SIMULATION OF THE FOUR-WAY AND TWO-WAY STOP SIGN CONTROLLED INTERSECTIONS

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Summary

The analytical complexity of most automobile traffic intersections strongly directs the researcher to simulation as his tool of analysis. This tool, through the General Purpose Systems Simulation language (GPSS), was used in a recent investigation of two different stop sign controlled intersections. On the basis of a comprehensive literature search, the authors feel that the simulation models reflect, as nearly as is presently possible, the "real-life" situations. This paper, an outgrowth of that investigation, includes discussions of:

(1) basic assumptions of the models,
(2) empirical probability distributions employed,
(3) comparisons for a selected objective function,
(4) comparisons of the models with existing literature, and
(5) conclusions and recommendations for further research.

Introduction

Automobile traffic control is one of the most rapidly growing problems in our increasingly urbanized society. The following presents a discussion of a recent study of stop sign control at traffic intersections. At the present time, the complexity of all but the simplest traffic intersections makes both analytical and empirical methods of analysis difficult. In order to avoid these difficulties, the authors have turned to simulation as a tool suitable for such an investigation. The primary advantages of this approach are that simulation is fast, comparatively inexpensive, and the circumstances and results associated with a simulated study are reproducible. Since the circumstances of the traffic situation are reproducible, the researchers may vary any one of the parameters affecting an intersection while keeping all others constant and, thereby, gain information and insight into the effect of that parameter.

The primary tool of this investigation is the General Purpose Simulation System language (GPSS). GPSS was chosen because of its flexibility and power when applied to a queuing situation such as that presented by a traffic intersection. A secondary reason was the automatic statistics gathering capabilities of GPSS.

Figures 1 (a) and 1 (b) present schematic representations of the two intersections which have been investigated. They are: (a) the four-way stop intersection, and (b) the two-way stop intersection. The four-way stop intersection consists of two, two-lane orthogonal streets with every approach controlled by a stop sign. The two-way stop intersection has the same configuration with only one street or two approaches controlled by stop signs. The other street is unconstrained except for possible conflicts provided by vehicles stopped in or moving through the intersection.

In an attempt to describe, as nearly as possible, the "real-life" situation at a traffic intersection, the simulation models have been constructed from a detailed microscopic viewpoint. This approach allows an automobile's elemental intersection crossing movements to be considered and, thus, greater flexibility and accuracy may be achieved.

Basic Assumptions

Certain basic assumptions are common to both the simulation models created in this investigation. They are the following:

(a) All drivers are cautious and law-abiding.
(b) Normal daylight driving conditions prevail.

(c) Low probability events such as accidents are ignored.
(d) Owing to the discrete nature of a digital computer, the basic unit of time employed is .01 second.
(e) Pedestrians are not considered in the models.
(f) The distribution of movements through the intersection in both of the models is 80 percent going straight, 10 percent turning left, and 10 percent turning right.
(g) Free-moving vehicles travel at 30 miles per hour or 44 feet per second.
(h) All vehicles are considered to be the same size.
(i) A vehicle has no effect on vehicular behavior in the intersection once it has exited the intersection.
(j) No passing in the intersection may take place on a two-lane, bi-directional street.

Probability Distributions Used in the Models

A literature search revealed that previous investigators had obtained, through empirical methods, most of the probability distributions necessary to simulate the selected intersections.

In order to simulate any traffic situation, some method must be employed to generate the arrival of vehicles. The probability distribution used for generating vehicles in both models is Schuhl's composite exponential distribution (16) as modified by Kell (7), (8).

The cumulative distribution of the composite exponential distribution is given by

$$P(h \leq t) = 1 - \left[(1-\alpha)\, e^{\frac{-t-\lambda}{T_1-\lambda}} + \alpha\, e^{\frac{-t-\tau}{T_2-\tau}} \right]$$

where

$P(h \leq t)$ = the probability of a headway, h (the distance between two cars), being less than or equal to t seconds.

α = proportion of restrained vehicles in the traffic stream.

$(1-\alpha)$ = proportion of unrestrained vehicles in the traffic stream.

T_1 = average headway of unrestrained vehicles.

T_2 = average headway of restrained vehicles.

λ = minimum headway of unrestrained vehicles.

τ = minimum headway of restrained vehicles.

The primary reasons for its use in the models are that it has achieved wide acceptance among other researchers (12), (14), (15), (17) and Kell has evaluated the parameters of the distribution so that it can be used for any volume approach rate. Both models generate vehicles for each approach independently of the other three approaches. Thus, any approach rate configuration to the intersection is possible.

A common problem drivers face at an intersection is that of deciding to cross the traffic flow or to enter it when a space in the traffic becomes available. In the two-way stop model, four "gap acceptance" situations must be considered. In the terminology of the traffic engineer, a gap is the time in seconds between two vehicles in a traffic stream. A lag is defined as the part of a gap that remains for observation when a vehicle arrives at the intersection, i.e., the time in seconds that separates a car from the entry point of an intersection. Bissell (2) has shown that gaps and lags are equivalent for the purpose of deciding whether to enter an intersection.

Bissell (2), Wagner (18), and Gerlough (5) have found that drivers faced with a gap acceptance situation accept or reject the gaps in the associated traffic flow according to a log normal distribution. The cumulative distributions used for gap acceptance in the two-way stop model are shown in Figure 2.

Another distribution required by the simulation models is that governing starting times from stop signs. Thomasson and Wright (17) conducted an extensive field study for the purpose of discovering the probability distribution associated with the time it takes a car to enter the intersection from a stop sign after the decision to move has been made. This starting time distribution was found to be a truncated normal distribution with a mean of 1.75 seconds, a standard deviation of 1.15 seconds and having approximately a 7 percent probability of a starting time of 0 second. This distribution has been incorporated directly into both of the models.

The intersection transit time distributions used in the models are also taken from Thomasson and Wright (17). They performed a regression fit to empirical data which indicated that intersection transit times are normally distributed for cars starting from stop signs. Each possible movement from a stop sign has its own distribution with all three sharing the same standard deviation of one second. The means of the distributions were found to be 3.15, 2.30, and 2.85 seconds for the straight, right and left movements, respectively.

Permissible Movements in the Intersection

Another major consideration in simulating any intersection is deciding what movements may be made through the intersection. Obviously, simultaneous movements may produce a conflict and some system of priority must be set up through the model logic to account for this contingency.

Only 12 possible single-vehicle movements exist at an intersection of two, two-lane orthogonal streets. If the consideration of direction is removed, the number of unique movements is reduced to three. Figure 3 illustrates these three movements with their associated "conflict" movements. The dashed lines show the basic movements, and the solid lines show the movements which conflict with them. Thus, six other movements conflict with any vehicle moving straight through the intersection. In a like manner, two conflict with a car turning right and six conflict with a car turning left.

From another viewpoint, one may observe the possible simultaneous movements that can be made through the intersection and select which the model logic will allow. Four possibilities are to be considered at each approach; no car, a car desiring to go straight, a car desiring to turn right, or a car desiring to turn left. Therefore, considering the intersection as a whole, there are 4^4 or 256 possible simultaneous movements.

If the consideration of direction is removed, many of these combinations become identical. Figure 4 illustrates the 18 different permissible simultaneous movements which the model logic allows. It is recognized that some of the simultaneous movements illustrated in Figure 4 may be prohibited by certain physical conditions at a particular intersection. If this is the case at some intersection of interest, the programming logic of the model may be altered to meet such a contingency.

Information Provided by the Models

The four-way stop sign model provides many useful and interesting statistics. Included in the normal GPSS terminal output are queue statistics for each of the four directions. The queue statistics include the average contents, the maximum contents and the average time each car spent in the queue. In addition, certain data on delay in entering the intersection are also collected. These data are presented in the form of GPSS tables and include the following information: mean delay with the associated standard deviation, sum of the delays, the number of cars contributing to the sum, and a histogram showing the number of cars whose delay fell in each of a selected number of equal intervals of a chosen length. This information is provided for the intersection average, for each directional average, and for each of the three movements from the four directions.

Further, statistics on the time spent from arrival to exit of the intersection are displayed in GPSS tables for the intersection average and the four direction averages.

The statistics provided by the two-way stop model are similar to those of the four-way stop model. The queue statistics and delay statistics prior to intersection entry are provided in exactly the same way as in the four-way stop model. In addition, GPSS tables are provided for main street and side street averages on delay prior to intersection entry and on total time from arrival to exit of the intersection.

The information provided automatically by the models in their present form is only a small subset of the complete set of information that can be obtained.

Resource Requirements

The GPSS programs have purposely been kept compatible with the GPSS III compiler so that they may be used on smaller computers which do not have the capacity for the GPSS/360 version. Although the programs have run successfully on an IBM 7040 computer, the runs for this investigation were made on the IBM 360/50 computer at the University of Arkansas. The computer time necessary for a successful simulation varies directly but not proportionally with the total vehicle approach rate to the intersection. In this study, the range of simulated time to real time ratios varied from 250 to 1 to 20 to 1. (Real time in this context includes compilation, execution, and I/O time.) The two-way stop model required more time for the same approach rate configuration.

Only a rudimentary knowledge of GPSS is required to use the models in their present form. However, a somewhat more sophisticated understanding would be needed for modifications of the programs to fulfill special needs. Given this increased understanding, it is quite easy to add capabilities such as additional lanes and left-turn lanes to the models.

Selected Results

The present discussion will be limited to an analysis of delay incurred by vehicles as they approach, arrive at, and pass through the intersection. In the models, delay is defined in one of two ways. Delay at an approach controlled by a stop sign is that time period from when the vehicle enters the stop sign queue until it departs the queue and begins crossing the intersection. Delay on the "main" street in the two-way stop model is incurred whenever a vehicle must slow or stop during its transit of the intersection.

It would appear that the primary objective of traffic intersection control is to minimize vehicular delay at the intersection consistent with vehicle safety and proper traffic control. In that context, we will say that the method of control which yields the lesser average delay per vehicle is the superior method. The authors are aware of certain extreme cases where inordinate individual delay at some approach outweighs the stated objective. This situation can be dealt with by computing a measure of effectiveness which is a function of both the mean delay and the standard deviation. This was done for the cases presented herein, but no difference in results was obtained.

Thirteen simulation "runs" with ten hours of simulated time in each run were made for each model. The only thing that was varied in the selected simulation runs was the approach rate configuration to the four directional approaches. With respect to the approach rate configuration, three sets of simulation runs were performed.

The first set of simulation runs was concerned with symmetric approach rates, i.e., each direction having the same approach rate. The volume of vehicles approaching the intersection in vehicles per hour for this set was incremented in steps of 50 from an initial value of 50 to a final value of 300 vehicles per hour. Thus, six runs of symmetric approach rate conditions were made for both models.

The second set of simulation runs dealt with nonsymmetric approach rate conditions where the main street had twice the volume of the side street. The main street, going east and west, had volumes of 100, 200, 300, and 400 vehicles per hour.

In the third set of simulation runs, the main street had three times the volume of the side street with selected volumes of 150, 300, and 450 vehicles per hour.

The results of these simulation runs give the clear indication that a two-way stop intersection yields less average delay than a four-way stop intersection when the intersection is considered as a whole. This statement is supported by the graphical presentation of selected symmetric approach rate results given in Figure 5. The upper graph shows the average delay per car without regard to direction or turning movement. Similarly, the two lower graphs show the average delay when only cars going west or south, respectively, are considered. Delay for westbound vehicles is much less for the two-way stop intersection than the four-way stop intersection. Clearly, this should be the case. The westbound vehicles in the two-way stop model are unconstrained except by interference in the intersection; those in the four-way stop model must stop at the stop signs.

The southbound vehicles, which must stop in either model, exhibit a nearly opposite tendency. In all but the extreme case of 300 vehicles per hour, the four-way stop model yields the lesser delay. The additional delay experienced by two-way stop vehicles is probably due to the gap acceptance situation they confront. However, the increase of delay on the north-south side street is outweighed by the decrease of delay on the east-west main street. Therefore, the average delay for the intersection as a whole is less for the two-way stop model.

The three graphs in Figure 5 are representative of the results which were obtained from the models. Similar graphs were prepared for each approach and for each of the three turning movements at each approach. Opposite approaches had the same vehicle approach rate and yielded very similar graphs. Further, the graphs of average delay for the turning movements at each approach were very similar to those for the approach average. Only one significant difference was noted: The left turning vehicles on the main street of the two-way stop model experienced a noticeable but small amount of additional delay when compared to those cars going straight or turning right. The graph of westbound vehicles turning left is pictured in Figure 6.

Figures 7 and 8 present the results obtained from the second and third set of simulation runs. The values given on the horizontal axes of the six graphs refer to the vehicle approach rate associated with the main street. It should be noted that they point to the same conclusions that are indicated by the graph obtained from the symmetric approach rate conditions. Further, the remarks contained in the previous paragraph apply equally well to the non-symmetric approach rate conditions.

The results from the upper graphs of Figures 5, 7, and 8 may be viewed in a slightly different way. Figure 9 presents the cited results with the average delay plotted against the total intersection approach rate, i.e., the sum of the approach rates at the four approaches. Viewing the results in this way permits another tendency to be observed. Apparently, average delay decreases as the approach rate becomes increasingly asymmetric. One contributing factor to this phenomenon may be interference in the intersection which is maximized by symmetric conditions.

To the best of the authors' knowledge, no published work concerning simulation of the four-way stop sign intersection has been attempted in the past. However, Lee and Vodrazka (11) performed an empirical study both of the four-way stop sign intersection and the two-way stop intersection. Because of various differences in intersection configuration and intersection parameters no direct comparison of results was attempted. However, the same major conclusions that (a) the two-way stop sign intersection yields lesser delay for the intersection as a whole, and (b) the average delay of side street vehicles is less for the four-way stop sign intersection were reached.

One minor difference was noted. The present results show a much more dramatic increase in average delay at higher vehicle approach rates for both types of intersections. This probably was due to Lee and Vodrazka's inability to collect data based on higher vehicle approach rates. Thus, their estimates of delay at higher approach rates were based on extrapolations from data gathered at lower approach rates.

Direct comparisons with published results concerning the simulation of two-way stop sign intersections (9), (12), (17) was not possible for a number of reasons. Primarily, these earlier models do not as fully describe the physical situation as the present model.

Thomasson and Wright (17) neither considered delay to main street vehicles nor left turning movements from the two stop sign controlled approaches. They did, however, perform meaningful field studies, some of which are incorporated into the present models.

Lewis and Michael (12) considered an intersection where the main street had four lanes. A strictly deterministic method was employed for gap acceptance and many analytical formulas governing vehicle behavior were incorporated into the model. Further, it was assumed that side street vehicles never interfere with the progress of main street vehicles. Kell (8), (9) formulated a rather complete model with the information available at that time. However, he was not able to include a starting time distribution from the stop signs or transit time distributions for the movements through the intersection.

In addition, in (17) and (12) a periodic scan was used to note events occurring in the simulation. In a <u>periodic scan</u>, the state of the model is observed only at specific predetermined times with a constant amount of time separating each scan. The present research employs the GPSS supplied <u>event scan</u> where the state of the model is observed each time an important or "moving event" occurs in the simulation. Unwarranted delay and erroneous statistics could be introduced if a periodic scan does not coincide with the occurrence of a moving event. This possibility is avoided through the use of an event scan. A limited event scan was used in (9) where the scan was triggered by the arrival of a main street vehicle.

Conclusions and Recommendations

The primary conclusion that may be gained from this study is that two-way stop intersections yield less average delay than four-way stop intersections. A secondary conclusion is that GPSS is an excellent method for simulating traffic situations.

In order that the models might represent the actual situation even closer, field studies need to be performed to determine the distribution governing the following things at an intersection:

(a) The distribution of the time to move from the second to the first position in a stop sign queue.
(b) The distribution of the time required for a driver to stop his car and recognize the intersection situation after having assumed the first position in a stop sign queue.

(c) The distribution of the difference in intersection exit times of two cars making the same move in the intersection when the first car has delayed the second.

This paper represents the first fruits of the efforts contained in (1) where five additional models of unique traffic intersections were formulated. Those readers interested in more detail are referred to reference (1) where listings of the programs and discussions of the models' logics are given. It is hoped that a more detailed analysis of results from the models presented here and of similar results from the other models cited might be performed in the future. A number of additional things should be done. First, more runs considering various turning movement percentages and approach rate configurations can be made with comparative ease. By doing this, curve fits to the obtained data should be possible. Work of this type might be extended to an investigation of possible analytical relationships between the various intersection parameters.

Certainly, other objective functions besides average delay should be considered. Perhaps the best "measure of effectiveness" would result from such an investigation. With these models and others that can be written, traffic networks could be formed using microscopic models as building blocks. In this way, an urban traffic network could be studied on a microscopic rather than a macroscopic level.

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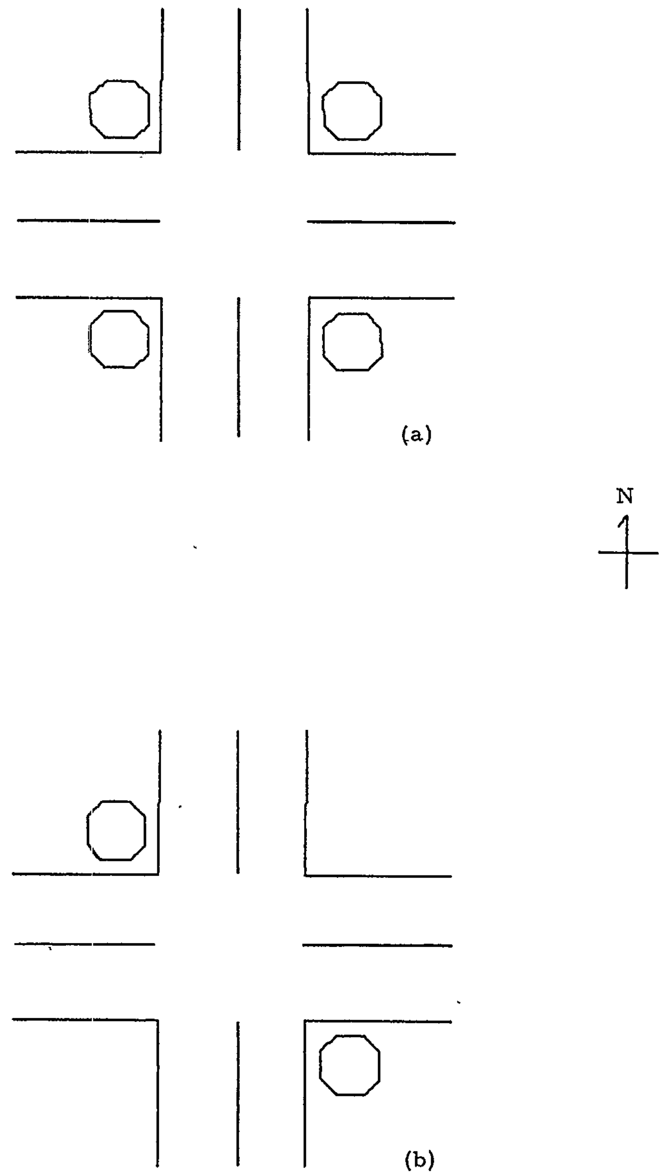


MODELS STUDIED

FIGURE 1

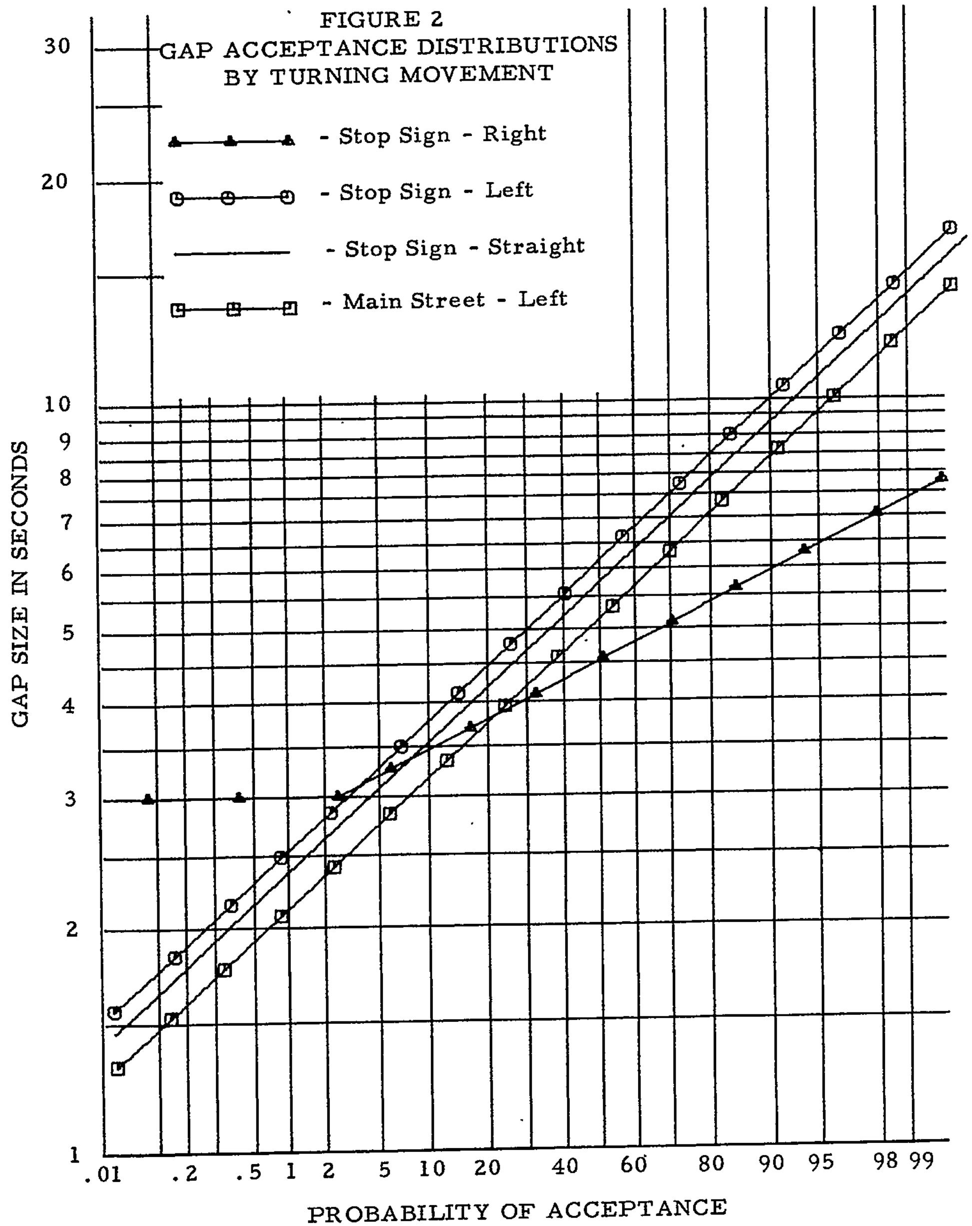


FIGURE 2
GAP ACCEPTANCE DISTRIBUTIONS
BY TURNING MOVEMENT

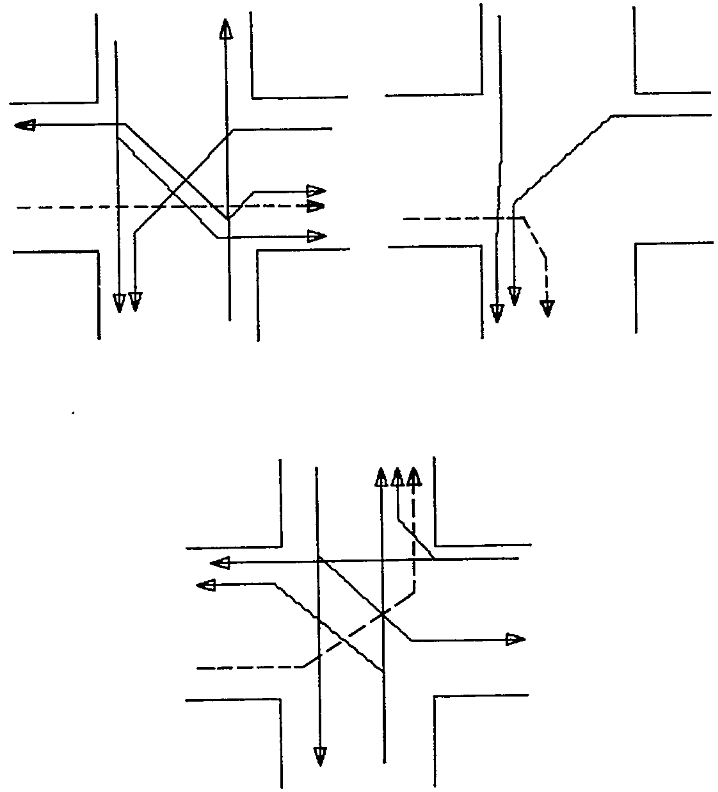
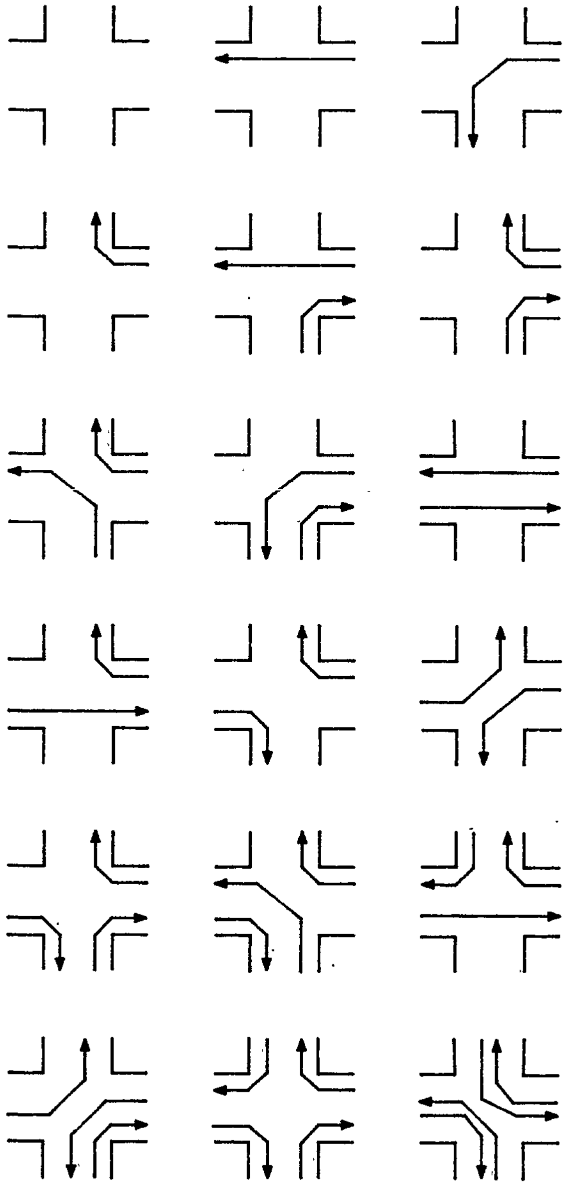
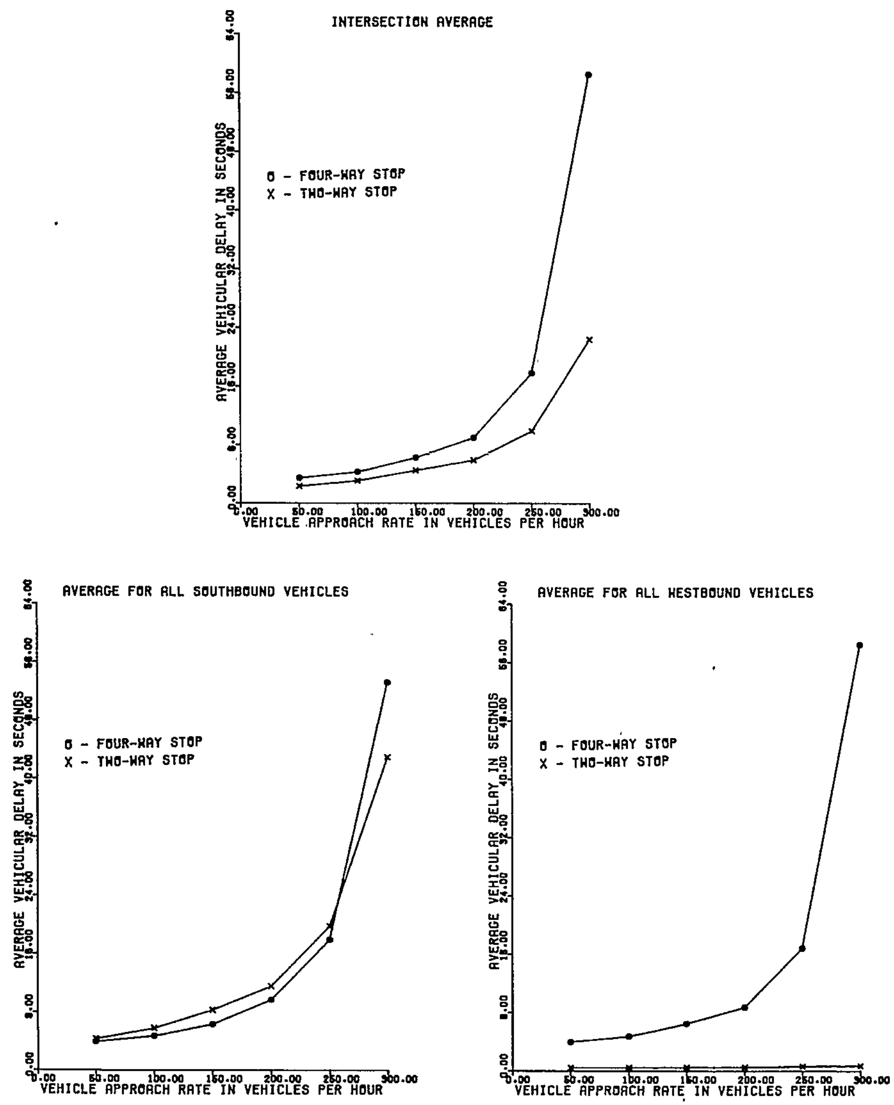


FIGURE 3
CONFLICT MOVEMENTS



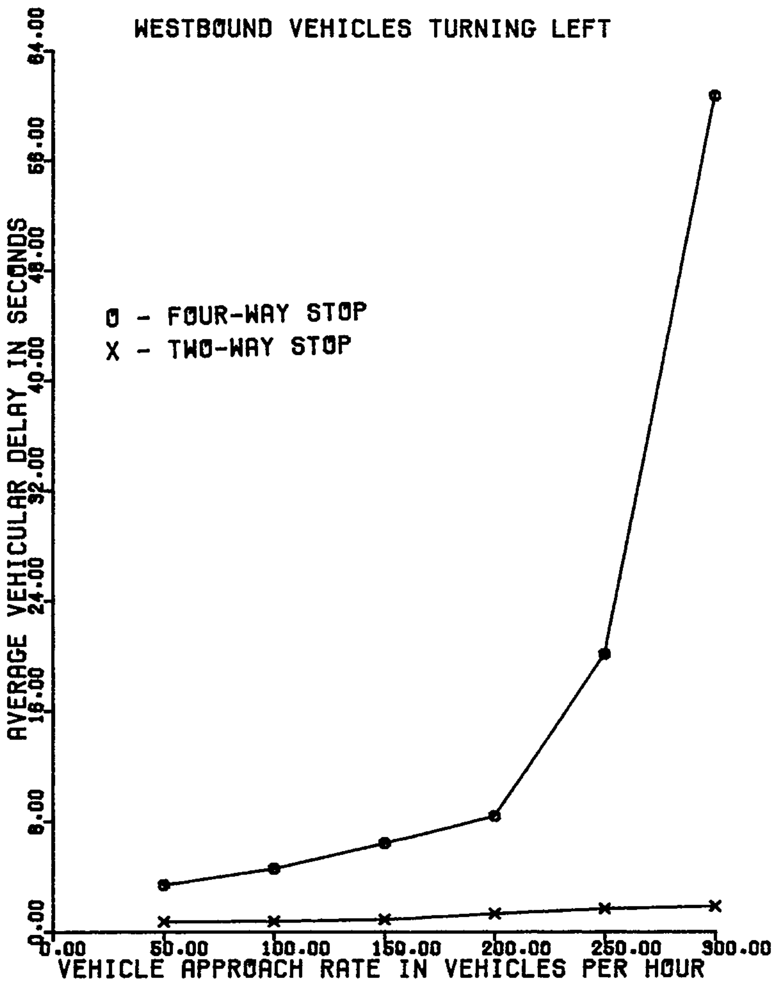
Possible Simultaneous Movements

Figure 4



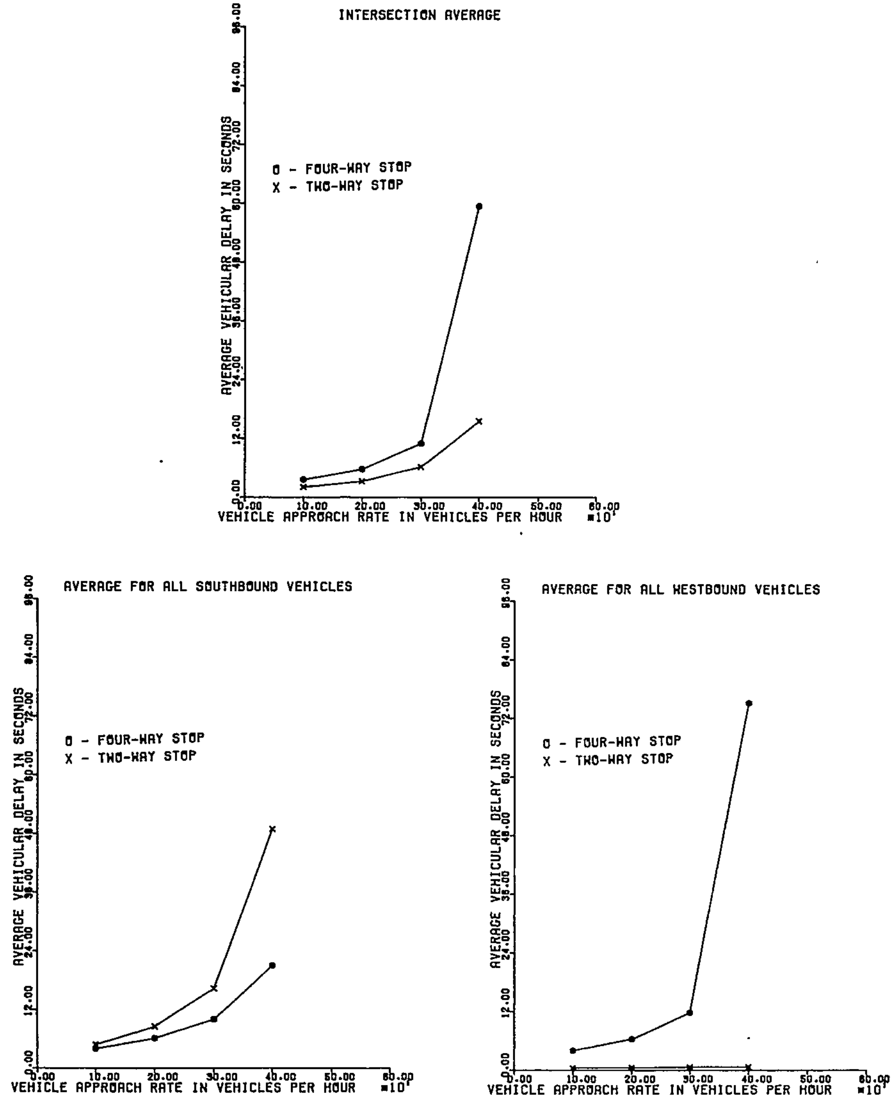
Delay Graphs for Symmetric Conditions

Figure 5



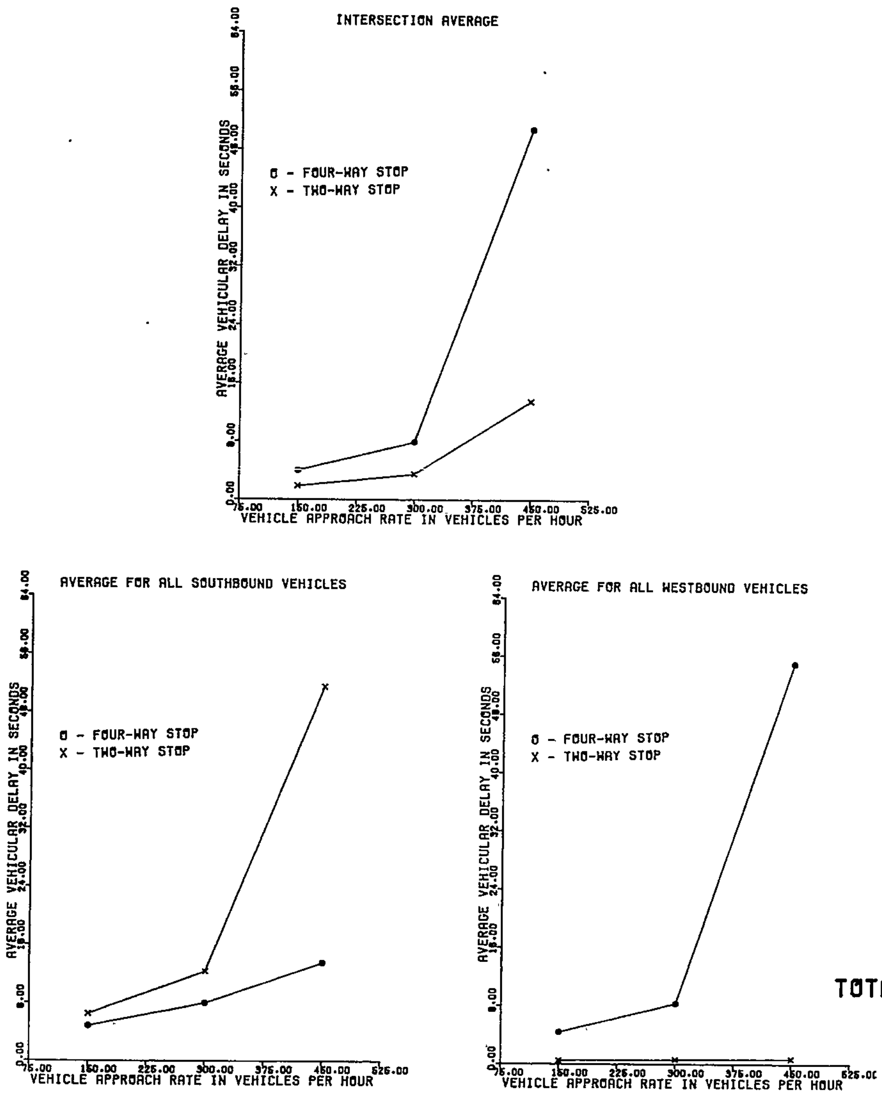
Delay Graph for Westbound Vehicles Turning Left

Figure 6



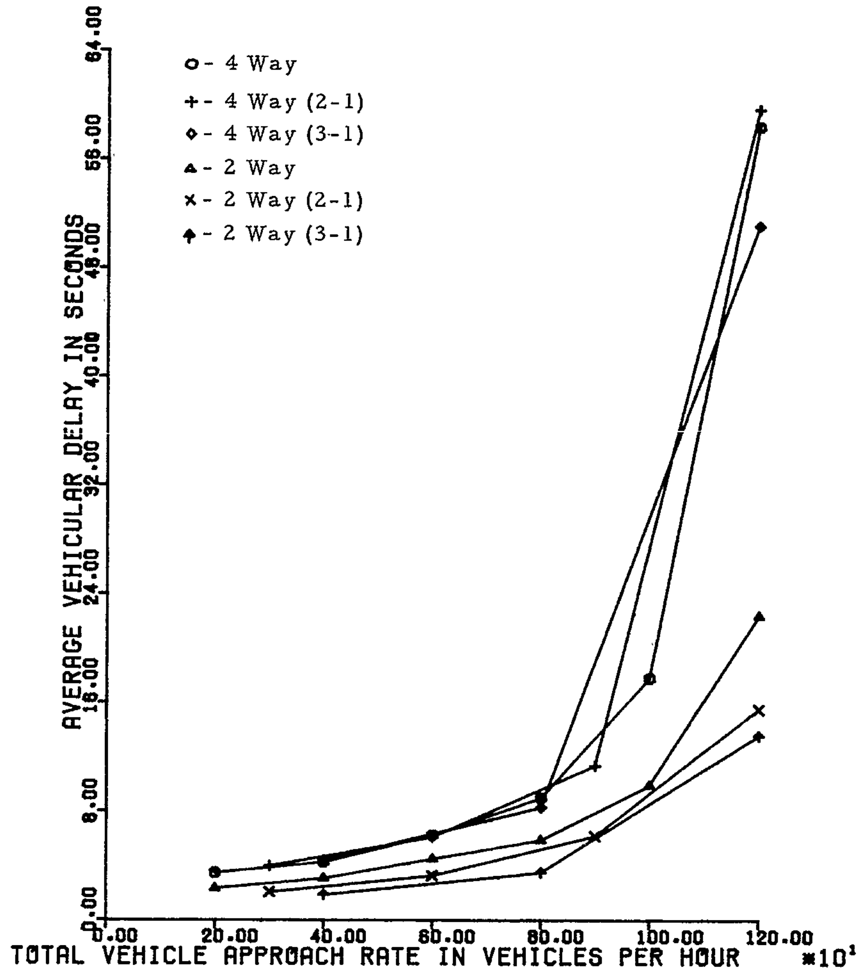
Delay Graphs for Asymmetric Conditions (2 to 1)

Figure 7



Delay Graphs for Asymmetric Conditions (3 to 1)

Figure 8



Delay Graph for Total Vehicle Approach Rate

Figure 9