VARIABILITY ASSUMPTIONS AND THEIR EFFECT ON CAPITAL INVESTMENT RISK

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Introduction

Conventional methods of analyzing risk in capital investment decisions fail to represent investment risk or variability in investment return because the calculation of rates of return using simple averages of the basic investment factors disguises the combined effect of external factor values on the rate of return. With Monte Carlo simulation, the sampling of investment factors according to their probability estimates allows the representation of factor variability in the rate of return calculations.

The usual procedure of assuming normally distributed factor probability is a simplifying assumption that influences the representation of variability and risk. According to statistical theory, Monte Carlo simulations with non-normal variability assumptions will produce simulated rates of return that are normally distributed but with greater degrees of variability. The purpose of this paper is to examine the results of simulations which utilize non-normal factors and to compare these results with those obtained using normally distributed investment factors and mean value investment factors.

A simulation approach rather than an analytical approach is used in the study because of the nature of the bounded statistical distributions used in the investment model. The model itself is complex to the extent that analytically handling the investment factors included in the model generates considerable mathematical tedium. Furthermore, the mathematics of combining statistical distributions of varying types can be extremely difficult and at times impossible.

Assumptions and Limitations

The scope of this study is restricted to those decisions involving one particular category of capital investment, the investment in initial production facilities. The type of investment under study is defined as a single cash outlay which produces varying revenues at varying costs over a future useful life. The study is further limited to a particular business environment in which the investment is influenced only by specified factors. These factors include the major marketing, investment, and production variables that contribute to the determination of profitability. It is further assumed that there is perfect positive correlation between the rate of return for this type of investment and that of the firm as a whole. Not included are such considerations as the opportunity for alternative investments, the cost of financing, or limitations on the amount of financing available. In general, the hypothetical investment decision included in this study is concerned only with the profitability of a single capital investment measured solely by its own earnings.

Other basic assumptions surrounding the study include:

(1) A reasonable approximation of risk is acceptable for confident, effective decision-making. Although other measures of risk are available, the standard deviation of the rate of return distribution is considered an acceptable measure of risk and will be used to measure risk in this study.

(2) Techniques of sales forecasting, cost projection, and investment prediction provide subjective probability estimates which are statistically valid.

(3) Computer programming and processing facilities are available at reasonable cost. (These costs are not included in the factors included in the investment model.)

(4) Monte Carlo simulation of the investment model reasonably approximates actual behavior in decision-making.

(5) The simulation model reasonably represents the pertinent factors and relationships of the investment decision.

(6) The internal rate of return is a reasonable measure of an investment's attractiveness.

Factors Influencing Investment Profitability

The analysis of risk in a proposed capital investment requires identification of the basic investment factors contributing to the determination of profitability and which have a significant effect on the risk of achieving expected profitability. Since the future values for investment factors may have different values than estimated, the final return on investment is subject to considerable variability. The key variable factors seem to be those that directly relate to investment earnings, such as sales revenue and production costs, or those that directly influence the nature of the investment, such as the amount and life of the investment.

In actual applications of risk analysis, the choice of significant factors will depend upon the particular market, production, and investment characteristics. For example, Hertz [1] selected as the key factors such variables as market size, selling prices, market growth rate, share of market, investment required, residual value of investment, operating costs, fixed costs, and useful life of facilities. Wagle [5] selected the same factors while Hillier [3] considered only revenue and cash flows. Hess and Quigley [2] used demand, price, fixed cost, variable cost, amount of investment, and plant capacity.

The simulation model here identifies and utilizes eleven investment factors as being significant. The selection of these particular factors results from the desire to construct a hypothetical investment that involves a relatively high level of risk. The factors are grouped into four classes and are defined as follows:

I. Marketing Factors

- A. PRCMKT - the dollar sales price of the investment product.
- B. VOLMKT - the number of product units of total industry sales.
- C. SHARE - the firm's percentage share of the total industry sales.

II. Production Factors

- A. VARCST - the variable manufacturing costs in dollars per unit, including taxes.
- B. FXDCST - the dollar amount of fixed manufacturing cost, including depreciation.
- C. SAEXP - the dollar amount of selling and administrative expenses.

III. Investment Factors

- A. RQDINV - the dollar amount of investment required for beginning production.
- B. INVLFE - the useful production life of the investment.
- C. SALVAG - the residual value of the investment at the end of its useful life.

IV. Dynamic Factors

- A. GROWTH - the percentage rate of change in market sales volume.
- B. RINFLA - the percentage rate of change in product prices and production costs and expenses.

The Hypothetical Investment Model

The hypothetical model used in this study to demonstrate Monte Carlo simulation and to evaluate different factor variability assumptions is constructed primarialy with the aim of revealing the risk involved in typical capital investment decisions. Emphasis in the model is on the number of probabilistic investment factors and the variability of their probability estimates. This emphasis allows the element of risk to be reflected in the rate of investment return without undue distortion from other sources.

Although basically an artificial construct, the hypothetical model is designed to represent reasonably realistic business conditions but not to parallel any particular capital investment. Actual applications of Monte Carlo simulation will necessitate a specific model tailored to the particular investment situation. Essential elements of the model are the rate of return function, the basic investment factors of the return function, the interrelationships between these factors, the estimated numerical values for the factors, and the type of variability in the factors.

The Rate of Return Function

Measuring profitability by the discounted cash flow method determines a rate of return which equates the sum of the present values of future period cash flows to the amount of initial investment. It is defined as that rate of return r which equates the initial investment I to the sum of expected cash flows C_1, C_2, ..., C_n as follows,

$$I = C_1\left[\frac{1}{(1+r)^1}\right] + C_2\left[\frac{1}{(1+r)^2}\right] + \ldots + C_n\left[\frac{1}{(1+r)^n}\right] \qquad (1)$$

in which the value of r is found by trial and error techniques [6].

Since the desired simulation model is one which represents the objective function, the hypothetical investment model is essentially the discounted rate of return equation. Expressed in the programming notation used for the computer simulation, the equation is:

RQDINV = PRVALU, in which

$$\mathrm{PRVALU} = \sum_{j=1}^{\mathrm{INVLFE}} \mathrm{CASHFL}(j)\left[\frac{1}{(1+\mathrm{RATE})^j}\right] + \mathrm{SALVAG}\left[\frac{1}{(1+\mathrm{RATE})^{\mathrm{INVLFE}}}\right] \qquad (2)$$

In the above relationship, RQDINV is the amount of initial investment required, PRVALU represents the present value of future cash flows, CASHFL(j) is the expected net cash receipts in period j, RATE is the rate of return which causes equality, and INVLFE is the useful life of the investment.

The net cash flow, CASHFL(j), consists of the sum of cash inflows less cash outflows. According to the discounted cash flow method, cash inflows include the sales revenue each period plus depreciation charges. Cash outflows are the costs of owning and operating the investment each period. The model computes the rate of return after taxes, with period taxes being treated as part of period costs. Considering the model equation with these component cash flows, then

$$\mathrm{CASHFL}(j) = \mathrm{CFSALE}_j - \mathrm{CFCOST}_j + \mathrm{DEPREC}_j \qquad (3)$$

in which CFSALE represents sales revenue each period, CFCOST is the total accounting cost in each period, and DEPREC refers to the depreciation charge for each period. The expression $(- \mathrm{CFCOST}_j + \mathrm{DEPREC}_j)$ is equivalent to the cash outflows per period and CFSALE (Sales revenue) is equivalent to cash inflows per period.

It is possible (and perhaps more desirable) to compute the period cash flows by subtracting period expenses from sales revenues and thus eliminating depreciation as a variable in the model. Such an approach presupposes that the cash flow information is available. Accounting records very often are the only source of input information for the model despite the fact that accounting "costs" may not be identical to "expenses." In such instances the accounting information must be properly adjusted to coincide with actual cash flows. For example, the two production variables FMFCST and VMFCST contain elements of depreciation which are not cash flow elements. The model thus recognizes the disparity between accounting costs and expenses and determines the period net cash flows in the manner indicated in equation (3).

The model uses the straight-line method of depreciation accounting for simplicity. Actual simulations would follow the particular convention of the user. By the straight-line method, the depreciation charge each period is the net amount of investment divided by the expected useful life of the investment. The equation for the model is

$$\mathrm{DEPREC}_j = (\mathrm{RQDINV} - \mathrm{SALVAG}) / \mathrm{INVLFE}. \qquad (4)$$

Other cash flows in the model are defined as

$$\mathrm{CFSALE} = \mathrm{PRCMKT} * \mathrm{SALVOL}, \qquad (5)$$

in which

$$\mathrm{SALVOL} = \mathrm{SHARE} * \mathrm{VOLMKT} \qquad (6)$$

and

$$\mathrm{CFCOST} = \mathrm{FSDCST} + (\mathrm{VARCST} * \mathrm{SALVOL}) + \mathrm{SAEXP} \qquad (7)$$

The model assumes that any time lags are constant over the life of the investment, and consequently the effect of time lags on rate of return variability is ignored for simplicity. Following this assumption, the investment begins production simultaneously with the investment, and sales occur at the time and rate of production.

The model recognizes two sets of functionally correlated investment factors, market price-sales volume and sales volume-variable manufacturing costs. Statistical interrelationship between these correlated factors is accomplished by utilizing multiple subjective probability estimates. For each possible value of one functionally correlated variable, there is a range of possible values for the other variable. These multiple estimates represent each factor as statistically independent although the variables themselves are functionally correlated.

Each time period of the investment life is also interrelated. To maintain statistical independence and to allow for functional correlation over time, the dynamic factors of growth and inflation serve to relate factors in earlier periods to the present simulation period. The model is constructed, and the simulation is performed so that the rates of growth and inflation determine a new range for the probability distribution of each investment factor.

To illustrate the sensitivity of rate of return variability to the assumption of the type of factor probability distribution, separate simulations of the hypothetical investment were performed using six different assumptions. They are a bounded standard normal distribution, a bounded peaked normal distribution, a bounded flat normal distribution, a bounded left-skewed distribution, a bounded right-skewed distribution, and a random selection of these five distributions.

These distributions are defined by varying the parameters of two basic functions. The normal function is symmetrical, and different standard deviations change the degree of peakedness for a given mean. Its equation is [4],

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \quad e^{-\frac{1}{2}\left[\frac{x-\bar{x}}{\sigma}\right]^2} \qquad (8)$$

The Beta function is skewed, and different parameters change both peakedness and skewness. Its equation is [4],

$$f(x) = \frac{(a+b+1)\,!}{a\,!\;b\,!}\, x^{a}\,(1-x)^{b} \qquad (9)$$

Figure 1 depicts the five types of distributions over the unit interval. The selection of these distributions is arbitrary and is intended only to reveal moderate departures from normal variability. Having assumed a type of variability, numerical parameters are necessary to completely define the factor subjective probability distributions.

Numerical Factor Values for the Model

The extremes of a distribution are a means to quantify and bound a function when the type of function between these extremes is known. Accordingly, simulation of the model utilizes the extreme high and low range estimates in conjunction with the assumed type of variability to provide subjective probability estimates for each investment factor.

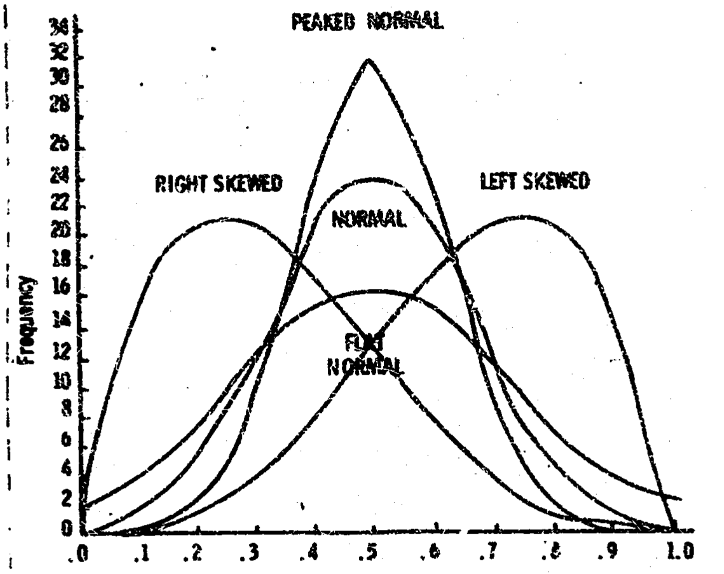

Figure 1 The Five Distributions of the Simulation Over the Unit Interval

The numerical values for the range estimates (bounds) of the eleven investment factors are arbitrary. The objective in the model is only to produce a measurable rate of return and to reveal the effect of factor variability upon rate of return variability. Whenever reference is made to any of the distributions used in the study, it is understood that they are bounded distributions and do not necessarily correspond precisely to their theoretical counterparts.

Simulation Flow Diagram

The logic flow diagram for simulating the hypothetical investment is shown in Figure 2. The basic flow is to simulate randomly different investment lives from the same conditions and to compute the return for each life. After all simulations, probabilities and statistical measures are calculated for the returns simulated, then, for comparison, the rate of return is computed by averaging factor estimates.

Simulation of an investment life first involves a definition of the hypothetical investment by determining the amount of investment (RQDINV), the life of the investment (INVLFE), and its residual value (SALVAG). Values for these factors, like all eleven factors, are selected by the Monte Carlo method. A value is randomly selected between the high and low range estimates according to the type of probability assumed for the investment. After this definition of the investment, each period of investment life is simulated.

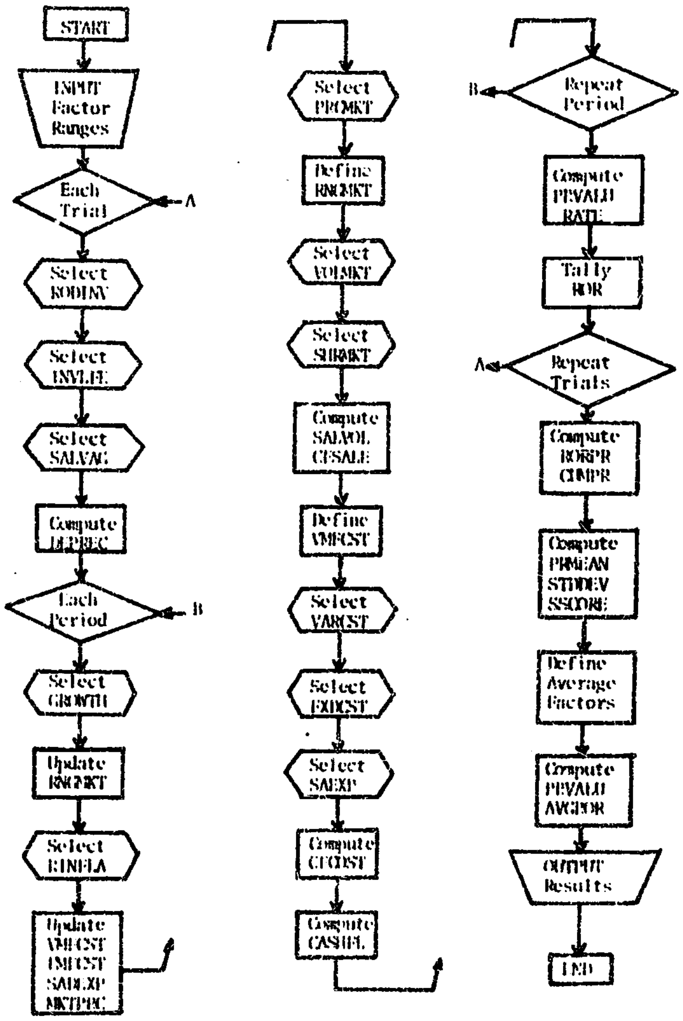

Figure 2 Program Flow Diagram

The rate of growth in market volume (GROWTH) and the rate of price-cost increase (RINFLA) are selected at the start of each period except the first. These rates update the ranges of market volume (RNGMKT) and the three costs. Values for the factors determining revenue cash flow are determined next. The selection of a market price (PRCMKT) also defines one of fifteen possible market ranges (RNGMKT) allowing for price-volume correlation. A value of market volume (VOLMKT) is then selected from the defined market range. After selecting a value for the firm's share of this market volume (SHRMKT), the sales volume (SALVOL) is computed as the product of share and volume. The revenue cash flow (CFSALE) is then computed from price and volume.

The next phase consists of selecting values for cash outflows. The value of sales volume, representing production volume, defines one of ten ranges for the correlated variable cost (VMFCST). After selecting this cost (VARCST), values for fixed manufacturing (FXDCST) and selling-administrative expenses (SAEXP) are selected. Cash outflow (CFCOST) is calculated by summing these costs and expenses. The net cash flow (CASHFL) is then calculated from the revenue, cost, and depreciation cash flows. This sequence is repeated for each period of the investment life.

When all periods have been simulated, the rate of return is computed from the period net cash flows and the residual value of the investment. The calculated rate of return is recorded, and the entire simulation is repeated for another trial simulating another investment with the same set of investment conditions but continuing to randomly select factor values.

After the desired number of investment trials has been simulated, statistical and probability measures are calculated for the returns simulated to provide an indication of the approximate risk in the hypothetical investment. The rate of return is computed using the conventional risk analysis method of averaging factor values for a comparison. First, the average value for each of the eleven investment factors is calculated from their high and low estimates. From these averages, the cash flows are computed for the average life of the investment. The rate of return is then determined by the same procedure as in the simulation.

Results of The Model Simulation

The computer simulation of the hypothetical investment produced a distribution of rates of return as the objective measure of risk in the proposed investment. The important results are:

1. The simulation using the assumption of normal variability in the investment factor estimates produced a distribution of returns with an expected rate of return which was much higher than the rate of return computed from the average of factor values.
2. The simulation that randomly used the five types of variability resulted in a distribution of returns with a lower expected value and a larger variation than obtained by the normal variability simulation.
3. The simulations involving four types of factor variability, which were non-normal, gave rates of return that were significantly different from the distribution of returns from the normal variability simulation. These distributions also directly reflected the type of variability distribution assumed for the factors.

These results suggest that, when measuring risk by the variability of the simulated rates of return, Monte Carlo simulation gives a different representation of risk depending upon the assumption of variability in the basic investment factors. These results further imply that the expected rate of return approximated by Monte Carlo simulation is likely to differ from the expected return computed by the conventional method of risk analysis.

The Rates of Return Simulated

The rates of return obtained by the computer simulation are presented in Figures 3 and 4. Figure 3 involves 1000 trial frequency distributions while Figure 4 involves 1600 trials. The curve for the simulation of normal variability conditions in Figure 3 is based upon the first 1000 trials whereas in Figure 4 the curve is based upon the total 1600 trials. The significance of these Figures is that they reveal how the distribution of returns varied over a range of possible returns from 0 to 270 percent.

Referring to Figure 3, the distribution for the five simulations varied over the range of returns according to the type of investment factor variability assumed during the simulation. The simulation assuming normal variability

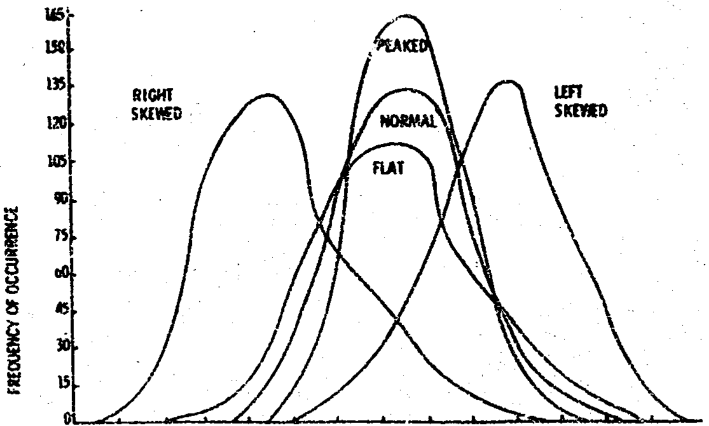

Figure 3 Simulation Rate of Return Frequency Distributions for 1000 Trials/Assuming Standard Normal, Peaked Normal, Flat Normal, Left-skewed, and Right-skewed Types of Investment Factor Variability

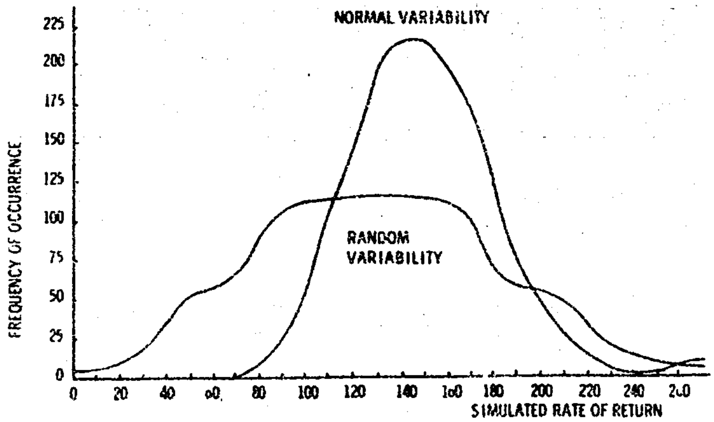

Figure 4 Simulation Rate of Return Frequency Distributions for 1600 Trials/Assuming Normal Variability and a Random Selection of Five Types of Variability

produced a distribution that was approximately symmetrical over the range of returns. The distribution for the simulation assuming peaked normal factor variability resulted in a distribution that was also symmetrical about the normal distribution but was more peaked than the normal. The distribution for the flat normal simulation resulted in a distribution that was also symmetrical but less peaked than the normal. The distributions for the right and left skewed simulations produced distributions that were skewed right and left respectively from the normal distribution with peakedness similar to the normal distribution. The character of these five curves corresponds very closely to the character of the five curves of the basic types of factor variability shown earlier in Figure 1.

Figure 4 shows the rate of return frequency curves for the simulations of 1600 trials using normal variability and a random selection of the five types of variability. The random variability distribution is clearly different from the normal distribution. The random curve is much more variable than the normal curve and is symmetrical at a much lower rate of return than the normal curve. Statistical measures provide a numerical description of the distribution characteristics.

The simulation also produced a distribution of simulated returns and computed a single rate of return by the conventional averaging method. The comparative advantage is shown in Figures 5 and 6. Figure 5 represents the "risk profile" of

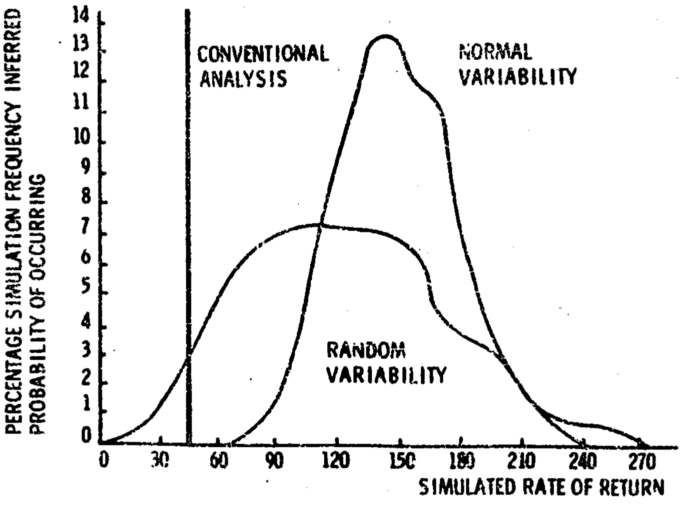

Figure 5 Risk Profile of Hypothetical Investment

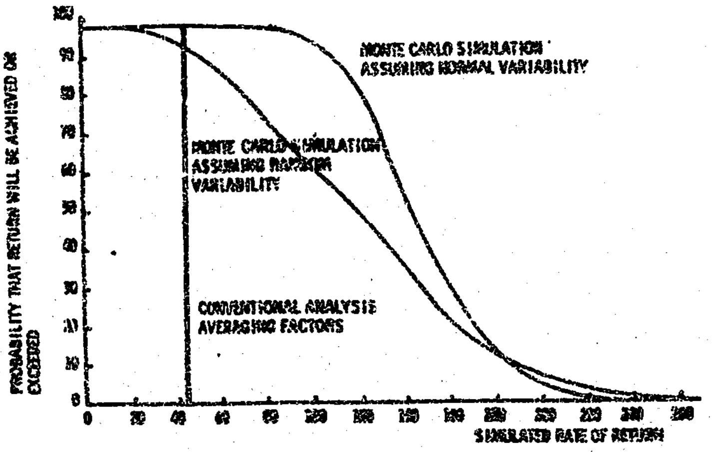

Figure 6 Comparison of Expected Investment Return Probabilities for Conventional and Monte Carlo Methods

the hypothetical investment for the conditions of normal and random factor variability. It is a percentage frequency distribution of the various simulated returns and indicates not only the variability in investment return but also the inferred probability of each rate of return. Figure 5 also includes the single expected rate of return computed by the averaging method. For the hypothetical investment, a significant difference in the anticipated investment returns is apparent.

Figure 6 shows the cumulative probability curves for the two conditions of investment and indicates the probability of realizing higher rates of return. It was drawn from cumulative probability values calculated from simulation results. The probability for the investment return using the averaging method is one since it is a single value. Figure 6 reveals a difference between the averaging and Monte Carlo methods. Figures 5 and 6 are also representative of the decision guide available to managers through Monte Carlo risk analysis. Management has a direct measure of the approximate risk and the chances of achieving various rates of return.

Analysis of Results and Conclusions

Table 1 shows the calculated statistical measures for the six rate of return distributions obtained from the computer simulation.

Statistical Measures

A comparison of the means and standard deviations supports the observations concerning the frequency curves for the simulated rates of return. The distributions from the normal, flat, and peaked variability simulations have approximately similar means (151.5, 154.5, 148.1). Their standard deviations bear a relationship corresponding to their assumed type of variability. The standard deviation for the distribution of the simulation using flat variability is larger than the standard deviation for the simulation using normal variability (37.62: 29.04). The standard deviation for the distribution assuming peaked variability is smaller than the normal distribution standard deviation (23.84:29.04). The distributions from the skewed simulations have means (95.4, 195.1) that are considerably different from the means for the normal, peaked, and flat variability simulations. Their standard deviations, however, are similar (30.39, 33.05). The simulation of random variability has a distribution with a

TABLE 1

STATISTICAL MEASURES FOR THE DISTRIBUTIONS OF SIMULATED RATES OF RETURN

Assumed Type of Variability	Mean	Standard Deviation	Skewness	Kurtosis	Sampling Error
Standard Normal	151.5	29.04	0.2	2.8	0.73
Peaked Normal	154.5	23.84	0.2	2.7	0.75
Flat Normal	148.1	37.69	0.3	2.7	1.19
Left Skewed	195.1	30.39	-0.1	2.8	0.96
Right Skewed	95.4	33.05	0.5	3.2	1.05
Random	124.6	50.99	0.2	2.6	1.28

mean (124.6), lower than the means from the three distributions of symmetrical variability but higher than the mean for the distribution of right skewed variability. The standard deviation for the random simulation (50.99) is much larger than the other standard deviations.

Comparing the peakedness and skewness of these distributions to the theoretical normal peakedness (kurtosis value of 3) and to the theoretical normal skewness (skewness value of 0), the distributions for the normal, flat, peaked, and random variability simulations are slightly positive or right-skewed and somewhat more peaked than a theoretical normal distribution. The simulation assuming left-skewed variability produced a distribution that was more left-skewed than the others. In contrast, the simulation assuming right-skewed variability produced a distribution that was more right-skewed and that had more peakedness. The simulation distribution using random variability had the lowest degree of peakedness. The measures of skewness and kurtosis are something of a cross-check on the reliability of the simulation since the calculated measures of skewness and kurtosis indicate approximately normally distributed sampling distributions.

Sampling error is another check on the simulation validity. It indicates the possible error in estimating the actual mean rate of return from the mean of the simulated rates of return because of the random simulated sampling used in the Monte Carlo method. A comparison of the sampling errors and means in Table 1 reveals that the differences between the means for the six simulations are greater than the possible errors which might result from the randomnesss of the simulation.

Analysis of Risk for the Hypothetical Investment

The purpose of Monte Carlo risk analysis is to determine for the proposed investment the variability in possible rates of return and the probability of various returns being achieved. Investment risk is defined as the likelihood of not achieving the expected rate of return and the variability in investment return is generally accepted as a measure of this risk. The frequency of individual rates of return occurring during simulation is a means of constructing an anticipated rate of return distribution. This distribution is the basis for determining rate of return variability and inferring the probability of achieving various rates of return.

The rate of return frequency distributions

TABLE 2

RATE OF RETURN FREQUENCIES AND PROBABILITIES FOR THE SIMULATION ASSUMING NORMAL FACTOR VARIABILITY

Rate of Return Intervals	Frequency 1000 Trials	Frequency 1600 Trials	Percentage 1600 Trials	Cumulative Probability 1600 Trials
0 or less	0	0	.000	1.000
1 to 10	0	0	.000	1.000
11 to 20	0	0	.000	1.000
21 to 30	0	0	.000	1.000
31 to 40	0	0	.000	1.000
41 to 50	0	0	.000	1.000
51 to 60	0	0	.000	1.000
61 to 70	0	0	.000	1.000
71 to 80	2	3	.002	.998
81 to 90	10	16	.010	.988
91 to 100	24	42	.026	.962
101 to 110	41	67	.042	.920
111 to 120	72	113	.071	.849
121 to 130	100	161	.101	.748
131 to 140	119	207	.129	.619
141 to 150	129	216	.135	.484
151 to 160	126	199	.124	.360
161 to 170	130	191	.119	.241
171 to 180	88	142	.089	.152
181 to 190	61	98	.061	.091
191 to 200	42	65	.041	.050
201 to 210	27	37	.023	.027
211 to 220	15	26	.016	.011
221 to 230	9	10	.006	.005
231 to 240	4	6	.004	.001
241 to 250	1	1	.001	.000
251 to 260	0	0	.000	.000
261 to 270	0	0	.000	.000
over 270	0	0	.000	.000

and their means and standard deviations have already been presented for the six simulations of different conditions of factor variability. The mean represents the most likely expected rate of return and the standard deviation represents the variability of investment return or the dispersion of returns about the mean. Considering the risk of the hypothetical investment measured by the mean and standard deviation, the hypothetical investment has generally lower anticipated rates of return with more variability if the factor subjective probability estimates are non-normally distributed than if they are normally distributed. Tables 2, 3, and 4 show the approximate probabilities inferred from the frequency of returns occurring during simulation.

The significant difference between the mean rates of return for the simulations of normal and random variability may be indicative of the weakness of the usual Monte Carlo method of risk

TABLE 3

RATE OF RETURN FREQUENCIES AND PROBABILITIES
FOR THE SIMULATION ASSUMING A RANDOM
SELECTION OF VARIABILITIES

Rate of Return Intervals	Frequency 1600 Trials	Percentage 1600 Trials	Cumulative Probability 1600 Trials
0 or less	5	.003	.997
1 to 10	3	.002	.995
11 to 20	4	.003	.992
21 to 30	15	.009	.983
31 to 40	30	.019	.964
41 to 50	56	.035	.929
51 to 60	60	.037	.892
61 to 70	80	.050	.842
71 to 80	99	.062	.780
81 to 90	108	.067	.713
91 to 100	114	.071	.642
101 to 110	115	.072	.570
111 to 120	116	.073	.497
121 to 130	83	.052	.445
131 to 140	113	.071	.374
141 to 150	111	.069	.305
151 to 160	112	.070	.235
161 to 170	79	.049	.186
171 to 180	62	.039	.147
181 to 190	57	.036	.111
191 to 200	58	.036	.075
201 to 210	44	.026	.048
211 to 220	24	.015	.033
221 to 230	16	.010	.023
231 to 240	15	.009	.014
241 to 250	6	.004	.010
251 to 260	11	.007	.003
261 to 270	1	.001	.002
over 270	3	.002	.000

TABLE 4

RATE OF RETURN FREQUENCIES ASSUMING PEAKED NORMAL, FLAT NORMAL, LEFT SKEWED, AND RIGHT SKEWED VARIABILITY

Rate of Return Intervals	Peaked Normal	Flat Normal	Left Skewed	Right Skewed
0 or less	0	0	0	0
1 to 10	0	0	0	0
11 to 20	0	0	0	3
21 to 30	0	0	0	9
31 to 40	0	0	0	18
41 to 50	0	2	0	37
51 to 60	0	3	0	80
61 to 70	0	4	0	102
71 to 80	0	14	0	113
81 to 90	0	31	0	131
91 to 100	6	52	1	132
101 to 110	21	63	3	97
111 to 120	46	81	2	82
121 to 130	92	101	15	61
131 to 140	145	98	25	37
141 to 150	155	107	34	29
151 to 160	160	114	62	23
161 to 170	140	78	73	25
171 to 180	99	65	105	10
181 to 190	66	44	128	5
191 to 200	33	38	136	4
201 to 210	30	41	120	2
211 to 220	5	30	91	0
221 to 230	2	17	79	0
231 to 240	0	11	60	0
241 to 250	0	6	36	0
251 to 260	0	0	22	0
261 to 270	0	0	6	0
over 270	0	0	2	0
Total Trials	1,000	1,000	1,000	1,000

analysis, since most studies on Monte Carlo analysis of investment risk assume normal variability in their factor estimates. From the probability aspect, the simulation using a random selection of the five types of factor variability should have a rate of return distribution with a mean not significantly different from the normal simulation mean and with a greater standard deviation than the simulation using normal variability. This expectation may be derived intuitively in that a random selection of the five types of basic variability should average out after many selections to a type of distribution that approximates a normal distribution but with a greater variability.

A possible explanation for this significant difference involves the complex effect of probabilistic relationships in the hypothetical investment model. There are eleven variables with two pairs intercorredated. Furthermore, not only are the periods in the rate of return equation intercorrelated, but time itself is a variable. It is possible for the probabilities to combine in an unusual manner not representative of the assumed probability because of the construction of the model.

For example, suppose in the random variability simulation a lower range of factor values of one particular investment factor has a greater influence on the rate of return than the higher range of values. The resulting rate of return distribution would then tend to be right-skewed. The hypothetical model does in fact contain certain factors with distinct ranges such as variable cost or market volume. There is also an upper limit to the quantity of sales and production volume, either of which could truncate the distribution of cash flows. Thus the differences in means could arise because of the model construction. If this is the case and the model is truly representative of typical investment situations, then the effect of non-normal probabilities is important. This effect suggests that the simulation using a random selection of non-normal probabilities is preferable to a simulation using only normal probabilities because it is a more representative measure of risk.

TABLE 5

COMPARISON OF PERCENTAGE DISTRIBUTION BY STANDARD SCORES

Standard Score Interval		Assumed Type of Factor Variability						Theoretical Normal Curve
		Standard Normal	Peaked Normal	Flat Normal	Left Skewed	Right Skewed	Random	
From	To Under							
-3.5	-3.0	.000	.000	.000	.000	.000	.000	.001
-3.0	-2.5	.001	.000	.002	.003	.000	.000	.004
-2.5	-2.0	.015	.013	.008	.022	.010	.008	.017
-2.0	-1.5	.042	.043	.044	.042	.028	.048	.044
-1.5	-1.0	.100	.094	.110	.092	.116	.114	.092
-1.0	-0.5	.168	.176	.169	.138	.177	.173	.150
-0.5	0.0	.190	.203	.195	.216	.204	.178	.191
0.0	0.5	.188	.169	.191	.178	.197	.169	.191
0.5	1.0	.139	.141	.114	.135	.104	.145	.150
1.0	1.5	.083	.080	.080	.098	.078	.089	.092
1.5	2.0	.044	.050	.059	.058	.040	.049	.044
2.0	2.5	.022	.027	.022	.013	.033	.018	.017
2.5	3.0	.007	.002	.006	.002	.008	.009	.004
3.0	3.5	.001	.002	.000	.000	.000	.000	.001
Totals		1.000	1.000	1.000	1.000	1.000	1.000	1.000

Cross-Checking by Standard Scores

An evaluation of the six rate of return distributions by means of standardizing their distributions serves as a cross-check on the validity of the simulation results. Standard score values were calculated for each of the six rate of return distributions and converted to percentage frequency. Table 5 shows these percentage frequencies by standard score and also the percentage frequency for the theoretical standard normal curve. A close similarity exists not only between the six simulations but also between the six distributions and the standard normal curve.

This standard tabulation helps to verify the statistical validity of the computer simulation. It indicates that, after correcting for the different means and standard deviations, there is no significant difference between the six simulations resulting from the simulation method.

REFERENCES

1. Hertz, David B. Risk Analysis in Capital Investment. Capital Investment Series, Harvard Business Review Reprints, January-February, 1964, pp. 95-107.
2. Hess, Sidney W. and Quigley, Harry A. Analysis of Risk in Investments Using Monte Carlo Techniques. Chemical Engineering Progress Symposium Series, No. 42, 1963, pp. 55-63.
3. Hillier, Frederick S. "The Derivation of Probabilistic Information for the Evaluation of Risky Investments." Management Science, IX, No. 3 (1963), pp. 443-457.
4. Sasieni, Maurice; Yaspan, Arthur; and Friedman, Lawrence. Operations Research. New York: John Wiley & Sons, Inc., 1967.
5. Wagle, B. "A Statistical Analysis of Risk in Capital Investment Projects." Operational Research Quarterly, XVIII, No. 1 (March, 1967), pp. 13-32.
6. Weston, J. Fred and Brigham, Eugene F. Managerial Finance. 2nd ed. New York: Holt, Rinehart and Winston, 1966.