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MODEL PREDICTIVE CONTROL IN OPTIMAL INTERVENTION OF COVID-19 WITH MIXED EPISTEMIC-ALEATORIC UNCERTAINTY

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ABSTRACT

Non-pharmaceutical interventions (NPI) have been proven vital in the fight against the COVID-19 pandemic before the massive rollout of vaccinations. Considering the inherent epistemic-aleatoric uncertainty of parameters, accurate simulation and modeling of the interplay between the NPI and contagion dynamics are critical to the optimal design of intervention policies. We propose a modified SIRD-MPC model that combines a modified stochastic Susceptible-Infected-Recovered-Deceased (SIRD) compartment model with mixed epistemic-aleatoric parameters and Model Predictive Control (MPC), to develop robust NPI control policies to contain the infection of the COVID-19 pandemic with minimum economic impact.

1 INTRODUCTION

The COVID-19 pandemic has taken a substantial economic and societal toll worldwide. Compartment models have been predominantly used in modeling the infection dynamics of epidemic outbreaks. The susceptible-infected-recovered-deceased (SIRD) model and its variants have been widely used to predict case counts for COVID-19 and the optimal design of NPIs. Model predictive control (MPC) has proven effective in controlling various real-world processes in which future dynamics are highly uncertain. Consequently, combining MPC with the compartment model is a promising approach to designing policies to contain the spread of diseases.

As in most complex systems, parameters in a compartmental model could be rather challenging to infer from the available data owing to a lack of knowledge, incomplete and inaccurate data, as well as computational issues. Stochastic compartmental models have been developed and applied to the investigation of the COVID-19 pandemic and quantify uncertainties in such models. In this study, we consider a mixed epistemic-aleatoric uncertainty for parameters in compartmental models in the context of COVID-19 for two reasons. First, the knowledge gap is narrowing as more studies are conducted on COVID-19 and its variants; and second, the uncertainty of infectious system parameters cannot be completely eliminated by acquiring new knowledge or data, due to inherent stochasticity and individual differences. Therefore, it is more reasonable to characterize the extensively studied COVID-19 pandemic in compartmental models using mixed epistemic-aleatoric parameters.

In this study, we propose a modified stochastic SIRD-MPC model to develop robust control policies to combat the spread of disease with minimal social and economic impacts. We use mixed epistemic-aleatoric parameters in a stochastic compartmental model with feedback to emulate the dynamics of human interventions. Subsequently, we apply the MPC technique to determine the optimal NPI control policy while considering uncertainty propagation using probability bounds analysis (PBA) for uncertainty quantification. It is noteworthy that our proposed model possesses general applicability beyond the scope of COVID-19.

2 MODEL

We propose a modified SIRD compartment model by incorporating the feedback control policy U_t and the conventional SIRD compartment model to capture the interventional evolution of the COVID-19 pandemic. The discretization of the modified SIRD model is adopted to characterize the compartment evolution, with time step $\Delta t = 1$ day:

$$\begin{gathered} S(t+1) = S(t) - (1 - U_t)\beta S(t)I(t) \\ I(t+1) = I(t) + (1 - U_t)\beta S(t)I(t) - \gamma I(t) - \alpha I(t) \\ R(t+1) = R(t) + \gamma I(t) \\ D(t+1) = D(t) + \alpha I(t) \\ U_t = C_t\left(1 - e^{(-\varphi \times I(t))}\right) \end{gathered} \tag{1}$$

Where mixed epistemic-aleatoric parameters β, γ, and α are the transmission rate, recovery rate, and death rate, respectively (The uncertainty about the probability distribution of a model parameter is expressed in terms of a particular form of the distribution function to an interval bounded by lower and upper bounds on the distribution function parameters. Therefore, we can use the interval to quantify the epistemic uncertainty to characterize the average infectivity under various ambient environments and use the normal distribution to represent the aleatory uncertainty. Specifically, $\beta \sim Normal([0.03,\ 0.90],\ 0.05)$ follows the normal distribution to represent the aleatory uncertainty (internal randomness of disease transmission) with mean $\mu = [0.03, 0.90]$ and standard deviation $\sigma = 0.05$, where $[a, b]$ represents an interval to quantify the epistemic uncertainty (knowledge gaps in understanding the contagion process of disease) of with lower bound a and upper bound b). Here, $S(t), I(t), R(t)$, and $D(t)$ represent the proportional population of compartment susceptible, infected, recovered, and deceased. $C_t \in [0.0, 1.0]$ represents the nominal level of control at time stamp t, which indicates a nominal discount on β; the effective control policy U_t is subject to the influence of the population's perception of infection risk and compliance with the intervention policy. For simplicity, we have $U_t = C_t\left(1 - e^{(-\varphi \times I)}\right)$; $\varphi > 0$ is the scaling factor to steer the strength of the feedback for infection rate I, which can capture the attitude of society and/or government encountering I. Large φ means that the public is prone to comply with the control policy and vice versa. Therefore, the $1 - e^{(-\varphi \times I)}$ represents the effective level of policy implementation, which can characterize overall compliance of control policy for the society and/or government in dealing with the pandemic.

In present work, the PBA method for uncertainty propagation is used to assess the possible outputs/compartment states and the effect of uncertainty on decision-making for our proposed model. Meanwhile, cumulative uncertainty makes effective optimal control impractical due to the time propagation of uncertainty with model dynamics. To address this issue, MPC is applied because it involves continuously updating the best strategy to make up for performance losses predicted over lengthy time horizons. We use a two-stage approach to implementing MPC in this investigation: (a) solving the optimization problem for a fixed predictive horizon N_p using the system's states $h(t_0) = (S(t_0), I(t_0),\ R(t_0),\ D(t_0))$, achieved from the collected epidemiological data (true data or estimated stochastic data), as the initial conditions at the start of the optimization problem and (b) putting into the first step of optimal design C_{t_0} for the compartment model (equation (1)) to achieve the next time stamp states $h(t_1)$ and starting the next new prediction horizon ($[t_1, t_{N_p+1}]$) until the final terminal time. Therefore, the MPC procedures aim at optimizing the objective within the predictive horizon N_p. The general MPC problem with cost function $f(h(t), U_t)$ starting from time stamp $t = k$ can be represented as:

$$\boldsymbol{J}\big(h(k)\big) = \min_{\boldsymbol{C(k)}} \sum_{t=k+1}^{k+N_p} f(h(t), U_t) \tag{2}$$

where $J(k)$ denotes the total estimated cost in MPC from time stamp k to $k + N_p$. $f(h(t), U_t)$ is the general form of the cost function, reliant on $h(t)$ and U_t at the time stamp t. $h(t) = h(t|k)$ is the predicted state at time stamp t given state $h(k)$ at time stamp k. $\boldsymbol{C(k)} = \{C_k, \dots C_{k+N_p-1}\}$ is a vector of manipulated variables in a prediction horizon N_p days start from time stamp k, and C_{min} and C_{max} are the minimum and maximum values of the nominal level of control C_t.