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CONDITIONAL IMPORTANCE SAMPLING FOR CONVEX RARE-EVENT SETS

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ABSTRACT

This paper studies the efficient estimation of expectations defined on convex rare-event sets using importance sampling. Classical importance sampling methods often neglect the geometry of the target set, resulting in a significant number of samples falling outside the target set. This can lead to an increase in the relative error of the estimator as the target event becomes rarer. To address this issue, we develop a conditional importance sampling scheme that achieves bounded relative error by changing the sampling distribution to ensure that a majority of samples lie inside the target set. The proposed method is easy to implement and significantly outperforms the existing approaches in various numerical experiments.

1 INTRODUCTION

In this paper, we consider the problem of estimating expectations over rare-event sets. This problem arises in a myriad of applications, where several studies improve the tractability and efficiency of simulation algorithms by focusing on particular forms of a target event set. Nevertheless, few studies consider general convex rare-event sets constructed by nonlinear constraints, which we will explore in this paper.

The primary motivation for this problem stems from distributionally robust rare-event simulation. According to Blanchet and Murthy (2019), the distributionally robust bound of the probability of a convex rare event can be represented as the probability of another convex rare event. In this case, given an uncertainty set of distributions, rare-event simulation of expectations over convex sets turns out to be useful not only to identify the new target set but also to estimate the distributionally robust bound. Furthermore, our problem has potential applications in pricing deep-out-of-the-money options and evaluating the reliability of transportation networks. It may also be relevant in cases where the structural information of the rare-event set is not available, but the decision maker believes that it is convex (Huang et al. 2018).

When estimating expectations evaluated on rare events, the crude Monte Carlo method produces significantly large simulation errors since it seldom generates samples belonging to the target set. To tackle this issue, we often resort to a well-known variance reduction technique, called importance sampling, which changes the sampling distribution to increase the likelihood of hitting the target set and assigns weights to the generated samples via a likelihood ratio to retain unbiasedness. Finding suitable sampling distributions is a challenging task that has been typically addressed by using the theory of large deviations. However, despite being provably effective in many settings, this traditional approach usually neglects salient geometric structures of the target set. As a result, the relative errors of the corresponding estimators could grow indefinitely as the target set becomes rarer.

In this paper, we propose to identify prominent geometric information of the target set and integrate them into the sampling distribution. Ahn and Zheng (2023) use this idea to estimate expectations defined on "polyhedral" rare-event sets and develop a conditional importance sampling method with bounded relative error based on sharp asymptotics. However, it is unclear how and under what conditions this method can be adapted to the setting with "general convex" rare-event sets in a way that the associated relative error remains bounded. In this regard, the main contribution of this paper lies in providing an easy-to-implement adaptation and proving that the bounded relative error property is preserved under mild conditions on the structure of the target set.

Our approach is related to Dupuis and Wang (2004), Glasserman and Li (2005), and Bucklew (2005) in terms of combining importance sampling and conditional Monte Carlo. However, those methods are designed only for specific problem setups such as the evaluation of portfolio credit risk and the computation of tail probabilities associated with random walks. Further, they do not guarantee bounded relative error. In contrast, our method is developed for fairly general objectives and achieves bounded relative error.

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