*Proceedings of the 2023 Winter Simulation Conference*
*C. G. Corlu, S. R. Hunter, H. Lam, B. S. Onggo, J. Shortle, and B. Biller, eds.*

# MINIMIZING MAKESPAN FOR A MULTIPLE ORDERS PER JOB SCHEDULING PROBLEM IN A TWO-STAGE PERMUTATION FLOWSHOP

Rohan Y. Korde
John W. Fowler

W. P. Carey School of Business
Arizona State University
300 E Lemon St.
Tempe, AZ 85281, USA

Lars Mönch

Mathematics and Computer Science
FernUniversität in Hagen
Universitätsstraße 47
Hagen, 58097, GERMANY

## EXTENDED ABSTRACT

The scheduling problem we study in this paper is known as a multiple orders per job (MOJ) (Mason et al. 2004) problem which is encountered in a few different industries including front-end semiconductor manufacturing. We look at the MOJ scheduling problem in a two-stage permutation flowshop with some real-world constraints with the goal of minimizing the makespan. We focus on the front-end semiconductor manufacturing operations that require the use of front opening unified pods (FOUPs) in an Overhead Hoist Transport (OHT) system. Orders from different customers (for the same product family) are packed together into FOUPs (also referred to as jobs) in such a way that the desired performance measures (such as makespan or total weighted tardiness) are minimized.

Mason et al. (2004) first studied the MOJ scheduling problem to minimize total weighted completion time on a single machine. In this research, we extend Mason et al. (2004) and Laub et al. (2007) by studying MOJ scheduling in a two-stage permutation flowshop to minimize the maximum completion time (Cmax) with machine bottleneck (Bk) constraints: $F2|moj(.), Bk(.), prmu|Cmax$. This problem is NP-hard. Each of the two stages in the flowshop has exactly one machine. There are two types of serial (non-batch) machines, namely item processing machine (IPM) and lot processing machine (LPM). There are four types of stages for moj(.): IPM-IPM, IPM-LPM, LPM-IPM, and LPM-LPM. The bottleneck Bk(.) attribute can take one of three values: in stage one (S1), in stage two (S2), and balanced (B).

In this study, our first goal is to understand how the performance of heuristics changes as we increase the size of problem instances and as we vary the types of problem instances. A second goal is to explore conditions under which certain heuristics work better than other heuristics for our MOJ problem. We have implemented the MILP in ILOG CPLEX MIP solver (v.22) to generate optimal schedules for small-sized problem instances (10 and 20 orders). For large-sized problem instances (50 and 100 orders), we have implemented the NEH heuristic (Nawaz et al. 1983), Johnson's algorithm (Johnson 1954), Slope heuristic (Palmer 1965), and heuristics from Mason et al. (2004). To compare the performance of the heuristics, we designed a full-factorial experiment with five factors to generate 1920 problem instances (192 factor combinations each with 10 replicates). Each problem instance is run through Mason et al. (2004) heuristics first, where the heuristics were created by combining order sorting rules, job filling directions, and bin packing rules. We then took the packed jobs from the Mason et al. (2004) heuristic that produced the smallest makespan and then ran these packed jobs through additional heuristics to see if the makespan could be reduced further. Finally, we calculated the Cmax performance ratios by dividing the makespan from each heuristic with the smallest makespan from all heuristics for each problem instance.

As summarized in Table 1, our results show that for moj(IPM-IPM), Cmax was minimized by the MIP solver for more than 90% of the small-sized problem instances regardless of the bottleneck type. When the

heuristics minimized Cmax, the Slope heuristic was the fastest and NEH heuristic was the slowest for over 90% of the large-sized problem instances. We plan to conduct additional experiments for other performance measures: total weighted completion time (TWC), weighted number of tardy orders (WNTO), and total weighted tardiness (TWT) and explore variable neighborhood search-based metaheuristics.

Table 1: Results from experiment to compare heuristics. In columns A/B/C/D, the 1st value is the worst Cmax ratio, the 2nd value is the average Cmax ratio, and the 3rd value is percent times of best Cmax ratio.

| # of orders | Stage | Bottleneck | NEH (A) | Johnson (B) | Slope (C) | Mason (D) |
|---|---|---|---|---|---|---|
| **10** | IPM-IPM | B | 1.28/1.13/2.5 | 1.28/1.13/2.5 | 1.28/1.13/2.5 | 1.46/1.17/0.25 |
| | | S1 | 1.22/1.08/0 | 1.22/1.08/0 | 1.27/1.16/0 | 1.48/1.17/0 |
| | | S2 | 1.25/1.09/7.5 | 1.25/1.09/7.5 | 1.29/1.16/0 | 1.48/1.17/0 |
| | IPM-LPM | B | 1/1/100 | 1/1/100 | 1/1/100 | 1.44/1.07/58.5 |
| | | S1 | 1/1/100 | 1/1/100 | 1/1/100 | 1.34/1.05/64.6 |
| | | S2 | 1/1/100 | 1/1/100 | 1/1/100 | 1.52/1.1/44.7 |
| | LPM-IPM | B | 1/1/100 | 1/1/100 | 1/1/100 | 1.45/1.07/55.1 |
| | | S1 | 1/1/100 | 1/1/100 | 1/1/100 | 1.55/1.13/37.3 |
| | | S2 | 1/1/100 | 1/1/100 | 1/1/100 | 1.37/1.05/64.4 |
| | LPM-LPM | B | 1/1/100 | 1/1/100 | 1/1/100 | 1.75/1.2/43.9 |
| | | S1 | 1/1/100 | 1/1/100 | 1/1/100 | 2.11/1.24/43.1 |
| | | S2 | 1/1/100 | 1/1/100 | 1/1/100 | 2.1/1.22/42.5 |
| **100** | IPM-IPM | B | 1/1/100 | 1/1/100 | 1/1/100 | 1/1/100 |
| | | S1 | 1/1/100 | 1/1/100 | 1.03/1.02/0 | 1.03/1.01/2.19 |
| | | S2 | 1/1/100 | 1/1/100 | 1.02/1.01/0 | 1.02/1.01/1.44 |
| | IPM-LPM | B | 1/1/100 | 1/1/100 | 1/1/100 | 1.36/1.05/31.8 |
| | | S1 | 1/1/100 | 1/1/100 | 1/1/100 | 1.15/1.02/56.1 |
| | | S2 | 1/1/100 | 1/1/100 | 1/1/100 | 1.43/1.11/5.5 |
| | LPM-IPM | B | 1/1/100 | 1/1/100 | 1/1/100 | 1.32/1.06/24.5 |
| | | S1 | 1/1/100 | 1/1/100 | 1/1/100 | 1.41/1.11/4.6 |
| | | S2 | 1/1/100 | 1/1/100 | 1/1/100 | 1.2/1.02/51.6 |
| | LPM-LPM | B | 1/1/100 | 1/1/100 | 1/1/100 | 1.4/1.1/23.25 |
| | | S1 | 1/1/100 | 1/1/100 | 1/1/100 | 1.42/1.1/20.5 |
| | | S2 | 1/1/100 | 1/1/100 | 1/1/100 | 1.39/1.1/21.9 |

## REFERENCES

Johnson, S. M. 1954. "Optimal Two and Three-Stage Production Schedules with Setup Times Included", *Naval Research Logistics Quarterly* 1:61-67.

Laub, J. D., J. W. Fowler, and A. B. Keha. 2007. "Minimizing Makespan With Multiple-Orders-Per-Job in a Two-Machine Flowshop", *European Journal of Operational Research* 182(1):63-79.

Mason, S. J., P. Qu, E. Kutanoglu, and J. W. Fowler. 2004. "The Single Machine Multiple Orders Per Job Scheduling Problem", *Working paper.*

Nawaz, M., E. E. Enscore, Jr, and I. Ham. 1983. "A Heuristic Algorithm for the m-Machine, n-Job Flowshop Sequencing Problem", *Omega – the International Journal of Management Science* 11:91-95.

Palmer, D. S. 1965. "Sequencing Jobs Through a Multi-Stage Process in the Minimum Total Time – A Quick Method of Obtaining a Near Optimum", *Operational Research Quarterly* 16:101-107.