Using an Extended Version of GERT To Simulate Priority and Assignment Rules In a Labor Limited Jobshop System

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ABSTRACT

In the literature, prior to 1965, most research on jobshop systems was on machine limited queueing systems. More recently this research has been directed toward labor and machine limited queueing systems. In this direction, the authors have developed and implemented an extended version of GERT called GERTS III QR (a GERTS model able to handle queueing systems with resource limitations). This model is further refined to handle both homogeneous and heterogeneous classes of labor.

This paper describes the GERTS III QR model and gives an illustration of its application. The example is a jobshop system with service centers in parallel. Three alternative priority and assignment rules, first-come-first-served, random and shortest operation time are evaluated.

The vast majority of the literature on jobshops has been addressed to the machine limited queueing systems. Recently, some analyses of actual jobshops suggest that machinery may not be the critical item but that available labor and its relative efficiency at various machine centers may be the limiting factor, (4).

With the introduction of the human element into this type of system, another dimension is added to the decision-making process. With a strict machine limited system, the problem was to determine "good" machine loading rules. Conway et al (3), Nanot (12) and others have developed such strategies. When the system is labor limited, however, the problem becomes more complex, i.e. one must also specify labor assignment rules.

Complete Labor Assignment Procedure. The complete labor assignment procedure includes the queue priority rule and its related labor assignment rule. In this paper, these systems are designated as DRC systems, e.g. Dual Resource Constrained systems. In DRC systems, moreover, the labor class may be either homogeneous or heterogeneous in nature. In homogeneous systems all laborers are equal and have identical efficiencies at each machine center. Many variations in labor efficiency patterns may be depicted in the heterogeneous systems, where the laborers do not have equal and identical efficiencies at each machine center. For the experiments described in this paper all DRC systems were homogeneous in nature only.

Labor Blocking. One of the phenomena that occurs in DRC systems is "labor blocking". This occurs where there are idle labors and there are jobs left in one or more queues. This can occur if there are empty machine centers and the only jobs waiting are in queues behind busy machine centers. If no jockeying between queues is allowed, we have both waiting jobs and idle workers. Such a situation can occur in reality where each machine center performs some special task that cannot be performed by the others. For example, a repair garage may have only one rig to realign the front-end of cars, one paint spray booth, etc.

The Model. The basic model described in this paper is a variation of GERT (Graphical Evaluation and Review Technique). Developed by Pritsker (18,19), GERT, which is similar to PERT, is a procedure for modeling stochastic decision networks. Since its inception, GERT has evolved through GERT II and GERTS III (a general purpose program written in GASP) for simulating stochastic networks. This was followed by GERTS III C, GERTS III Q and GERTS III R.

Basic GERTS. The general features of the basic GERTS simulation models include:

a) Network branches - characterized by the probability of being selected, the time required to complete the activity represented by the branch (it may have any one of several probability distributions), and the efficiency of each resource required to perform that activity (optional).

b) Nodes - characterized by number of "releases" before the node is realized or reached for the first time and after the first time, which activities must be completed for the node (event) to be achieved (since some branches have a probability of being selected, not all branches incident upon a node need be required), method of scheduling the activities emanating from the node, and the statistics to be collected (if any) at the node.

For network modeling purposes, each node may be classified in one or more of 10 categories. In this work only the following were used. (See [17] for a description of all types).

SOURCE Nodes: Nodes which initiate activities at the origin time of the project.

SINK Nodes: Nodes which may be the terminal node of the network. Normally SINK nodes only receive flow, but it is also possible to have activities leaving a SINK node if so desired.

MARK Node: This node is used as a time frame reference point for an item being processed. The point in time at which an entity passes a MARK node is recorded as an attribute to that entity. MARK statistics are collected at INTERVAL STATISTICS nodes and constitute the time spent in passing between the MARK node and the INTERVAL STATISTICS nodes.

QUEUE Nodes: QUEUE nodes are those which provide a storage capacity for items in progress. Items are automatically held at a QUEUE node until a service activity is performed on that item. Statistics are automatically maintained on QUEUE nodes.

STATISTICS Nodes: STATISTICS nodes are those at which statistical quantities are collected. There are five basic statistics which can be collected. Any node except START, QUEUE, or MARK node is a candidate for a STATISTICS node.

Figure 1 shows the node symbolism for normal GERT nodes and Figure 2 gives the notation for QUEUE nodes.

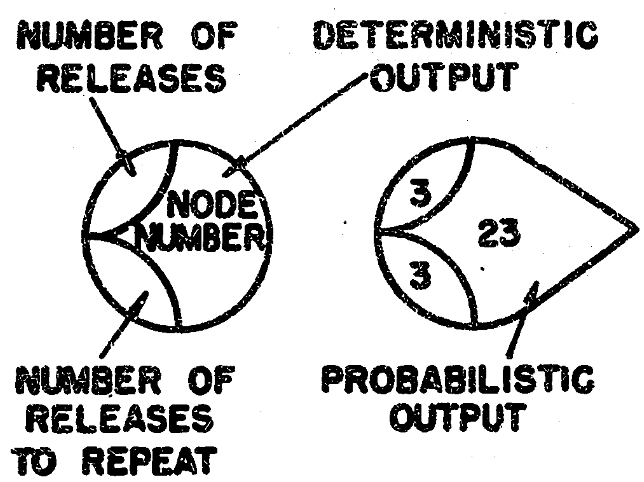

FIGURE 1

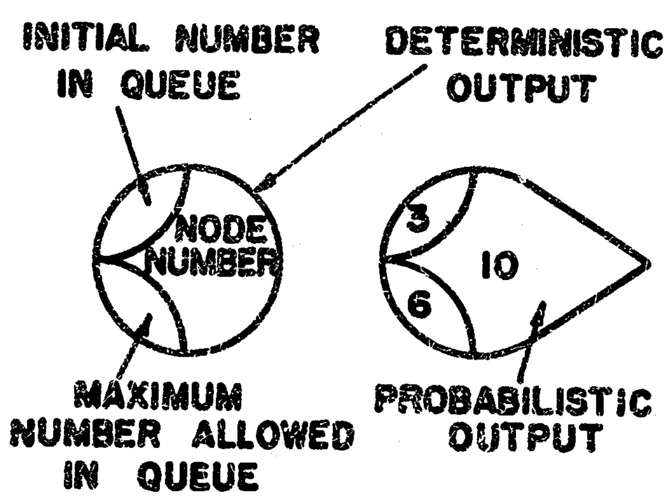

FIGURE 2

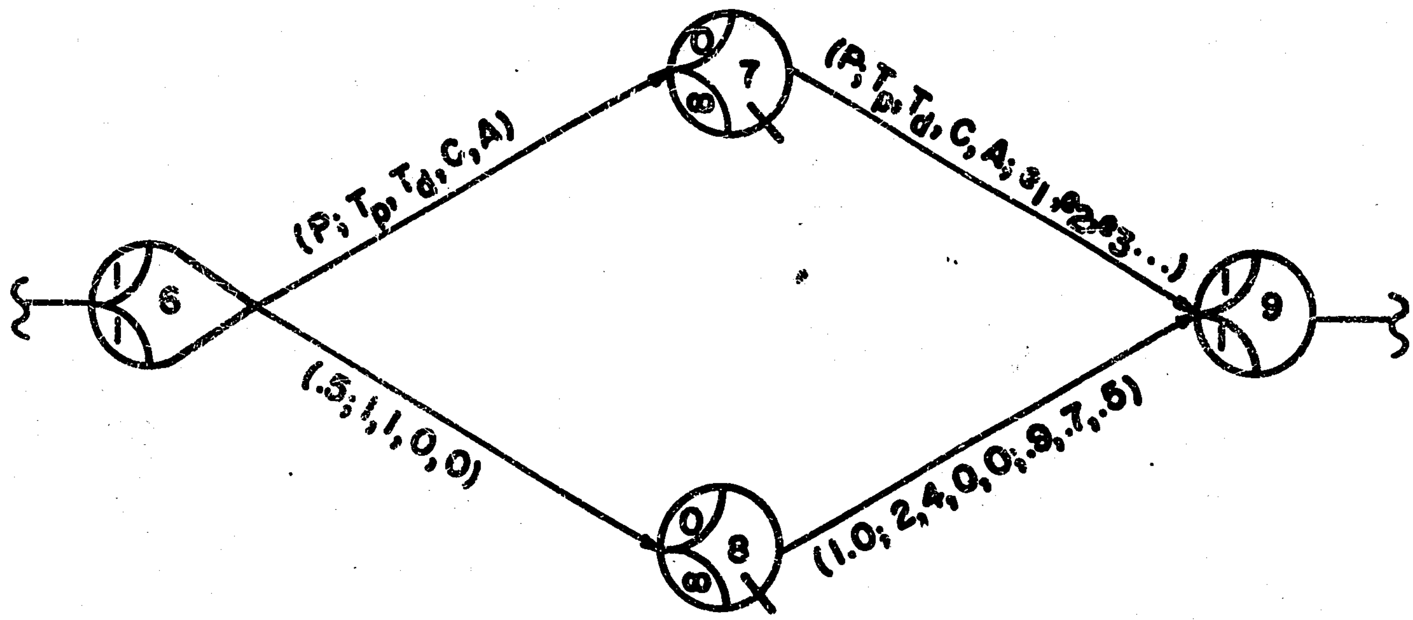

FIGURE 3

Figure 3 further illustrates the node symbolism and also displays the method for specifying the arc parameters. Figure 3 shows a variety of node and arc types, for which a brief explanation follows. Node 6 is a node which has probabilistic output, thus the probability of selection (P) is less than one on arcs (6,7) and (6,8). The parameter set format is defined on arc (6,7) and a typical numerical example is given on arc (6,8). Here the probability of selection is .5; the time of traversal is given by the first parameter set and first distribution type, and there is no counter or activity number assigned. Both nodes 7 and 8 are queue nodes with initial queue length of zero and infinite queue capacity. Note that the output arcs of these nodes have additional parameters, $e_1, e_2, \ldots, e_n$; these parameters give the efficiency of each resource in performing the activity. The parameters of arc (8,9) show that the first laborer has an efficiency of .9; the second .7; and the third .5 when performing this activity. Further explanation will be given in the context of the GERT III QR network examples in the next section.

The GERTS III QR simulation model was constructed using GERTS III Q as a framework, and integrating the concepts of GERTS III R with some modification. GERTS III R was designed for the study of activity networks with limited resources; i.e., most arcs represent activities which are resource constrained. So in the processing of the network, GERTS III R assumes that the resource constraints act upon every arc. In most queueing networks, the only arcs which might require resources are the output arcs of Q-nodes, which represent the service activity associated with the queue. Most often the other arcs of a queueing network merely represent flow paths for the items flowing through the system. Thus, considering every arc to be labor constrained could lead to much useless monitoring; therefore, GERTS III QR considers only output arcs and nodes in the normal GERTS III manner.

Another important change in the basic concepts of GERTS III R is the incorporation of the facility to deal with heterogeneous labor forces. In GERTS III R a number of resource types is specified (in the present version as many as three, although a special application to be reported later uses ten resource types) and an activity requires a fixed number of units of each type. The resource types are not interchangeable and each type is equally efficient on all activities. In GERTS III QR resources are interchangeable; that is, if an activity requires units of some resource any type can be used. Thus the type merely identifies the resource. Further, the time required to complete an activity is a function of the resource type (laborer) which performs the activity and the identity of activity (service facility). Hence, complex patterns of labor and machine efficiency can be studied.

Some modification of the basic concepts of GERTS III Q was also required. In DRC queueing systems, items may be detained in the queue not only because the service facility is busy but also because no labor is available, so that a service facility may be idle while items are in queue. Thus the basic queueing concepts had to be modified to deal with this dichotomy which exists in the DRC situation.

Hence, it is inaccurate to say that the GERTS III QR model is formed by mere superposition of GERTS III R upon GERTS III Q. Although the GERTS III QR simulator was written for DRC systems it will simulate ordinary machine limited systems with little or no loss in computational efficiency compared with GERTS III Q. However, for networks where practically all arcs are resource constrained and no queues are involved the GERTS III R simulator is more effective. GERTS III QR represents an integration of all the standard features of GERTS III R and GERTS III Q.

A standard feature of GERTS III QR is the QUEUE node. The QUEUE node is one which provides a storage capability for on-going items. The concepts of first and secondary releases are not appropriate for a QUEUE node, and so the QUEUE node is characterized by: (1) the number of items initially in the queue, and (2) the maximum number of items allowed in the queue. Other parameters associated with a QUEUE node are the order of processing and the node to which an item would balk if it arrives when a queue is full.

In the version used in this study, GERTS III QR was further modified to include priority rules other than the standard rules, LIFO and FCFS. To implement the SOT rule additional attributes had to be added to the queue list since job processing times were assigned to the job upon entry into the system. Therefore, a job selection sub-routine was also now required.

Implementation of the RANDOM priority rule required only minor modification of the SOT subroutines. The initial randomly assigned job times were used as the random variable for job priority selection using the SOT procedure, however, a different job processing time was then assigned when the job underwent actual processing.

For STATISTICS nodes, GERTS III QR obtains estimates of the mean, standard deviation, minimum, maximum and a histogram associated with the time a node is realized. Five types of time statistics are possible:

- F. The time of first realization of a node;
- A. The time of all realizations of a node;
- B. The time between realizations of a node;
- I. The time interval required to go between two nodes in the network; and
- D. The time delay from first activity completion on the node until the node is realized.

The nodes on which statistics are to be collected and the type of statistics desired are part of the description given to a node by the input to GERTS III QR.

A distribution type and parameters are assigned to an arc through the specification of a parameter set number and a distribution type. Each parameter set defines parameters from which the mean, variance, maximum value and minimum value for each of the above distribution types can be computed. This description is part of the data input to the GERTS III QR program.

A powerful device when using the GERTS III QR program to analyze complex activity networks is the ability to modify the network while an activity is in progress. Specification of an activity number allows network modifications based on the completion of specified activities within the model. An activity may or may not be numbered. However, only those activities which are numbered are candidates for network modification.

Standard output of the GERTS III QR programs consists of the following:

1. An echo check of the input data, consisting of:
   - A. Node characteristics
   - B. Branch characteristics
   - C. Listing of the SOURCE, SINK, and STATISTICS nodes
   - D. Network modifications
2. Statistical summaries consisting of:
   - A. The probability of node realization during the simulation period. The mean, standard deviation, number of observations, maximum, and minimum time units to realize a STATISTIC node during the simulation.
   - B. All of the above statistics for counter types.
   - C. Mean, standard deviation, minimum and maximum of the queue length, waiting time, busy time of processors (service

activities), and balkers per unit time, for all QUEUE nodes.

D. Histograms of the time to realization for each STATISTICS node, and the queue lengths for each QUEUE node. Histograms reflect the underlying probability distributions associated with All, Interval, Delay, Between, and First realization statistics.

It should be noted that the above output quantities are common to all GERTS simulation programs, and the interpretation of each statistic has been previously discussed in prior publications (1,2,20). Hence, in the examples which follow, only the output statistics unique to GERTS III QR will be discussed.

A GERTS III QR Network Model for Multi-Queue, Multi-Channel, Single-Phase, DRC Queueing System: An Example

In this section an application of the GERTS III QR model will be illustrated; the systems modeled are multi-queue, multi-channel, single phase systems. Each service facility contains one machine and has its own queue. Balking or switching between queues and reneging are not allowed. The arrival process is Poisson with a mean arrival rate of $\lambda = 1.0$. Each job is routed to a specific service channel with equal probabilities, $1/m$, upon arrival. Every job requires only a single processing operation at the service facility to which it is routed. The mean processing times are assumed to be identical and exponentially distributed. The number of laborers, n, is less than the number of machine centers, m. Thus the system is constrained by both labor and machines. Labor is allocated to available jobs by assigning the most efficient laborer to the machine center with the job having the highest priority per the selected priority rule. In this example the labor force is assumed to be homogeneous and all laborers' efficiencies are equal to 1.0 (100%).

The Problem Statement

The problem can be stated as follows. Given the system described above, determine the proper job selection rule (machine loading) and its related labor assignment rule so as to optimize some measure of system performance. The key measures used in this study are job waiting time characteristics.

The machine loading and labor assignment rules to be evaluated are: (1) select jobs from the queue per the first-come-first-serve (FCFS) rule and assign idle laborers accordingly, (2) select the job from the queue with the expected shortest-operation-time (SOT) and (3) select jobs at random (RANDOM). A schematic representation of the experimental systems is given in Figure 4.

This illustration, representing a typical labor limited (DRC) system, is merely one example of the application of GERTS III QR. Certainly many other system descriptions could

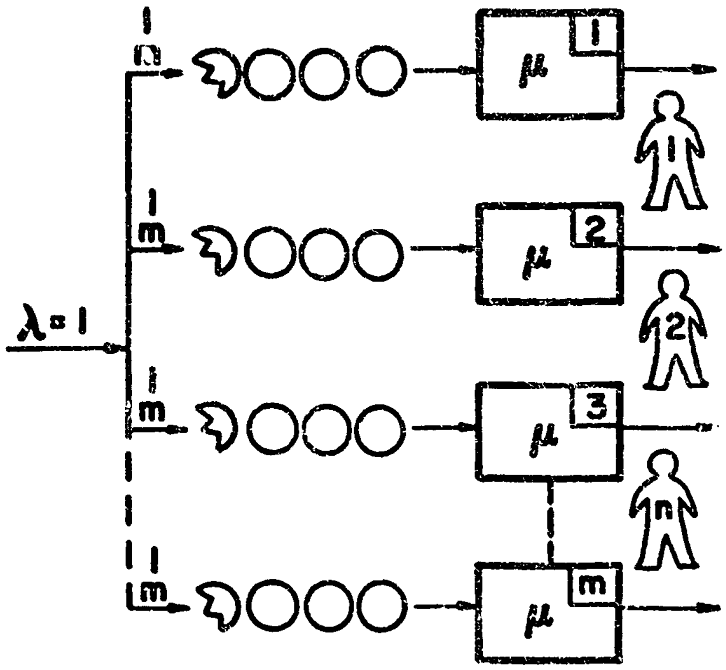

FIGURE 4

have been chosen as the technique is very general in structure. However, this particular system was chosen for three major reasons: (1) there are many actual situations which are closely approximated by this model; (2) the system is complex enough to illustrate the power of GERTS III QR and yet simple enough for reasonably compact discussion; and (3) a study was conducted on systems of this type using Nelson's (9) simulator; so a comparison of simulator effectiveness can be made, as well as some verification of the model.

Upon further examination of Figure 4 one may see why the network approach was applied to systems of this type: the system schematic itself suggests a network model. Thus the network description is conceptually appealing and investigation in this area seems natural. The GERTS III QR network model for a 3 service facility and 2 laborer system is depicted in Figure 5.

Figure 5 depicts a GERTS III QR network model for a system with three machines and two laborers. The accompanying set of system and arc parameters for this example are given in Table 1. The data in this table and the figure specify that this system has three machines and two laborers. For convenience this is referred to as a 3-2 system; the first number being the number of machines, m, and the latter being the number of laborers, n.

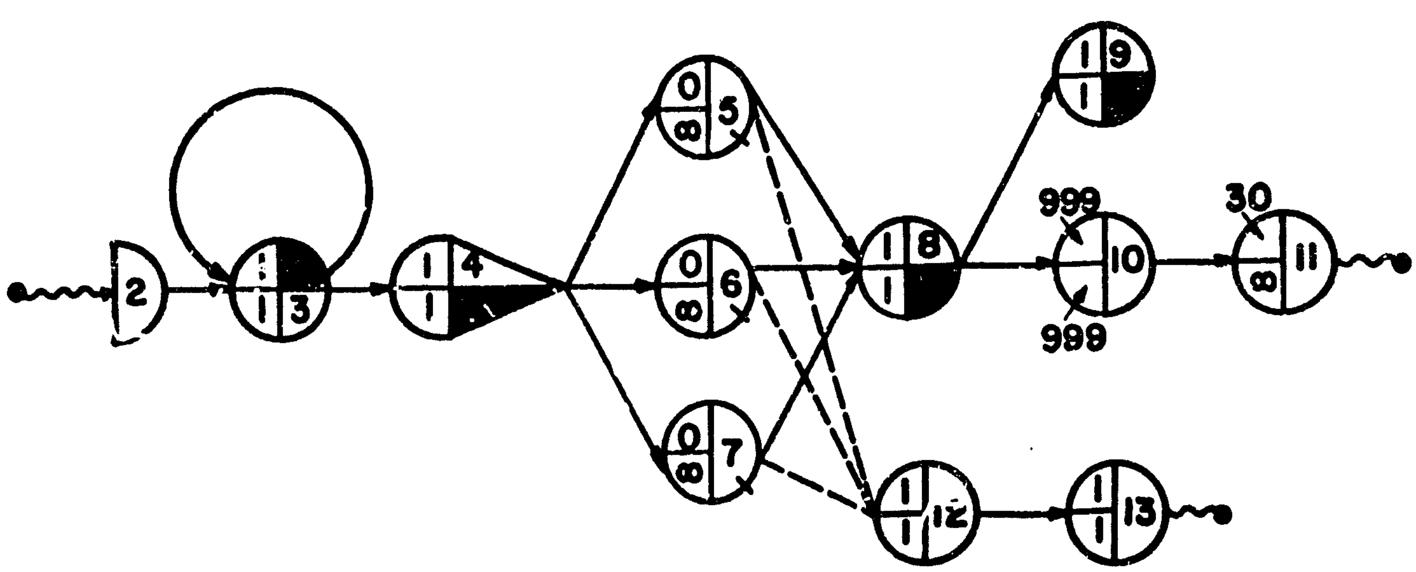

FIGURE 5

TABLE I

System and Arc Parameter for Example 1

System Parameters

Total number of nodes = 12
Number of sink nodes = 2
Number of nodes to realize the network is 1

Number of resources (laborers) = 2
Number of source nodes = 1
Statistics collected on 5 nodes

Arc Parameters

Parameter Sets

(Probability of realization; distribution of traversal time; labor efficiency)

Arcs	
(2,3)	(1.0; constant time of zero; no resources)
(3,3)	(1.0; exponentially dist. with mean = 1.0; no resources)
(3,4)	(1.0; constant time of zero; no resources)
(4,5)	(1/3; constant time of zero; no resources)
(4,6)	(1/3; constant time of zero; no resources)
(4,7)	(1/3; constant time of zero; no resources)
(5,8)	(1.0; exponentially dist. with mean = 1.8; e_1 = 1.0, e_2 = 1.0)
(6,8)	(1.0; exponentially dist. with mean = 1.8; e_1 = 1.0, e_2 = 1.0)
(7,8)	(1.0; exponentially dist. with mean = 1.8; e_1 = 1.0, e_2 = 1.0)
(8,9)	(1.0; constant time of zero; no resources)
(8,10)	(1.0; constant time of zero; no resources)
(10,11)	(1.0; constant time of zero; no resources)
(12,13)	(1.0; constant time of zero; no resources)

Experimental Design

The two basic building blocks in these experiments are the two machine, one laborer system and the three machine, two laborer systems. The 2-1/FCFS System can be solved analytically since it is equivalent to the 1-1/FCFS system. In fact, it was one of the methods used to validate the simulation model. The 2-1/SOT system was also solved analytically and used as a check of the model. (The analytical solution for this system is not readily found in the literature. An equivalent form was found in Saaty (21) and is attributed to the work by Phipps.)

The 3-2 system is the smallest truly labor limited system. All subsequent systems were designed to be multiples of these two basic systems.

For each system (2-1, 3-2, 4-2, etc.) a total of 26,000 jobs were simulated with the first thousand jobs discarded to initialize the system. In order to compare systems, at equivalent levels of utilization, the arrival rate was held constant and the mean service rate was adjusted to compensate for the varying number of labors. System utilizations of 0.75, 0.90 and 0.95 were selected for study.

As proposed by Nelson (14), system utilization, e.g. average labor utilization, may be estimated by equation 1.1 (in a simplified form):

$$1.1 \quad \hat{\beta}_{\ell} = \lambda/n\mu$$

Measures of System Performance

System performance is measured in terms of the "normalized" system mean waiting time, $\bar{w}/\bar{s}$, i.e., the actual mean waiting time, $\bar{w}$, measured in units of one mean service time, $\bar{s}$, as obtained from the simulations. As the experimental labor and machine limited systems currently defy analytical solution, the normalized mean waiting time values were obtained from the results of the simulations.

The Experimental Results

In Table II, the results using the FCFS, SOT and RANDOM machine loading and labor assignment rules are summarized. The main control variables were system configuration and labor utilization.

From the Table, it is evident that the SOT rule is more efficient in terms of normalized mean waiting time and mean flow time, than the FCFS and RANDOM rules. These results are intuitive as is the result that SOT yields a higher variance of flow time than FCFS. This rule quickly processes the short jobs and thus reduces both wait and flow times. However, those jobs with long estimated processing job times usually have a long wait, hence the higher variance. In comparing the variances for SOT and RANDOM, SOT has lower variance for systems of size 4-3 or greater. With respect to labor blocking, RANDOM had the greatest amount followed by SOT with FCFS having the lowest amount.

TABLE II

Waiting, Flow Time and Labor Blocking Characteristics of the Selected Priority and Assignment Rules for the Experimental Systems

			FCFS				SOT				RANDOM			
Run Number	SYST Configuration m-n	Estimated Utilization $\rho\ell$	Normalized Mean Waiting Time w/s	System Mean Flow Time MFT	Variance System Flow Time σ^2_F	Labor Blocking Hours B	Normalized Mean Waiting Time w/s	System Mean Flow Time MFT	Variance System Flow Time σ^2_F	Labor Blocking Hours B	Normalized Mean Waiting Time w/s	System Mean Flow Time MFT	Variance System Flow Time σ^2_F	Labor Blocking Hours B
1	2-1	.75	2.8170	2.9434	8.632	0	1.5839	1.9342	11.37	0	2.4627	2.5576	11.8626	0
2	2-1	.90	8.9722	9.0322	108.36	0	3.2375	3.8201	156.44	0	6.9245	7.1327	106.6822	0
3	3-2	.75	1.6540	4.0163	12.785	3,862	1.1074	3.1718	17.9	7,207	1.6477	3.9704	22.3177	9,646.44
4	3-2	.90	3.8347	8.2767	54.56	2,383	2.1676	5.7146	186.11	5,334	4.7777	10.3923	378.6786	6,573.49
5	3-2	.95	7.5662	16.3209	194.36	1,436	3.2044	8.0028	744.[illegible]	3,157	8.9884	18.9937	1407.6952	3,523.96
6	4-2	.75	1.7144	4.1791	15.72	2,483	2.9758	2.9741	15.25	4,995	1.3918	3.5742	[illegible]	6,795.01
7	4-2	.90	4.2369	9.5089	69.50	1,427	2.0108	5.0327	160.[illegible]	3,797	3.3636	7.8485	147.0993	4,681
8	4-2	.95	8.2942	17.7143	183.60	678	3.0524	7.7142	662.41	2,231	7.4744	16.3012	1014.8307	2,789.02
9	4-3	.75	1.5098	5.6777	28.33	15,247	1.0930	4.6351	[illegible]	12,704	1.5543	5.7418	56.7077	41,408.55
10	4-3	.90	3.6994	12.7286	124.35	8,233	1.9670	8.0104	262.35	20,018	3.8068	12.9790	741.0416	23,157.13
11	4-3	.95	11.6595	16.1677	1263.81	3,178	2.8624	11.0301	1101.90	11,344	7.2490	23.5234	4219.2482	12,419.25
12	6-3	.75	1.1407	4.9436	19.23	7,612	0.7892	4.6351	17.06	42,704	1.0473	4.6030	30.1960	22,447.76
13	6-3	.90	2.9908	10.8921	74.81	4,102	1.5379	6.8270	165.25	[illegible],147	2.2549	8.7786	192.2334	13,516.07
14	6-3	.95	4.1905	14.9073	124.41	2,875	2.1111	9.4592	647.41	7,658	4.7083	16.2617	1371.9761	8,669.71
15	6-4	.75	1.2064	6.6849	36.79	24,292	.8753	5.6471	44.19	63,639	1.1162	6.3464	61.6117	86,828.87
16	6-4	.90	3.0821	14.7855	171.95	12,422	1.5557	9.2272	259.99	18,799	2.8622	11.9128	760.7598	40,689.83
17	6-4	.95	5.2560	21.8385	356.26	6,845	2.2636	12.4106	940.82	20,270	4.6681	21.5474	1142.8013	22,382.82
18	8-4	.75	0.8992	5.7761	28.24	18,048	0.5814	5.0651	31.74	39,161	0.6583	5.5665	40.2196	53,931.64
19	8-4	.90	2.3019	12.0246	97.44	9,569	1.3073	8.3126	181.61	25,878	2.1318	11.3070	537.8106	29,231.72
20	8-4	.95	3.8669	18.5936	213.04	5,202	1.9666	11.1025	680.42	14,717	3.5549	17.2971	1733.1064	17,197.83
21	8-6	.75	1.3788	10.7384	111.32	110,604	0.9231	8.6866	106.98	372,308	1.1969	9.94	195.7367	564,578.13
22	8-6	.90	2.6991	20.0477	289.74	42,726	1.5646	13.8891	502.80	190,453	3.0855	22.1091	2375.6958	133,778.41
23	8-6	.95	5.0651	34.6755	702.96	13,210	2.1852	18.1993	1022.31	54,534	4.2840	30.1082	7454.2246	76,452.48
24	9-6	.75	1.0223	9.1184	73.61	85,517	.7892	8.0839	81.05	270,232	0.9699	8.8256	129.5426	163,357.71
25	9-6	.90	2.6499	19.8192	284.41	30,943	1.3480	12.7133	353.11	139,150	2.0591	16.5338	1130.5102	157,303.25
26	9-6	.95	3.6460	26.5558	455.17	17,818	1.8917	16.5241	1209.32	54,008	4.1337	29.1176	11148.0962	67,011.67

A somewhat surprising result is that the amount of labor blocking is higher for the SOT rule than the FCFS rule. The authors have no readily intuitive explanation for this phenomenon, and it is a subject for further research. Another effect is that for a given system with all rules, labor blocking drops as system utilization increases. Again, this is an intuitive result since labor blocking is a phenomena associated with idle laborers and this (idleness) decreases with higher systems utilization.

<u>Labor Effectiveness</u>

To compare selected experimental results with computed analytical results let's refer to Figure 6.

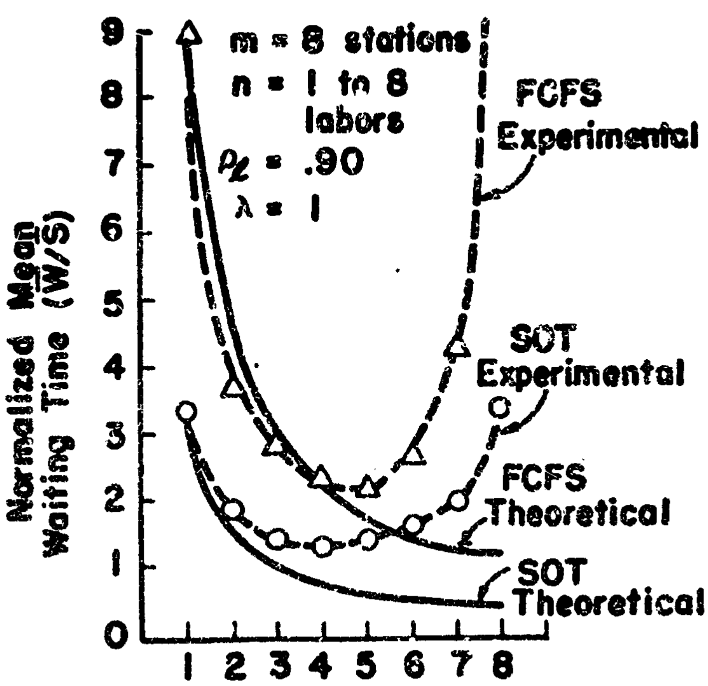

FIGURE 6

In Figure 6, the number of service centers, m, is fixed at 8, $\hat{\rho}_\ell$ = .90, and the number of laborers, n, varies from 1 to 8. The experimental change in $\bar{w}$/s resulting from a change in n is depicted by the u-shape curves. In addition to the experimentally obtained u-shape curves, curves for the "theoretical normalized mean waiting time" using FCFS and SOT are shown. The data points for these curves are given by the equation:

$$1.2 \quad (\bar{W}/\bar{S})_n = \bar{W}_1\mu_1/n$$

where

$(\bar{W}/\bar{S})_n$ = the "theoretical" normalized mean waiting time for a system with n laborers.

$\bar{W}_1$ = the mean waiting time for a system with one laborer

μ_1 = the mean service rate when the system has only one laborer.

n = the number of laborers.

The "theoretical" values given by this formula assume that each laborer added to the system is equally as effective as the first laborer; i.e., this formula neglects the effects of both labor blocking and "flexibility" in the system. Here "flexibility" is measured by the number of machine centers to which an idle laborer may be assigned. By comparing this theoretical curve with the simulated values of normalized mean waiting time, one may get some conception of the effects of blocking and "flexibility" on the system performance.

If additional laborers were as effective as the first, the normalized mean waiting time would decrease as shown by the theoretical curve. Since the effect of labor blocking is present, one would expect that as the number of laborers increases, their effectiveness would decrease, i.e. labor blocking increases because there are fewer machine centers to which an idle laborer can be assigned. A single laborer, in fact, can greatly affect system performance. If one is very busy then two things happen: (1) he is not often available to help cover the remaining work space and (2) jobs in this queue are blocked from the other laborers. A true measure of "labor effectiveness" remains to be formulated and appears to be an interesting area for future research. Nevertheless, we can see from Figure 6, that the SOT rule consistently performs the best with respect to $\bar{W}/\bar{S}$.

In Table II are shown some comparisons between multiples of a basic system at the same level of utilization. (4-2, 6-3, 8-4 and 3-2, 6-4, 9-6). As can be seen, the normalized mean waiting time decreases as the size of the multiple increases. This is due to the fact that, for example, an 8-4 system is more efficient than two 4-2 systems since laborers can cross over to any of the 8 machines in the first, while constrained to stay within each set of four machines in the second system. The same effect and analysis applies to mean flow time. Among these three rules, again SOT gives the best results.

Computer Experience

The simulations were run on a CDC 6600 computer at the University of Texas at Austin. A total of 26,000 jobs were simulated in each run. The first thousand jobs were not used in calculating the statistics but were required to initialize the system. The simulation run times ranged from 142 seconds for a 2-1 system to a maximum of 175 seconds for a 9-6 system. This includes 13.5 seconds for compile time. The run times for the FCFS and RANDOM systems were generally 20% higher than for the SOT rule. The core storage requirements were 40,200 words. For identical small systems the GERTS III QR simulator requires less than half the computation time required by the prior SIMSCRIPT simulator (9,14). For larger systems the savings is even more significant.

Conclusions

The results of this study show that for labor and machine limited queueing systems the GERTS III QR model is not only a feasible model but that it is a very efficient model. With respect to the example problem, the Shortest Operating Time rule is found to be superior to the First-Come-First-Served and RANDOM rules. This is also the case with purely machine limited systems. However, in the DRC systems a phenomenon exists in the form of labor blocking. This causes a change in normalized mean waiting time from the theoretical values.

The GERTS III QR simulator represents a powerful tool to simulate labor limited job shops and to evaluate various machine loading and labor assignment rules.

While the discussion in this paper centered upon parallel channel, single phase, homogeneous DRC experimental systems, GERTS III QR has been extended and used in a real world application. At a major air force maintenance base this model has been used to plan the work flow of aircraft engines through overhaul. The network consisted of three major maintenance lines in parallel with each line being a network of service centers in parallel and series, requiring up to ten resource classifications. Each line consists of at least 40 nodes and the entire network includes 132 service centers.

The preliminary runs were used to identify both bottlenecks and greatly underutilized service centers. This facilitated a shift of resources throughout the network in order to achieve a more balanced workload.

The experiments reported in this paper are being duplicated for heterogeneous DRC systems. These latter investigations have led to the evolution of some interesting and practical labor assignment rules and they will be the subject of a future paper.

In summary, GERTS III QR is a major improvement over the more traditional simulation packages for ease of structuring systems and in savings of computer time.

* * *

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REFERENCES

1. Burgess, R. R. and A. A. B. Pritsker, "Definitions and Listings for GERTS III, GERTS III Q, GERTS III C, and GERTS III R," NASA/ERC Contract NAS-12-2113, Virginia Polytechnic Institute, (May, 1970).

2. Burgess, R. R. and A. A. B. Pritsker, "The GERTS Simulations Programs: GERTS III, GERTS III Q, GERTS III C, and GERTS III R," NASA/ERC Contract NAS-12-2113, Virginia Polytechnic Institute, (May, 1970).

3. Conway, R. W., "An Experimental Investigation of Priority Assignment in a Job Shop," Memorandum RM-3789-PR, The Rand Corporation, Santa Monica, California, February, 1964.

4. Harris Roy D., "An Empirical Investigation and Model Proposal of a Jobshop-Like Queueing System," Western Management Science Institute, Working Paper No. 84, University of California, Los Angeles, July, 1965.

5. Harris, Roy D., "A Manpower Limited Service Center Model," Working Paper No. 68-14, Graduate School of Business, University of Texas at Austin, November 1967.

6. Harris, Roy D. and Michael J. Maggard, "Empirical, Analytical and Simulation Studies of Labor Limited Production Systems," Working Paper No. 70-16, The Graduate School of Business, The University of Texas at Austin, November, 1969.

7. Hoge, James, "A GERT Simulation of Labor Limited Queueing Systems Using the Shortest Operating Time Priority Rule," Unpublished Professional Report, Graduate School of Business, The University of Texas at Austin, July, 1972.

8. Hogg, Gary L., "An Analysis of Labor Limited Queueing Systems with a GERT Simulation," Unpublished Ph.D. dissertation, University of Texas at Austin, 1971. Available from University Microfilms, Ann Arbor, Michigan.

9. Maggard, Michael J., "An Evaluation of Labor and Machine Limited, Parallel Queueing Systems," Unpublished Ph.D. Dissertation, University of California, Los Angeles, 1968. Available from University Microfilms, Ann Arbor, Michigan.

10. Maggard, Michael J., "An Evaluation of Labor and Machine Limited, Parallel Queueing Systems: Part 1: Design Implementations," Working Paper 69-63, Graduate School of Business, The University of Texas at Austin, February, 1969.

11. Maggard, Michael J., "An Evaluation of Labor and Machine Limited, Parallel Queueing Systems: Part 2: Economic Models," Working Paper 69-68, Graduate School of Business, The University of Texas at Austin, April, 1969.

12. Nanot, Yves R., "An Experimental Investigation and Comparative Evaluation of Priority Disciplines in Job Shop-Like Queueing Networks," Management Sciences Research Project, Research Report No. 87, University of California, Los Angeles, December, 1963.

13. Nelson, Rosser T., "An Empirical Study of Arrival, Service Time, and Waiting Time Distributions of a Job Shop Production Process," Management Sciences Research Project, Research Report No. 60, University of California, Los Angeles, June, 1959.

14. Nelson, Rosser T., "Labor and Machine Limited Production Systems," Management Science, Vol. 13, No. 9, pp. 640-671, May, 1967.

15. Nelson, Rosser T., "Dual Resource Constrained Series Service Systems," Operations Research, Vol. 16, No. 2, pp. 324-341, March-April, 1968.

16. Nelson, Rosser T., "A Research Methodology for Studying Complex Service Systems," AIIE Transactions, Vol. I, No. 2, pp. 97-105, June, 1969.

17. Phillips, Don T., Gary L. Hogg, and Michael J. Maggard, "GERTS III QR: A GERTS Simulator for Labor Limited Queueing Systems," Research Memorandum No. 72-2, School of Industrial Engineering, Purdue University, Lafayette, Indiana, April, 1972.

18. Pritsker, A. A. B. and W. W. Happ, "GERT: Graphical Evaluation and Review Technique; Part I - Development," The Journal of Industrial Engineering, Vol. 17, No. 5, May, 1966.

19. Pritsker, A. A. B. and G. E. Whitehouse, "GERT: Graphical Evaluation and Review Technique; Part II - Probabilistic and Industrial Engineering Applications," Journal of Industrial Engineering, Vol. 17, No. 6, June, 1966, pp. 45-50.

20. Pritsker. A. A. B. and Don T. Phillips, "GERT Network Analysis of Complex Queueing Systems," Research Memorandum No. 72-1, School of Industrial Engineering, Purdue University, Lafayette, Indiana, January, 1972.

21. Saaty, Thomas L., Elements of Queueing Theory with Applications, McGraw-Hill Book Co., Inc., New York, 1961.

22. Takacs, Lajor, "Two Queues Attended by a Single Server," Operations Research, Vol. 16, No. 3, pp. 639-650, May-June, 1968.