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STOCHASTICALLY CONSTRAINED LEVEL SET APPROXIMATION VIA PROBABILISTIC BRANCH AND BOUND

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ABSTRACT

This paper investigates a simulation optimization problem with both stochastic objective and constraint functions with a discrete solution space. Our objective is to identify a set of near-optimal solutions within a specific quantile, such as the top 10%. To achieve this goal, we first employs a probabilistic branch-and-bound algorithm to find a level set of solutions. Then, we combine a penalty function approach with the probabilistic branch-and-bound algorithm to handle stochastically constrained problems. Both convergence analysis and experimental results are provided that demonstrate the superior efficiency of our proposed approaches over existing methods.

1 INTRODUCTION

Simulation is a popular approach for complex decision-making problems involving uncertainty. For discrete decision variables, several simulation optimization (SO) methods exist, including ranking and selection procedures, retrospective optimization, and random search algorithms (Hong, Nelson, and Xu 2015). Research has also focused on discrete SO with stochastic constraints, where Park and Kim (2015) developed penalty function with memory (PFM). PFM uses a penalty parameter and a measure of violation that can handle boundary solutions. Most SO research aims for finite-time or asymptotic convergence guarantees, but the probabilistic branch-and-bound (PBnB) algorithm takes a different approach, seeking a level set of solutions reaching a specific quantile (Zabinsky and Huang 2020). The existing PBnB lacks consideration of stochastic constraints and is limited by conservativeness in critical parameters. Our contributions include extending PBnB for discrete SO problems with an asymptotic convergence guarantee, incorporating penalty function techniques, and developing a benchmark algorithm for comparison in extensive experiments.

2 METHODOLOGIES

The SO problem considered in this study is formed as $\min_{x\in\Omega} g(x) = E[\mathrm{G}(\mathrm{x})]$, where Ω is the feasible solutions bounded by stochastic constraints $h_\ell(x) \le Q_\ell$, for all $\ell = 1, \dots, m$. The goal of this study is to approximate the target level set $L(\delta, \Omega)$, which is the top 100δ percent of solutions in the feasible region. We first introduce one of the PFM, PS_2, from Park and Kim (2015), which convert the SO problem into

$\min_{x \in \Theta} z(x)$. Θ is the set of all solutions including feasible and infeasible ones, and $z(x) = \lim_{k \to \infty} \bar{Z}_k(x)$. $\bar{Z}_k(x) = \bar{G}_k(x) + \sum_\ell \lambda_{\ell k}[S_{\ell k}(x)]^+$ is the penalized objective function calculated based on PS_2.

Using $\bar{Z}_k(x)$ as the objective function, we proposed the discrete version of PBnB to approximate the target level set for discrete SO problems with stochastic constraints. The discrete PBnB follows a similar framework with the original PBnB (Zabinsky and Huang 2020). PBnB iteratively uses samples solutions to estimate the quantile bounding the target level set, and it categorize subregions of the domain space into maintained, pruned, and current subregions. These categories representing the part of the solutions is part of the target level set or not, where maintained subregions are considered to be part of the target level set. There are three phases of the discrete PBnB: (1) Sampling & Quantile Estimation; (2) Subregions Updating; (3) Branching & Terminating. Specifically, the discrete PBnB randomly allocates different size of observations in different types of subergions, where more observations are allocated to the undecided current subregions. Then, using the observed samples, the discrete PBnB estimates the quantile which is used as the threshold for the categorization of subregions. Also, the discrete PBnB allows returning to current subregions, but the original PBnB does not.

3 RESULTS AND DISCUSSIONS

Both convergence analysis and experimental results are provided for the PS_2 incorporated discrete PBnB. First, the convergence analysis leads to the maintained subregions provided by the PS_2 incorporated discrete PBnB converging to the target level set as the iteration $k \to \infty$. Second, in the numerical experiment, Goldstein-Price function with two types of constraints are the implemented as the testing environment. The the PS_2 incorporated discrete PBnB is compared with a two-stage PBnB algorithm. The two-stage PBnB algorithm implements a feasible region detection PBnB (Tsai et al. 2018) first and use the discrete PBnB to identify the target level set. Two metrics are used in the comparison, ratio of the target level set maintained and the accuracy of the maintained subregion. As shown in Table 1, PS_2 incorporated discrete PBnB and the two-stage PBnB performs slightly better with the easy constraint, where the difference between the ratio maintained expanded during the hard constraint. The difference could be caused by the incorrect feasibility identification for the two-stage PBnB during the first stage, that loses part of the target level set.

Table 1. Numerical Experiment Results

Goldstein-Price	PS_2 PBnB		Two-Stage PBnB	
	Ratio Maintained	Accuracy	Ratio Maintained	Accuracy
Easy constraint	0.9943	0.9950	0.9659	0.9586
Hard constraint	0.9731	0.9731	0.9012	0.9421

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