	7	8	9	Str. Lay* (ins.)
1	3	3	0.9	1.4	1.2							24R
2	9	9	1.8	2.7	6.2	2.0	3.8	1.7	2.9	5.5	2.2	24L
3	15	3	1.1	1.2	1.5							24R
4	20	5	7.1	3.0	4.6	2.5	3.6					24L
5	25	5	0.9	1.5	1.2	1.8	1.4					24R
6	30	6	6.2	2.9	2.0	4.4	3.4	2.4				24L

* An effect of stranding lay is to possibly change relative twist lengths of two pairs. This adjustment is made in defining "effective twist lengths" which are used in the analysis to follow.

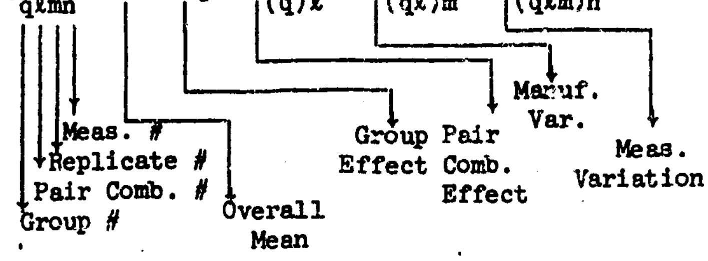
TABLE 2

Breakdown of the 2450 Measurements

Group	No. of Pair Combinations*			
	Distinct	With Information on Manuf. Var.	With Repeat Meas.	Total
Within layers	267	104	49	420
Between layers I ($D_{ij} \leq 3$)	301	1294	87	1682
Between layers II ($D_{ij} > 3$)	87	261	0	348
Total	655	1659	136	2450

D_{ij} is the minimum distance† between the two pairs involved, and (iii) between layers when $D_{ij} > 3$. The 17 pairs common in the two 50-pair sets provide repeat measurements which in turn yield information on the measurement variability. Similarly, identification of distinct pair combinations (see Figure 1) and an analysis of different realizations (if more than one) of each distinct pair combination provides an estimate of the variability due to the manufacturing process. The following model expresses crosstalk measurements (in dB) as sums of several components:

$$C_{q\ell mn} = \mu + G_q + P_{(q)\ell} + \delta_{(q\ell)m} + \epsilon_{(q\ell m)n}$$



* The numbers in the middle two columns are more properly labeled as degrees of freedom.

† The unit of distance is the diameter of a pair,

where $\delta_{(q\ell)m} \sim N\left(0,\sigma^2_{(q)\delta}\right)$ and $\epsilon_{(q\ell m)n} \sim N\left(0,\sigma^2_{(q)}\right)$. Table 3 shows the mean squares due to (i) distinct pair comb., (ii) manufacturing variability, and (iii) measurement variability in each of the three groups. It is clear that manufacturing variability is quite large.

Next we fitted regression functions which attempt to describe average crosstalk for distinct pair combinations as functions of the following design variables:

D_{ij} = minimum distance between pairs i and j where the unit of distance is the diameter of a pair,

T_i = twist length of pair i (in inches),

T_j = twist length of pair j, and

$L_i = \begin{cases} 1 & \text{if pair i is in layer 6} \\ 0 & \text{otherwise} \end{cases}$

A regression function for group q may be written as

$$C_{(q)ij} = f_q(D_{ij},T_i,T_j,L_i)$$

TABLE 3

A Breakdown of Variation by Group

Group	Mean Square*		
	Distinct Pair Comb.	Manufacturing Variability	Measurement Variability
Within layers	122.23 (266)	33.91 (104)	5.39 (49)
Between layers I ($D_{ij} \leq 3$)	225.91 (300)	42.14 (1294)	6.71 (87)
Between layers II ($D_{ij} > 3$)	129.02 (86)	23.24 (261)	-

where $C_{(q)ij}$ = crosstalk from pair j to pair i in group q.

The functional form of f was explored by plotting $C_{(q)ij}$ against the design variables and some simple functions of these variables. Let us illustrate the fitting procedure by considering the within-layer group. For this group the only variable which indicated a significant relationship with C_{ij} (q, which is equal to 1 for within-layer group, is omitted for convenience) is D_{ij}. In other words, if we consider only one variable at a time (and consequently ignore the other variables), we are not able to detect any relationship of C_{ij} with the other design variables. After fitting $C_{ij} = 57.04 + 18.33 \log(D_{ij})$, and plotting C'_{ij} $\left(= C_{ij} - 57.04 - 18.33 \log(D_{ij})\right)$ against the twist length variables, we noticed a negative relationship with $T_i + T_j$ which led to the new regression:

$$C_{ij} = 61.62 + 18.69 \log(D_{ij}) - 6.64 \log(T_i+T_j).$$

Again we computed the new residuals

$$C''_{ij} = C_{ij} - 61.62 - 18.69 \log(D_{ij}) + 6.64 \log(T_i+T_j) ,$$

and made a plot of C''_{ij} against other variables which suggested the addition of $\log(|T_i-T_j|)^{\dagger}$ and L_i. No additional terms, which make further improvement in the explanation of C_{ij}, could be found. Thus, the best fitted statistical model for within-layer crosstalk is

$$C_{ij} = 64.08 - 3.53L_i + \log_{10}\left\{\frac{D_{ij}^{18.94}\ (|T_i-T_j|)^{2.45}}{(T_i+T_j)^{8.83}}\right\} + e_{ij} \qquad (1)$$

where

$$e_{ij} = \delta_{ij} + \varepsilon_{ij} + \text{lack of fit, and}$$

$$e_{ij} \sim N(0,49.05).$$

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* The numbers in parentheses are degrees of freedom.

† If $|T_i-T_j| < 0.05$, then we substitute $|T_i-T_j| = 0.05$.

Lack of fit represents the discrepancy between the fitted model and the true relationship (if one exists) between C_{ij} and the design variables. The above fitted model does not fit the average crosstalk for distinct pair combinations perfectly. However, the lack of fit is small and the above fitted model is quite useful in describing crosstalk as a function of design variables.

Similarly, the best fitted models for the between layers groups were the following:

Between layers I ($D_{ij} \leq 3$):

$$C_{ij} = 62.35 + 3.47L_i + \log_{10}\left\{\frac{D_{ij}^{23.34}\cdot(T_i+T_j)^{17.06}}{(T_i\cdot T_j)^{12.68}\cdot(T_j)^{2.16}}\right\} + e_{ij} \quad (2)$$

where $e_{ij} \sim N(0,49.42)$.

Between layers II ($D_{ij} > 3$):

$$C_{ij} = 59.97 + 7.53L_i + \log_{10}\left\{\frac{D_{ij}^{40.49}\cdot(|T_i-T_j|)^{1.96}}{(T_i+T_j)^{17.30}\cdot T_j^{5.33}}\right\} + e_{ij} \quad (3)$$

where $e_{ij} \sim N(0,27.49)$.

It may be pointed out that $j < i$ and since the pairs are numbered from the center outward, j is in the outer layer relative to i.

3.2 Simulation of Pair-to-Pair Crosstalk

The statistical models developed above can be used for Monte Carlo simulation of pair-to-pair crosstalk as follows:

(a) Specify D_{ij}, T_i, T_j, and L_i for the pair combination.

(b) Compute the expected crosstalk from the appropriate one of the above three fitted models (without e_{ij} term).

(c) Add e_{ij} to the expected crosstalk where $e_{ij} \sim N\left(0,\sigma_e^2\right)$, and

$$\sigma_e^2 = \begin{cases} 49.05 & \text{for within-layer} \\ 49.42 & \text{for between-layer I } (D_{ij} \leq 3) \\ 27.49 & \text{for between-layer II } (D_{ij} > 3). \end{cases}$$

We generate a random number, r_{ij}, from $N(0,1)$ through the use of a random number generator on a computer and define $e_{ij} = (r_{ij})\cdot(\sigma_e)$.

By generating 5151 pair-to-pair crosstalk "measurements" among 102 pairs in the cable we can develop a synthetic cable of a specified twist length design.

4. Exploration of New Cable Designs

4.1 Power Sum Distribution

Before exploring new cable designs for the 102-pair cable, it is necessary to assess the performance of the design used in the experimental cable. A measure of the overall crosstalk interference in a pair is called the power sum, which is defined below. Let C_{ij} = crosstalk from pair j to pair i, $j \neq i$. Then

$$P_i = \text{Power sum for pair } i = -10\log_{10}\left\{\sum_{j\neq i} 10^{-C_{ij}/10}\right\}$$

Unfortunately, we do not have all C_{ij} measurements for $j \neq i$. Therefore, we cannot compute all P_i's from the 2450 crosstalk measurements for the experimental cable discussed in Section 2. First we simulate the unmeasured crosstalk as described in Section 3 and then compute P_i(i=1 to 102). Figure 3 shows a plot of P_i on normal probability apper. The power sum distribution is an indication of the overall performance of the cable. For example, the worst pair has a power sum of 39 dB and the best pair has a power sum of 48 dB. The left-hand tail is the important tail of the distribution because the crosstalk requirement is stated as follows:

$$\text{Prob.}(P_i < P_0) \leq \alpha.$$

Since the simulated crosstalk and consequently simulated P_i are random variables, the plot in Figure 3 represents one realization of the power sum distribution for the design used in the experimental cable. If we make another simulation of the unmeasured crosstalk, recompute P_i and plot the resulting power sum distribution, we will get another realization of the power sum distribution. Figure 4 shows 20 realizations of this distribution (one of which is shown in Figure 3) for the experimental cable design. It can be seen that the tails of the power sum distribution are quite variable and consequently it is not efficient to assess the performance of a given design on the basis of one realization of its power sum distribution. The average of the 20 realizations displayed in Figure 4 is shown in Figure 5 along with three other similar average distributions. It can be seen that the four average distributions are quite close to each other. Therefore, we can use the average of 20 realizations of power sum distributions for comparing one cable design with another cable design.

4.2 Validation of Simulation

The simulation of power sums discussed above is based on the statistical models developed in Section 3 which were judged to be quite useful in describing pair-to-pair crosstalk. Can we investigate the validity of the simulation of power sums more directly? Figure 6 shows four realizations of the average power sum distribution for each of the following two cases:

(i) When only unmeasured pair-to-pair crosstalk is simulated from the fitted regression models (also shown in Figure 5), and

(ii) When all pair-to-pair crosstalk are simulated from the fitted regression models.

The above two sets of distributions have a considerable amount of overlap which indicates that the power sum distribution is not affected if we replace available measurements of pair-to-pair crosstalk by corresponding values simulated from the fitted models. In other words, the simulation of the power sum distribution appears to be valid for practical purposes.

4.3 New Cable Designs

As discussed above, the statistical models fitted to measurements of crosstalk in the experimental cable lead to a proper simulation of pair-to-pair crosstalk and power sum distribution for this cable design. In this section we will assume that the fitted models are also valid for new (i.e., different from that in the experimental cable) 102-pair layer type cable designs. To minimize the lack of applicability of the fitted models, we will not consider twist lengths outside the range of those of the experimental cable.

There are 26 distinct twist lengths in the experimental cable. Since it is expensive to keep an inventory of a large number of distinct twist lengths, the cable manufacturing organization would prefer to use fewer twist lengths if crosstalk can be kept within specified limits. The exploration of new cable designs is discussed below.

First, let us consider the "tuning up" of the design used in the experimental cable. What changes in twist length assignment will improve the power sum distribution? To help answer this question let us examine Figure 7 which shows a plot of the average (of 20 "synthetic" cables) power sum for each of the 102 pairs. It can be seen that the worst power sums correspond to the pairs in layer 4. Reviewing Table 1, Figure 1 and Model (1) given earlier, the following facts may explain why the pairs in layer 4 have the worst power sums:

(i) Layer 4 pairs have the largest number of neighbors in adjacent layers.

(ii) Layer 4 pairs have long twist lengths which result in large within-layer crosstalk.

We cannot change the number of neighbors in adjacent layers for pairs in Layer 4. However, we can assign short twist lengths to Layer 4, thereby improving crosstalk within this layer. This was done by selecting short twist lengths for layers 2, 4 and 6 and long twist lengths for layers 1, 3 and 5. In addition, the number of distinct twist lengths in layer 2 was reduced from 9 to 5 and the longest twist length (7.14") was eliminated. The resulting design, labeled as Design (1) (see Table 4), improved the worst average power sums by 1 dB as shown in Figure 8 and Table 5. The number of distinct twist lengths has been reduced from 26 for the original design to 23 for Design (1).

Next, let us consider the exploration of designs with considerably fewer distinct twist lengths. By assigning the same twist lengths to alternate layers, we can do with 10 distinct twist lengths. Unlike the original design and Design (1), stranding lays of alternate layers are made different to make "effective twist lengths" different for these layers. Among the 10-twist length designs considered, Design (2) (see Table 4) was found to be the best. The average power sum distribution corresponding to Design (2) was still better than that for the original design with 26 distinct twist lengths (Table 5).

TABLE 4

Twist Length Assignments

	Original Design		Design (1)		Design (2)	
Layer No.	Range of TL	Str.	Range of TL	Str.	Range of TL	Str.
1	0.90-1.39	24R	2.96-6.25	24L	1.18-5.46	25L
2	1.77-6.25	24L	1.07-2.17	24R	0.90-4.36	35R
3	1.07-1.48	24R	2.43-5.46	24L	1.18-5.46	35L
4	2.50-7.14	24L	0.90-2.05	24R	0.90-4.36	20R
5	0.90-1.77	24R	2.30-4.90	24L	1.18-5.46	25L
6	2.05-6.25	24L	1.07-2.17	24R	0.90-4.36	35R
No. of Dist. Twists	26		23		10	

5. Summary

The crosstalk measurements performed on the experimental cable were chosen to provide estimates of inherent variability due to the measurement and manufacturing processes as well as to provide sufficient data for fitting regression models. Choice of regression variables was aided by plotting crosstalk and residual crosstalk against design variables. Resulting fitted models have a small lack of fit and were judged adequate for describing pair-to-pair crosstalk.

These fitted models and an appropriate random component of variability were used to simulate pair-to-pair crosstalk. Crosstalk in each pair from the remaining 101 pairs were "added" to get an overall index of interference called the power sum. The distribution of power sums for all pairs of a cable is a measure of the performance of the cable design which it represents. The power sum distribution was found not to be sensitive to replacement of measured crosstalk by simulated crosstalk, which fact supported the validity of the Monte Carlo simulation.

New designs have been developed for the 102-pair layer type cable by building and analyzing "synthetic" cables. Two of these are reported herein. Recall that there were 26 distinct twist lengths in the experimental cable. The first of the two new designs uses 23 distinct twist lengths and yields an improvement of 1 dB in the worst power sums, an improvement of engineering significance despite appearing to be small. The development of the second design

TABLE 5

Simulated Power Sum Distributions
For Several Designs

Ordered Power Sum No.	Simulated Power Sum (dB) For Length 1000 Ft.		
	Original Design	Design (1)	Design (2)
1	36.20	37.22	36.90
2	36.70	37.67	37.37
3	37.66	38.42	38.04
4	38.05	38.92	38.39
5	38.54	39.24	38.66
.	.	.	.
.	.	.	.
10	39.75	40.02	39.55
.	.	.	.
.	.	.	.
20	40.97	40.91	40.77
.	.	.	.
.	.	.	.
30	41.69	41.63	41.43
.	.	.	.
.	.	.	.
50	42.83	42.63	42.69
.	.	.	.
.	.	.	.
102	49.90	48.41	48.96

indicates that it may be possible to reduce the number of distinct twist lengths from 26 to 10 without degrading crosstalk power sums for the worst pairs. Such a reduction holds promise for the cable manufacturing organization for reasons of cost.

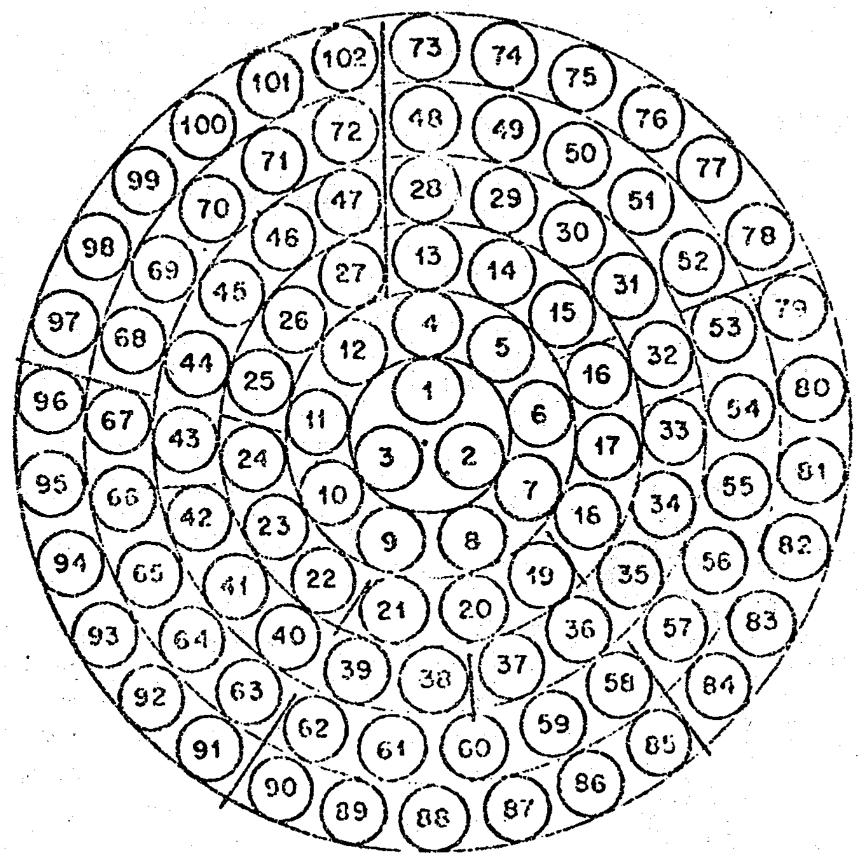

FIGURE 1 CONFIGURATION OF 102-PAIR LAYER CONSTRUCTION CABLE

NOTES 1. In each layer there is a <u>group</u> of <u>distinct twist lengths</u>. Let T_i = twist length of pair i. Then in layer 3: $(T_{13},T_{14},T_{15}) = (T_{16},T_{17},T_{18}) = \ldots = (T_{25},T_{26},T_{27}) = (1.1'',1.2'',1.5'')$.

2. Pair combinations (13,14) and (16,17) are not considered <u>distinct pair combinations</u>. Similarly, (15,31) and (18,36) are not considered distinct pair combinations, whereas (15,16) and (16,17) are distinct pair combinations.

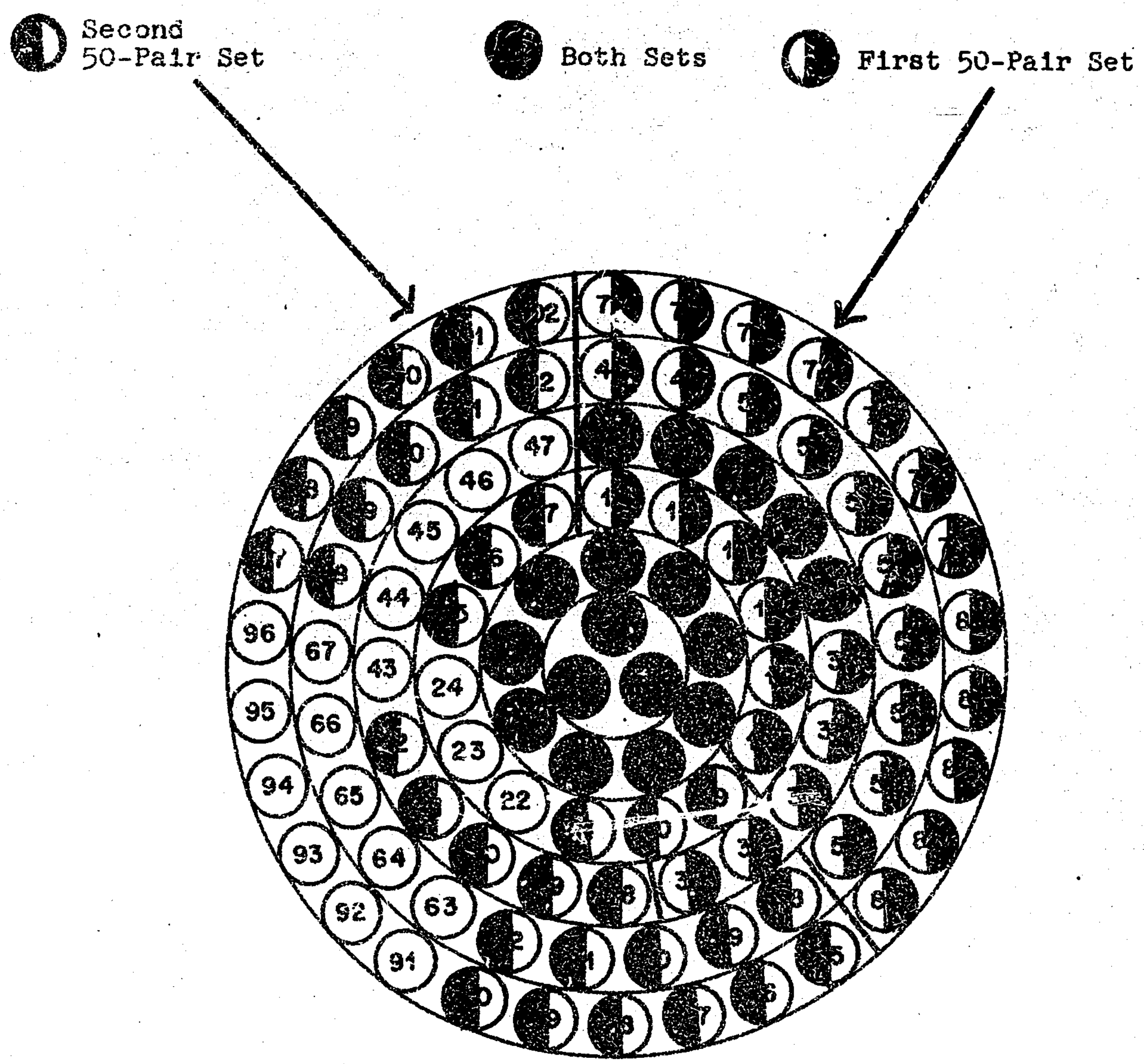

FIGURE 2 SELECTION OF TWO 50-PAIR SETS

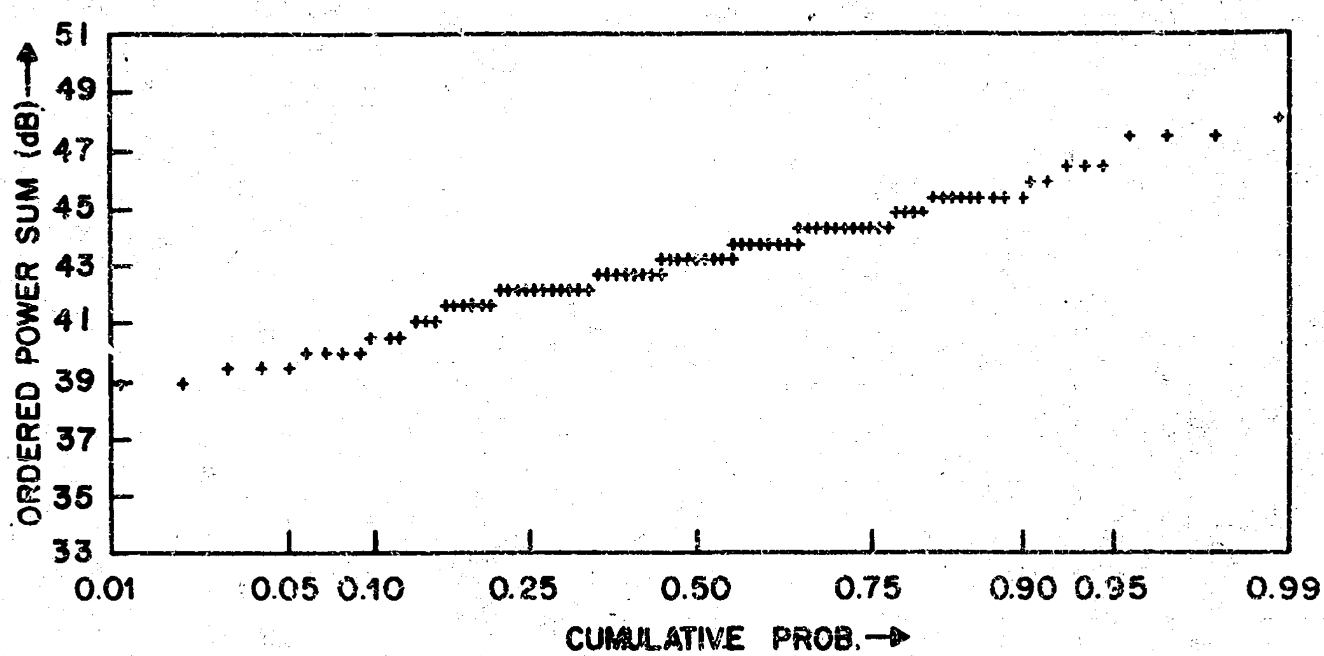

FIGURE 3 A SIMULATED POWER SUM DISTRIBUTION

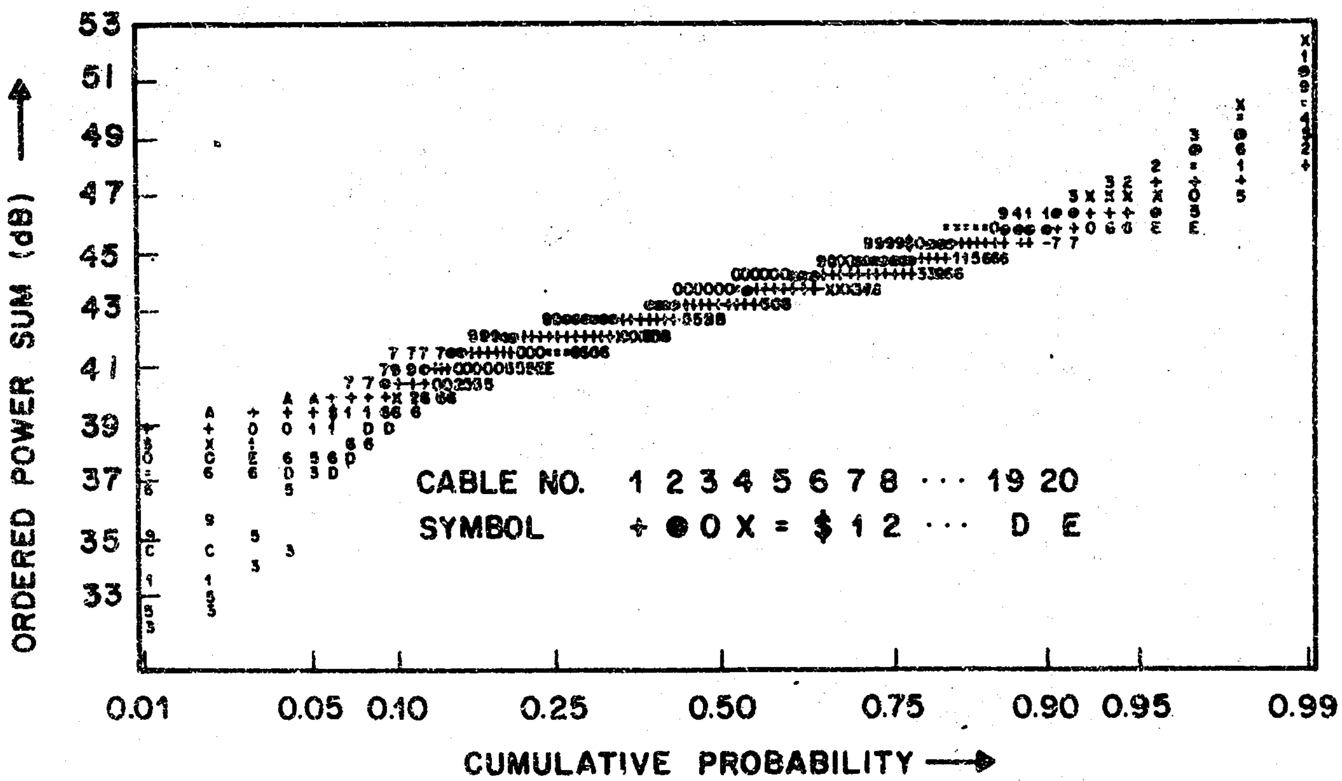

FIGURE 4 TWENTY REALIZATIONS OF THE POWER SUM DISTRIBUTION

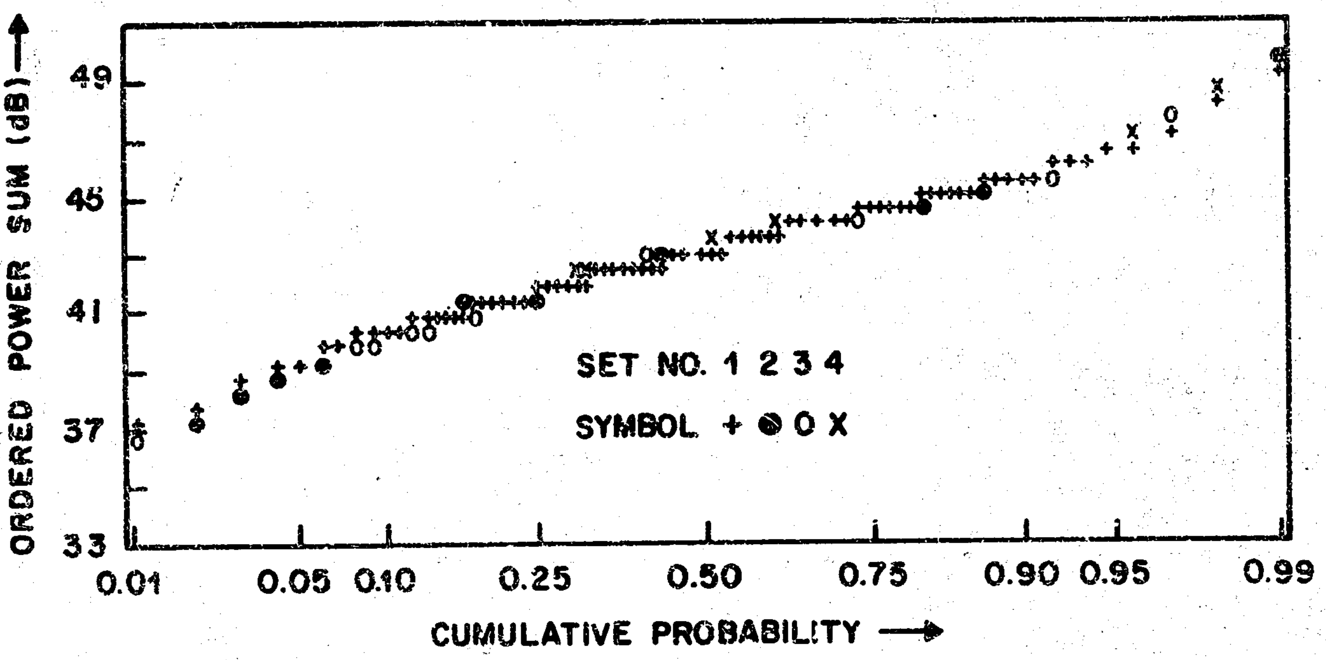

FIGURE 5 AVERAGES OF TWENTY REALIZATIONS OF THE POWER SUM DISTRIBUTION

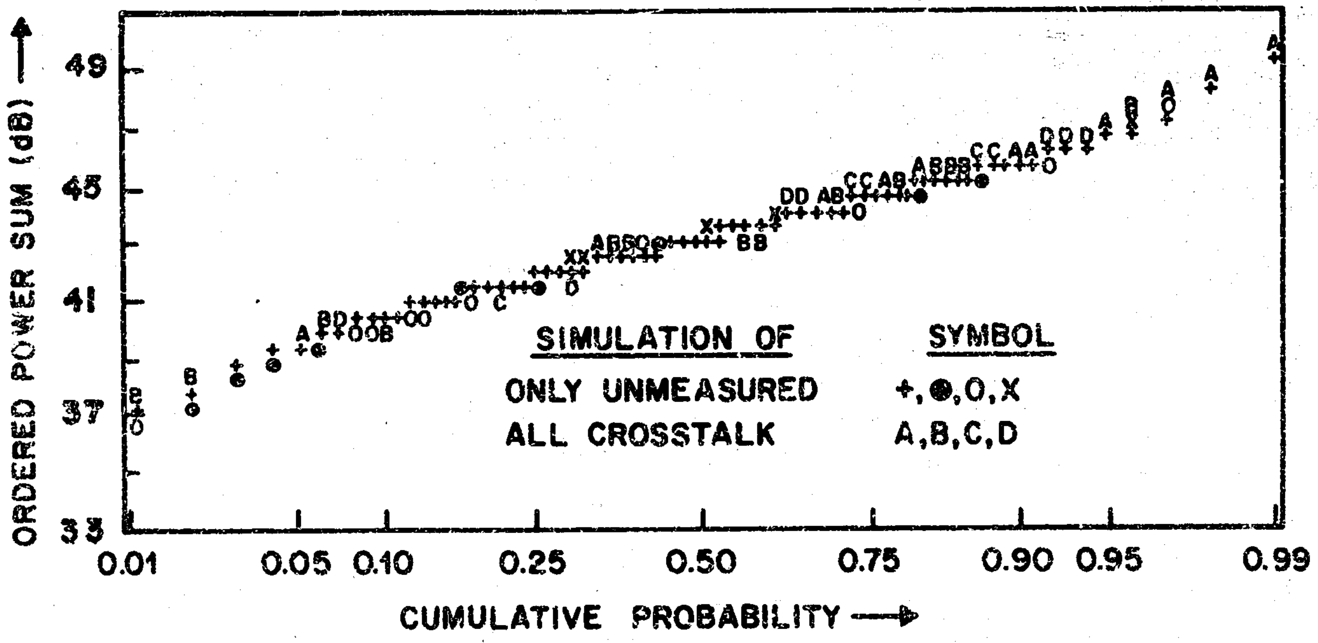

FIGURE 6 AVERAGE POWER SUM DISTRIBUTIONS WHEN ALL (OR ONLY UNMEASURED) CROSSTALK IS SIMULATED

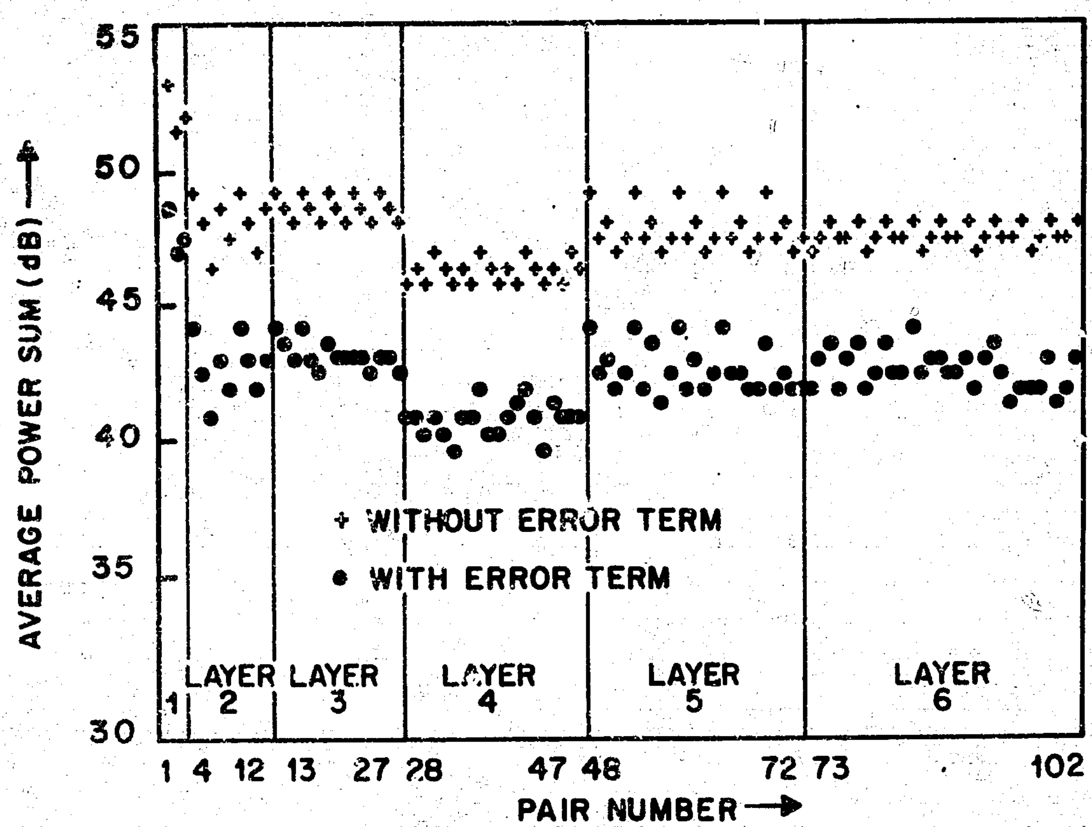

FIGURE 7 AVERAGE POWER SUM BY PAIR NUMBER

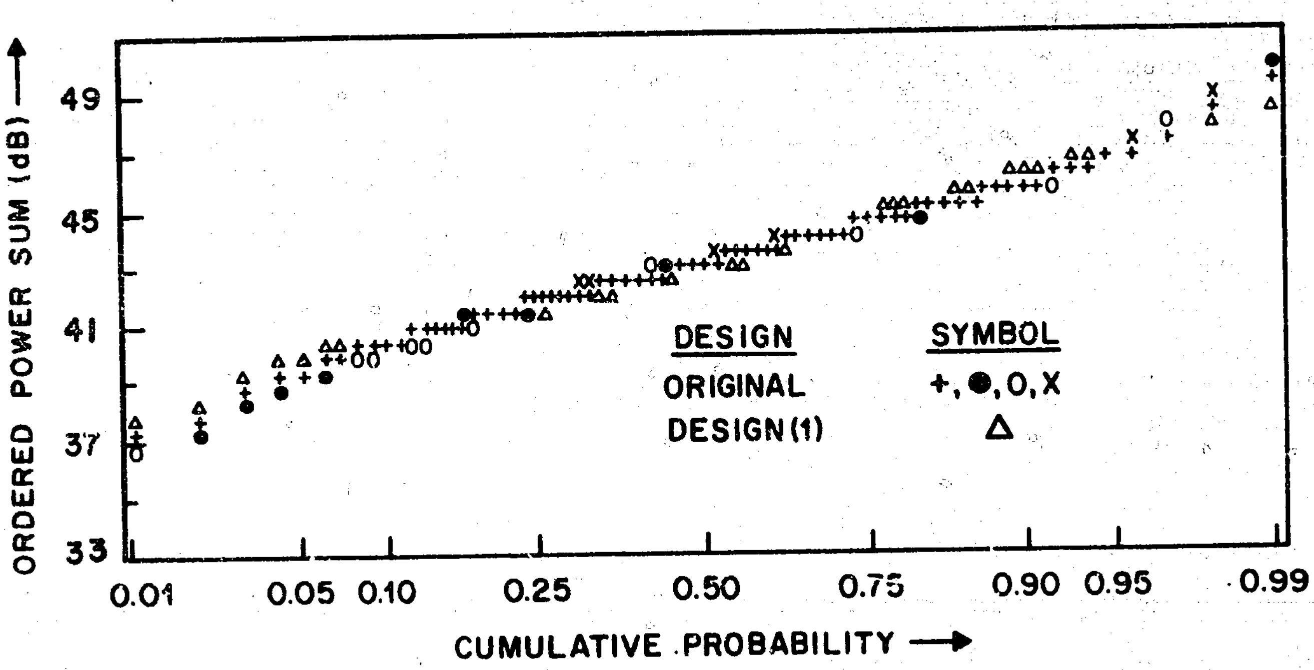

FIGURE 8 SIMULATED POWER SUM DISTRIBUTIONS FOR ORIGINAL AND NEW DESIGNS