APPLICATION OF THE GERTS II SIMULATOR IN THE INDUSTRIAL ENVIRONMENT[1]

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Summary

GERT was originally developed to aid in the analysis of stochastic networks. GERT can be used to graphically model and analyze complex systems. Recently a simulator model, GERTS II, has been developed to solve GERT Networks. The simulator language used in the development of this model was GASP IIA. This paper discusses the possible application of GERTS II to model and analyze (1) assembly line operations, (2) project management networks, (3) conveyor systems and (4) inventory systems. Finally, an actual application dealing with a job shop loading problem is presented.

Introduction

In recent years, networks and network analysis have played an increasingly important role in the description and improvement of systems. This is primarily because of the ease with which systems can be modeled in network form. However, there are other reasons for using networks; they often serve as a communication mechanism to discuss significant features of the system or an excellent means to specify the data requirement for analysis of the system. GERT: Graphical Evaluation and Review Technique[5] is a new technique which has been developed to analyze a class of network known as stochastic networks. The elements of a GERT network are directed branches or activities and logical nodes or events. A directed branch has associated with it one node from which it emanates and one node at which it terminates. Two parameters are associated with a branch: (1) the probability that a branch is taken, given that the node from which it emanated is realized; and (2) a time required to accomplish the activity which the branch represents. The time can be a random variable. The GERT Simulation Program[4] (referred to as GERTS) was written by A. Alan B. Pritsker and Philip C. Ishmel to analyze GERT network models through the technique of simulation. It is the purpose of this paper to demonstrate how this program might be applied to model and analyze certain industrial systems such as inventory, project management, conveyor and assembly line systems. Finally, an actual application to the modeling of a cable shop will be presented. The cable shop was modeled to perform a job shop loading study.

GERTS

Due to space limitation, it will not be possible to give a detailed explanation of either GERT or GERTS. This paper, therefore, assumes that the reader has some knowledge of both GERT and GERTS. The theory and applications of GERT are discussed by Pritsker, Happ and Whitehouse[5 6 12], and Pritsker[4] gives a listing of the required input to the GERTS package and a discussion of the program's characteristics. GERTS has been programmed using GASP II[7].

Briefly, GERTS can accommodate networks which have OR and AND logical operations associated with the input side of a node and deterministic or probabilistic operations associated with the output side of a node. As in GERT, each activity has associated with it a probability that a branch is taken, given that the node from which it emanates is realized and a probability density function of the time to realize the branch. With GERTS, two additional characteristics can be associated with an activity. These are a counter type and an activity number. The counter type number specifies the counter to be increased by 1 every time the activity is realized. Activity numbers are given to activities to permit network modifications based on the realization of the activity. Network modification involves the replacing of a node by another node. The node to be replaced is deleted from the network when and if it is realized. The activities then caused to occur are from the node that is inserted. In this way a node can be changed many times before it is actually realized.

GERTS also has a queue node available which allows the modeling and collection of statistics for situations involving waiting lines at particular nodes. The queue nodes include such parameters as maximum queue length allowed before being shunted to another node and queue discipline.

The results obtained from the program are:

1. The probability that a node is realized.
2. The average, standard deviation, minimum and maximum time to realize a node.
3. A histogram of the times to realize a node.
4. The average, standard deviation, minimum and maximum counts observed during the time to realize a node.

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5. The average, standard deviation, minimum and maximum of the waiting time and queue length at each queue node.

For the purpose of discussion in this paper, several specialized notations have been used. They are given below:

1. The symbol N5 is to be read "node 5".

2. The symbol P(2,3) represents "the path from node 2 to node 3".

3. C1 refers to counter number 1.

4. M1 refers to network modification number 1.

5. The symbol QN3 is to read "queue node 3".

Possible Industrial Applications of GERTS

The following examples are presented to demonstrate the capabilities, advantages and disadvantages of GERTS in the industrial environment. It is hoped that the following section will stimulate thought and discussion relating to this new simulation package.

Assembly Line Operations

Components for a subassembly in a manufacturing process are produced on assembly lines A, B and C. The time to produce a part on a given assembly line is given below:

Assembly line	Distribution of time per part (in minutes)
A	normal, mean = 3, std. dev. = 1
B	uniform, between 3 and 5
C	normal, mean = 4, std. dev. = 2

If all three components must be present for the assembly to take place, what would be the expected distribution of the number of units assembled per hour? It is assumed that there is no in-process inventory; that is, no new production is begun on lines A, B or C until the previous assembly is complete.

The GERTS model of this hypothetical assembly line is shown in Figure #1. The nodes in this figure contain two pieces of information previously not discussed. The two numbers inserted in the left-hand portion of each node represent the number of times that activities ending at that node must be realized (called releases) before the node is realized. When the node is realized, the activities emanating from that node may begin. The top number represents the number of releases for the first realization of the node, and the bottom number represents the number of releases for all succeeding realizations of the node. For example, N8 needs three releases each time it is realized while N3 can only be realized once and needs one release. The parameters for each path are shown in the insert in the figure. P(5,8) is realized with probability 1.0 given that node 5 is realized. The time to realize the path is an observation from a normal distribution with mean of 3 and standard deviation of 1. This insert also includes information about counters and modifications. P(8,4) activates C1 when it is realized.

In Figure #1, P(5,8), P(6,8) and P(7,8) represent subassembly lines A, B and C respectively, and N8, which requires 3 releases, can only be realized if all three paths leading into N8 have occurred. Thus, N8 represents the final assembly of parts from lines A, B and C. Note that P(8,4) is tagged with a counter, C1, which records the number of parts that have been assembled. The simulation is terminated by the use of a "timer path", P(2,3), exactly 60 time units after the simulation begins; thus, C1 will be the number of units assembled in 60 minutes.

The results of the model are shown below:

Final Results for 50 Simulations

Node	Prob./ Count	Mean	Std. Dev.	Min.	Max.
3	1.0/	60.0	0.0	60.0	60.0
3	1	11.76	0.744	10.0	13.0

The mean of the number of assemblies produced is 11.76 with a standard deviation of 0.744 units. The minimum number of assemblies in an hour will be 10 while the maximum will be 13.

Many assembly lines, unlike the one just discussed, allow in-process inventory. Consider the following example similar to one suggested by Pritsker and Kiviat[7]:

> The maintenance facility of a large manufacturer performs two operations. These operations must be performed in series; operation 2 always follows operation 1. The units that are maintained are fairly bulky, and there is limited space available for the maintenance facility. Currently, only eight units including units being worked on, can be handled at one time. A proposed design leaves space for two units between work stations for operations 1 and 2, and space for four units before work station 1. Current company policy subcontracts maintenance of a unit if it cannot gain access to the queue preceding work stations 1 or 2.
>
> Historical data indicates that the time interval between requests for maintenance is exponentially distributed with a mean of 0.4 time units. Service times are approximately exponentially distributed with the first station requiring on the average 0.50 time units and the second service station, 0.25 time units. It takes 0.2 time units to move a part from work station 1 to 2.

This problem can be modeled using GERTS as shown in Figure #2. The queue nodes, are used to model the queueing behavior in front of each service facility. The number in the small circle attached to the queue node represents the maximum capacity of the queue. If a unit arrives while the queue is at maximum capacity, it will be shunted along the doublepath emanating from the small circle. Thus, if QN1 has 4 people in its queue, an arriving part will be shunted along P(1,8) which is representative of subcontracting of maintenance. Similarly, if QN2 is full, the unit is shunted to P(2,9). P(1,6) and P(2,7) represent the service time at service facilities 1 and 2. P(5,1) is the transit time from service facility 1 to service facility 2. The interarrival time of units is represented by P(4,5). The simulation is terminated by P(3,10) exactly 1000 time units after the start of the run. The statistics of interest for this problem would be those related to -

C1 - the number of units shunted to subcontract for total maintenance

C2 - the number of units shunted to subcontract for the second portion of maintenance

C3 & C4 - the number of units serviced by facilities 1 and 2

QN1 & QN2 - the in-process inventory preceding service facilities 1 and 2

Conveyor Systems

The modeling of conveyor systems is similar to that of modeling assembly lines. The particular example that we are going to consider is one from a class of problems studied by Pritsker[8], Burbridge[1] and Disney[2].

The conveyor system of interest has units arriving at a service facility from some outside source. The service facility may have provisions for the storage of a limited number of units prior to service. If the units cannot enter the service area due to all the storage space being occupied, they recirculate and reenter the arrival stream at some later time. If the units can enter the service area, they must then wait their turn for service. The specific problem will be assumed to have three servers. The service time for each server will be assumed to be normal with mean of 1 time unit and a standard deviation of 0.5. There is assumed to be room for storage of 2 units in front of each server. Units arrive with an exponential interarrival time with a mean equal to 0.5. The travel time from station to station is 1 time unit and recycle time is 25 time units. This example is modeled in Figure #3. QN1, QN2 and QN3 represent the queues preceding the service facilities. The service times are represented by P(1,7), P(2,7) and P(3,7). P(5,6) represents the interarrival of parts. When units find a facility filled, they take P(1,2), P(2,3) or P(3,1) which represent movement on the conveyor. The model will simulate until N7 is released, which represents the servicing of 1000 units. The output statistics of interest are those collected on -

C1, C2, C3, which are the number of units serviced by service facilities 1, 2 and 3

C4, which is the number of units recycled and

QN1, QN2, QN3, which are the queue characteristics preceding each service facility

Inventory Systems

To date, we have not demonstrated the use of the network modification feature of GERTS. The following example demonstrates this feature in addition to showing how GERTS might be used to model inventory systems.

Consider the case of a retail company which stocks a product that has a demand per week which is normal, mean of 100 units and standard deviation of 10 units. The reorder quantity for this particular product has previously been established at 404 units; however, the company's inventory policy is such that the inventory level is only determined at the end of the week. (Only if there is less than 404 units on hand at the end of the week does the company reorder more of the product.) If the lead time required to receive shipment from the supplier is a random variable such that:

Lead Time (in weeks)	Probability
2	0.15
3	0.50
4	0.30
5	0.05

What is the probability that the company has a stockout of this particular product?

There are three separate variables interacting in a random manner which must be represented in any GERTS model which could realistically simulate this problem. The first is the inventory level at the time the reorder is placed; the number of units on hand can be 404 or less but it is extremely difficult to describe the actual distribution. The second is the number of units which customers demand during each week of the replenishment period. The third is the number of weeks until the new shipment of units arrives. A model which incorporates the interaction of all three of these random variables is shown in Figure #4.

The parameter along each path is the number of units. The combination of P(5,6) and P(2,3) is used to simulate the random effect that the policy of checking only at the end of each week has on the number of parts in inventory when the reorder takes place. P(5,6) generates weekly demands for the product. When N3 is realized, there are exactly 404 units left in inventory and the realization of N3 triggers a modification which replaces N6 with N7 the next time N6 is realized. P(3,4) keeps track of the number of units left in inventory. N8 accounts for the probabilistic nature of the lead time required before the new order arrives by varying the number of releases supplied to N11. P(9,10) generates customer demand during the lead time and each time N10 is realized, another release is acquired by N11. Thus, if N11 is realized, it must mean that the new shipment has arrived. If N4 is realized, it must be that the customer demand generated by P(9,10) was greater than the number of units in inventory when the reorder was placed, and since N11 has not been realized, no new units have arrived. Thus,

Prob. (N11 is realized) = Prob. (new shipment arrives before inventory level equals zero)

and

Prob. (N4 is realized) = Prob. (stockout occurs)

Project Management

Network methods have become popular as aids in the management of projects. One of these techniques is PERT. The PERT node is realized when all paths incident to it are realized. The PERT approach to analyzing networks is subject to a number of assumptions. One of which is that the longest (or critical) path will completely define the project duration. Van Slyke[17] used simulation to show that if the times associated with the activities of a project are random variables then the critical path will depend upon the particular observations of activity time. He showed that the PERT approach consistently underestimated the mean time to finish a project and overestimated the variance.

The PERT network in Figure #5 can be modeled in GERTS as shown in Figure #6. The times in both figures are assumed to be normal. The number of releases for all GERTS nodes is set equal to number of paths incident to the node. Thus, N6 has three releases. The statistics of interest will be those collected on N6 and represent the mean, variance and histogram of the time to finish the project. Pritsker[6] has shown that GERTS gives the project manager great flexibility over standard project management techniques. The GERTS approach allows the inclusion of logical-or nodes, feedback and network modification.

Other Areas

Other examples of the use of GERTS in the industrial environment can be found in applications to quality control[13], reliability systems[3] and job shop loading[10].

An Industrial Application of GERTS

In the preceding section, we have discussed some possible applications of GERTS in the industrial environment. In this section we will discuss a study using GERTS just completed by an engineer with the Western Electric Company[10].

The manufacturing process which was investigated was the manufacture of cable. The cable manufacturing system which was used here is one which produces three types of cable. These cable types are polyethylene insulated cable, paper insulated cable, and polyvinyl chloride insulated cable. As can be seen in Figure #7, this process consists of many distinct operations. These facilities each contain production stations, test and repair facilities and storage locations. The time to produce the cable and quality of the product is determined by the nature of the individual operations and their interconnection.

There are six main phases in the manufacturing process being investigated. When the raw copper is received it is drawn to the desired size. The product then moves through the operations to produce the desired type of cable. These operations in order are: (1) insulating, (2) twisting, (3) stranding, (4) cabling, (5) sheathing. The test facilities follow each operation to determine if the product is complying with the specifications required.

This study was concerned with the operation of sheathing for polyethylene insulated cable. This operation is the last operation in the diagram shown in Figure #7. This operation is a system in itself. It consists of five stages involving up to three types of machines in each stage.

It was decided to use historical data in the development of the GERTS network. The data which was used for this study is the wage incentive rates for the individual products. This data gives information as to time required for an operation and the sequence of operations used for this product. The units used by this data are carried throughout this study. A unit of product is defined as one-thousand linear feet of cable. The time value is the number of hours to process this one-thousand linear feet.

The use of this data to create the GERTS network is the next step. By searching the data concerning the first sheathing operation it is found that machine types 1, 2 and 3 are used. Material flow upon entering the system goes to one of these stations. This leads to branches P(1,2), P(1,3) and P(1,4) on Figure #8 being drawn. Flow must then be found from each of these stations. Searching the data, it was found that from machine type 1, sheathing operation 1, the unit will flow to either machine type 1 in sheathing operation 2, machine type 6 in sheathing operation 2, or the product leaves sheathing and continues into the system. In the example system given, the operation following sheathing would be the final test before shipping. Branches P(2,5), P(2,7) and P(2,16) of Figure #8 now exist and machine type 2 in sheathing operation 1 is investigated to determine product flow from it. In this manner each machine type of each operation is investigated with the end result being that of Figure #8. From this data the probability and time for each path was found. Nodes 2-15 are queue nodes with maximum capacity equal to the available in-process inventory space. These nodes can be assumed to shunt to a node which could be interpreted as a system overload.

The model was used to find such information as:

(1) Expected in-process inventory level and throughput flow time for the system at a given arrival rate.

(2) Sensitivity analysis to determine nodes most effected by an increase in arrivals to the system or modification of facilities internal to the system ("bottleneck" analysis).

The major concern of the engineer was the "bottleneck" analysis. The first "bottleneck" in this system was detected when QN7 reached capacity with a mean interarrival time of 0.4. The network was modified to place another type 6 machine in sheathing operation 2. The modified system would accommodate interarrival times up to 0.34. The analysis continued in this manner over interarrival rates up to and including 0.1.

The model constructed seemed to predict in-process inventory, and throughput time in a consistent manner with respect to the historical data available. There are, however, some strong criticisms of this model: (1) The model did not distinguish between products when choosing service times on particular machine types, and (2) The probabilistic selection of paths from the service facilities lead to some job sequences which did not correspond to historical data. These criticisms are not as serious for problems of "bottleneck" analysis as they would have been if the engineer's major goal had been a throughput analysis by product.

Discussion

GERTS, a new idea in network modeling, has been presented. We have tried to demonstrate the techniques, features and potential. In certain systems such as assembly line and conveyor systems, we feel that it shows great potential. Its promise in project management has already been demonstrated[9]. The industrial application reported, although open to criticism, demonstrates the ability of GERTS to model a large scale industrial system.

The authors are anxious to find out the opinions of the participants of the 4th Conference of Applications of Simulation in the following areas:

(1) Comparison of GERTS to GPSS, Simscript, etc. for the problems discussed.

(2) Other areas of possible application for GERTS.

(3) Features that might be added to GERTS.

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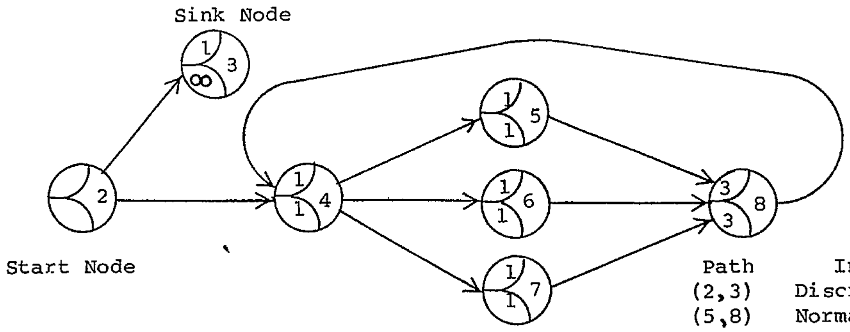
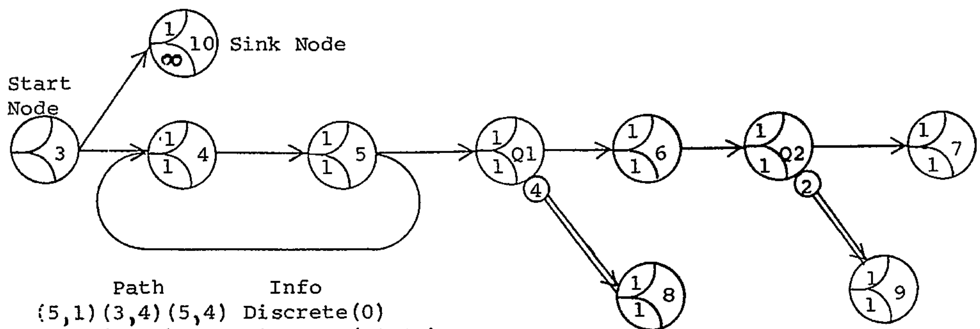
Note: Since this paper was written, a new GERTS computer package, GERTS III, has been developed. GERTS III is superior to GERTS II in its ability to collect statistics. All problems discussed in this paper have been modeled using GERTS III, and these models will be discussed during the presentation of this paper. The GERTS III system is available from Dr. A. Alan B. Pritsker, Department of Industrial Engineering, Purdue University, W. Lafayette, Indiana.



Path	Info
(2,3)	Discrete(60.0)
(5,8)	Normal(3,1)
(6,8)	Normal(4,2)
(7,8)	Uniform(3,5)
(8,4)	Discrete(0) C1
All Other	Discrete(0)

The probabilities are all 1.0

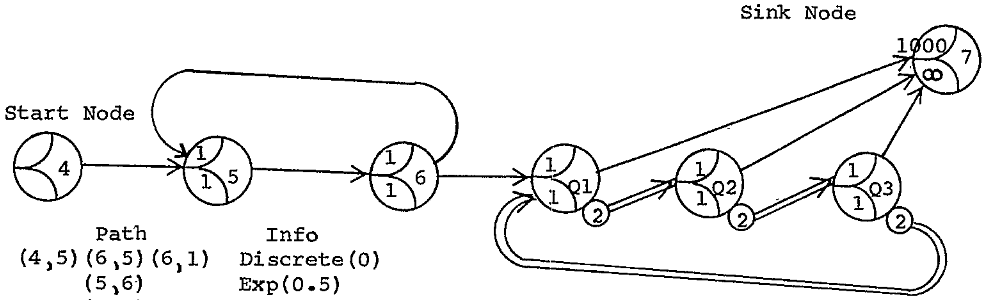
Figure 1 GERTS Model of a Production Line



Path	Info
(5,1)(3,4)(5,4)	Discrete(0)
(3,10)	Discrete(10000)
(4,5)	Exp(0.4)
(1,8)	Discrete(0) C1
(1,6)	Exp(0.5) C3
(6,2)	Discrete(0.2)
(2,9)	Discrete(0) C2
(2,7)	Discrete(0.25) C4

The probabilities on all paths are 1.0

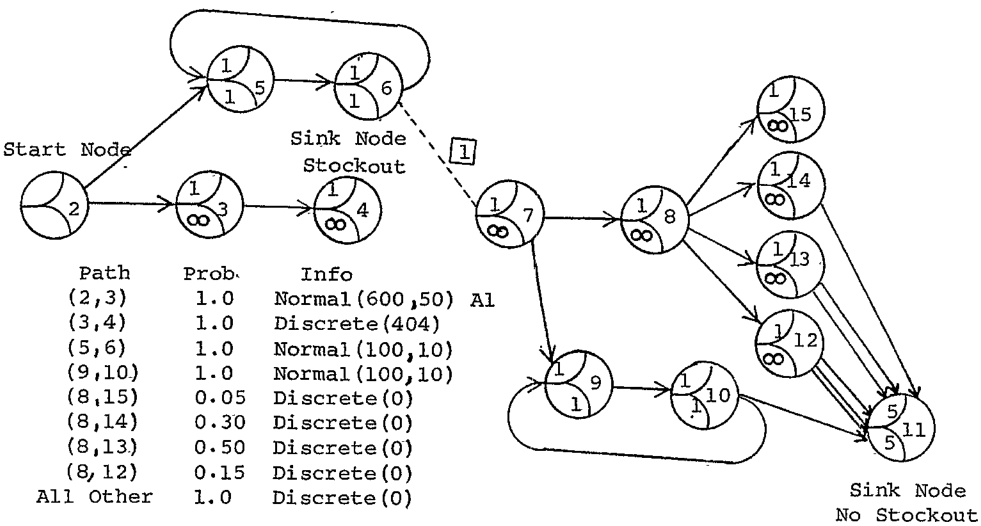
Figure 2 GERTS Model of a Production Line with Queueing



Path	Info
(4,5) (6,5) (6,1)	Discrete(0)
(5,6)	Exp(0.5)
(1,7)	Normal(1,0.5) C1
(2,7)	Normal(1,0.5) C2
(3,7)	Normal(1,0.5) C3
(1,2) (2,3)	Discrete(1)
(3,1)	Discrete(25) C4

The probabilities for all paths are 1.0

Figure 3 GERTS Model of a Conveyor System



Path	Prob.	Info
(2,3)	1.0	Normal(600,50) A1
(3,4)	1.0	Discrete(404)
(5,6)	1.0	Normal(100,10)
(9,10)	1.0	Normal(100,10)
(8,15)	0.05	Discrete(0)
(8,14)	0.30	Discrete(0)
(8,13)	0.50	Discrete(0)
(8,12)	0.15	Discrete(0)
All Other	1.0	Discrete(0)

Figure 4 GERTS Model of an Inventory System

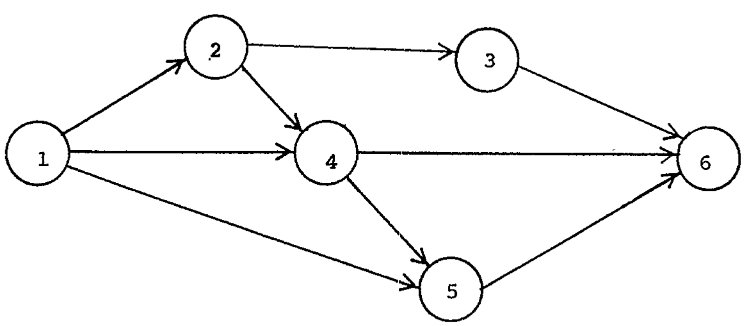


Figure 5 PERT Network

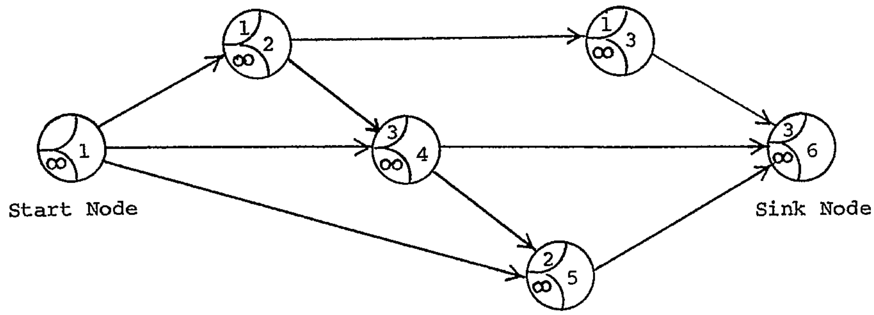


Figure 6 GERTS Model of the PERT Network in Figure 5

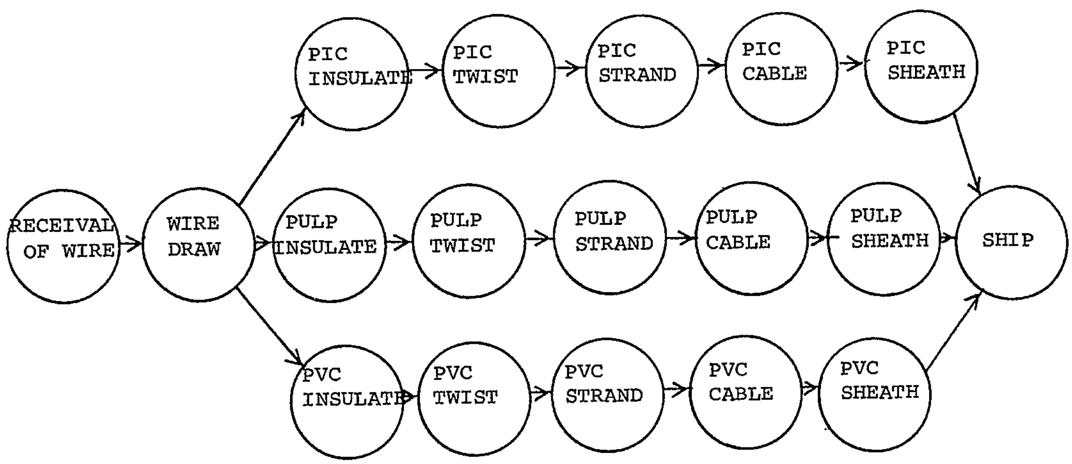


Figure 7 Total Operation

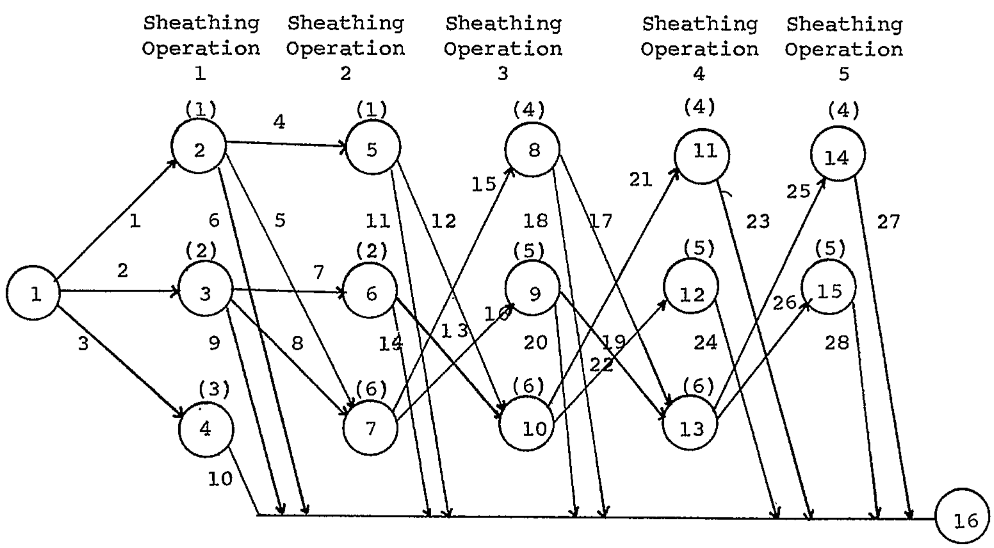


Figure 8 GERTS Model of the PIC Sheathing Operation