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SIMULATING JUSTICE: SIMULATION OF STOCHASTIC MODELS FOR COMMUNITY BAIL FUNDS

Sophia Gunluk
Yidan Zhang
Jamol Pender
Operations Research and Information Engineering
Cornell University
Ithaca, NY 14853, USA

ABSTRACT

Bail funds have a long history of helping those who cannot afford bail in order to wait for trial at home. They have also had a large impact on the verdict of the defendant. In this paper, the first stochastic model for capturing the dynamics of a community bail fund is presented. The bail fund model integrates traditional queueing models with classic insurance/risk models to represent the bail fund's intricate dynamics. Since the blocking model does not have closed-form expressions for its moments, stochastic simulation is used to assess Gaussian-based approximations that estimate the probability of a defendant being denied access to the bail fund when it lacks adequate funds to support them. Additionally, a new simulation-based algorithm is proposed and it uses a deterministic infusion of capital as a control variable to stabilize the probability that defendants have access to the bail fund. Our simulation results reveal that our Gaussian-based approximations are suitable for moderately and highly active bail funds.

1 INTRODUCTION

On August 4^{th} 1964, Attorney General Robert F. Kennedy said "the rich man and the poor man do not receive equal justice in our courts. And in no area is this more evident than in the matter of bail". In the United States judicial system, when an individual is said to be allegedly involved in a crime and is arrested, they are sent to jail. The individual then remains in a pre-trial detention facility until their case is brought to trial. Once at trial a judge and/or jury must decide whether or not the individual is guilty or not guilty of the crime of which they are charged. Individuals can be released from jail before their trial if they post **bail**, which is set at the time of the individual's official arrest. If an individual shows up for their trial, whatever amount of money that was posted for bail will be returned. However, if they do not show up for their trial date, the bail amount is seized from the individual. Thus, bail is meant to prevent individuals from going on the run and not coming back for their trial. Despite the purpose of bail to ensure defendants show up for their trial, it has enabled wealthy individuals to avoid jail time while individuals from low-income backgrounds often have no choice but to stay in jail until their trial, which can be anywhere from days to years. The tragic suicide of Kalief Browder is just one example of the impact that excessive jail can have on a defendant and their family, see for instance Jones (2015) and Johnson (2018).

Pre-trial detention can have severe impacts on individuals' lives, disrupting their ability to work, care for their families, and even leading to job loss or separation from their children. Beyond these challenges, bail can also influence the outcome of a trial. Joseph E. Krakora, a New Jersey public defender and joint committee member, explained that defendants are often given a choice: wait for a trial, potentially months away, while in jail, or plead guilty and go home Dabruzzo, D. (2019). This coercive dynamic can result in innocent defendants pleading guilty simply to be released from detention. In response to these issues, bail funds have been established to help low-income individuals get out of jail and maintain their lives while awaiting trial, aiming to reduce the disparity between wealthy and poor defendants.

Bail funds have a rich history in the United States, dating back to the 1920s when the American Civil Liberties Union (ACLU) established a bail fund to free individuals arrested for sedition during the First Red Scare (Steinberg et al. 2018; Goldman et al. 2021; Simonson 2016). However, the popularity of community bail funds has grown exponentially since 2012. This growth can be attributed, in part, to the passage of a law in New York State that legalized charitable bail funds posting bail amounts of $2000 or less. Although these bail funds aimed to assist those in need of financial support to secure bail, some critics viewed the law as flawed since it did not eliminate cash bail, the primary objective. As a result, some individuals believe that the New York State bail reform package has, in essence, institutionalized bail funds as a permanent fixture of the criminal justice system. In the Charitable Bail Organizations Act, it specifies that bail funds must

1. Only deposit money as bail in the amount of two thousand dollars or less for a defendant charged with one or more misdemeanors, provided, however, that such organization shall not execute as surety any bond for any defendant;
2. Only deposit money as bail on behalf of a person who is financially unable to post bail, which may constitute a portion or the whole amount of such bail;
3. Only deposit money as bail in one county in this state. Provided, however, that a charitable bail organization whose principal place of business is located within a city of a million or more may deposit money as bail in the counties comprising such city; and
4. Not charge a premium or receive compensation for acting as a charitable bail organization.

1.1 History of Bail Funds in Ithaca

In response to the rapidly growing local jail population, the Tompkins County Bail Fund was established as a non-profit organization in the 1980s, initially relying solely on community donations. Over time, the fund expanded its operations and demonstrated to Tompkins County that releasing more individuals on reconnaissance or supporting them in securing bail would be a cost-effective alternative. According to county estimates, the daily cost of housing an individual in jail, excluding staff and food expenses, is around $160. By helping to reduce the number of individuals held in pre-trial detention, the Tompkins County Bail Fund has been able to save the county hundreds of dollars per day.

Around nine years ago, Tompkins County officials were considering the construction of a new jail due to overcrowding issues resulting from an increasing number of defendants being held in the county. However, instead of building a new facility, the county made a donation of $10,000 to Opportunities, Alternatives, and Resources (OAR), a community organization that provides rehabilitation services to individuals in the justice system. An additional $800,000 was also given to support other rehabilitation programs, including probation tools like ankle monitoring. Together with community donations, these funds have enabled the OAR bail fund to operate without charging fees to clients, and it has not required further support from the county since. Since its establishment, the fund has helped bail out hundreds of non-convicted individuals annually, including over 300 people in both 2021 and 2022. Through its efforts, the bail fund has made a meaningful impact on the lives of many citizens of Tompkins County. Figure 1 provides an illustration of how OAR operates its bail fund.

While the bail fund's mission is to assist all defendants, there are instances where the fund may be unable to fulfill a request. One common reason for rejecting a request is if the individual has a history of not showing up for court dates, although this is a rare occurrence. If a defendant fails to appear in court, the bail fund loses the money it provided to the individual, and the court takes the money instead. Since the establishment of the bail fund, the highest forfeiture rate in one year has been 7%, which indicates that 7% of defendants failed to appear in court. However, overall the OAR bail fund maintains a low average forfeiture rate of approximately 2%. In some cases, a request may be rejected due to insufficient funds, but the OAR bail fund has not turned down any bail requests on this basis since they have a line of credit that they can utilize if necessary.

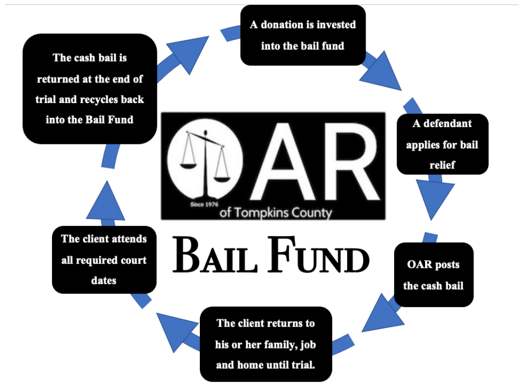

Figure 1: A Picture of the Tompkins County Bail Fund Operation.

1.2 How a Bail Fund Works

This section outlines the key operations of a bail fund, which are illustrated in Figure 2. Typically, bail funds consist of four primary components. First, the bail fund solicits donations from the public or corporations to support its activities. It is important to note that, in most states, these donations cannot be used to cover expenses such as personnel and wages. Second, once a bail request is made, the bail fund provides the necessary amount to the courts to hold for the defendant. This decreases the available funds in the bail fund. Third, at the beginning of the trial, it is assessed whether the defendant will appear for trial, which is a crucial step because the bail fund could forfeit the money if the defendant fails to appear in court. Finally, after the trial, the defendant's guilt or innocence is determined. If found guilty, the bail fund usually pays a poundage fee to the court, whereas if found not guilty, the bail fund receives the entire bail loan back to assist other defendants.

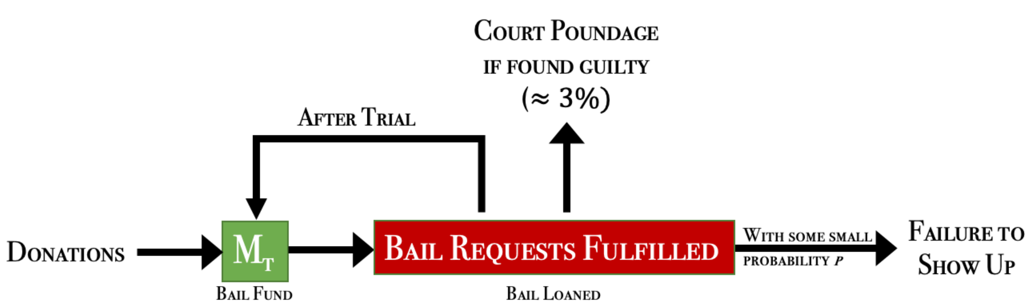

Figure 2: A Diagram of a Bail Fund Operations.

In summary, the bail fund's primary objective is to help defendants avoid jail time for minor offenses while they await trial. To achieve this, the bail fund must collect donations, provide the bail money to the courts, and monitor the defendant's attendance at trial. After the trial, the bail fund's funds are either refunded or reduced depending on the outcome of the case and the defendant's ability to show up for trial. The rest of this paper is devoted to analyzing the operations of bail funds with these components.

1.3 Contributions of the Paper

This paper makes the following contributions to the literature:

- This paper constructs the first and only stochastic process model for a community bail fund.

- This paper derives many properties of the bail fund under the assumption that everyone can be bailed out regardless of the monetary capacity of the fund. This provides insight for understanding how the bail fund would operate when it has unlimited resources.
- Finally, using simulation, this paper analyzes a blocking model and develops a simulation based algorithm for stabilizing the probability that a defendant will have access to the bail fund using deterministic capital infusions or donation rates as control variables. Simulation and Gaussian based approximations are needed as exact dynamics are not known for the blocking model in closed form.

1.4 Organization of Paper and Notation

In Section 2, we describe two stochastic models for the bail fund dynamics that we will analyze in this work. We compute the mean, variance, and moment generating function for the infinite resource model. We also provide new Gaussian based approximations for the blocking model. In Section 3, we describe a new algorithm for stabilizing the performance of the bail fund when there is blocking. Finally, Section 4 provides a conclusion and new areas of future research. Table 1 refers to all of the notation used in the paper for the convenience of the reader.

Table 1: Notation.

$Q^B(t), Q^\infty(t)$	Number of people receiving bail help at time t (blocking, infinite resource)
$M^B(t), M^\infty(t)$	Money left in (blocking, infinite resource) bail fund at time t
p_j	Poundage proportion taken by courts for j^{th} bail request
a_j	Arrival time of j^{th} bail request
s_j	Time between bail request and end of trial of j^{th} defendant
b_j	Bail request amount of j^{th} defendant
d_j	Amount of j^{th} donation to bail fund
$N^{(b)}(t), N^{(d)}(t)$	Number of (bail requests, donations) received by time t
M_0	Initial capital of bail fund

2 STOCHASTIC MODEL OF COMMUNITY BAIL FUND

In this section, the first model for a community bail fund is introduced. As a modeling assumption, it is assumed that if there is not enough money to fully support a defendant, then that defendant is blocked from the bail fund. Thus, defendants do not wait or retry for funds and are immediately sent away. As such we consider a bail fund model with a blocking mechanism.

2.1 The Blocking Bail Fund Model

In our bail fund model, two stochastic processes are critical to consider. The first process determines the amount of money available within the bail fund. The second process tracks the number of defendants currently being served by the bail fund, known as the queue length process. In our model, with blocking, it is unnecessary to model the number of people waiting as they are blocked if there is insufficient money to serve them immediately. The main challenge in our paper is that the remaining money process and the queue length process are coupled and interdependent. The two processes interact, and their coupled evolution determines the money available in the community bail fund and the number of defendants that can be supported by the bail fund.

2.1.1 The Remaining Money Process

The first process modeled is the allocation of the remaining funds to support the community. This process is defined as $M^B(t)$, the remaining amount of money available to support additional defendants at time t.

However, before providing a mathematical expression for $M^B(t)$, it is useful to describe all the primitives necessary to construct it.

The bail fund starts with an initial capital amount of M_0 and receives donations according to a Poisson process $N^{(d)}(t)$ with rate $\lambda^{(d)}$. The i^{th} donation has size d_i. An infusion of capital from an exogenous source is represented by the integral $\int_0^t c(s)ds$. Furthermore, the community bail fund receives bail requests from defendants according to a Poisson process $N^{(b)}(t)$ with rate $\lambda^{(b)}$. The i^{th} bail request has size b_i, and the arrival time of the i^{th} request is given by the random variable a_i. The i^{th} time to trial is given by the random variable s_i and has cumulative distribution function $G(x) = 1 - \overline{G}(x)$. When the i^{th} defendant is found guilty of the crime or fails to show up for the trial, the bail fund pays a penalty of $p_i b_i$ to the courts, which is a percentage of the bail amount. With these primitives, we can represent $M^B(t)$ as follows:

$$\underbrace{M^B(t)}_{\text{Total Money Available}} = \underbrace{M_0}_{\text{Initial Capital}} + \underbrace{\int_0^t c(s)ds}_{\text{Infusion of Capital}} + \underbrace{\sum_{i=1}^{N^{(d)}(t)} d_i}_{\text{Total Donations}} - \underbrace{\sum_{j=1}^{N^{(b)}(t)} b_j \cdot \{M^B(a_j-) > b_j\}}_{\text{Total Bail Paid Out}} + \underbrace{\sum_{j=1}^{N^{(b)}(t)} (1-p_j) \cdot b_j \cdot \{t > a_j + s_j\} \cdot \{M^B(a_j-) > b_j\}}_{\text{Total Money Returned}}. \quad (1)$$

The first two terms on the right-hand side represent the initial capital and the infusion of exogenous capital. The third term represents the bail paid out for defendants who are awaiting trial. The last term represents the penalties paid out for defendants found guilty or who fail to show up for trial.

In addition to describing the remaining money in the bail fund, it is possible to describe the queue length process for the blocked bail fund. The queue length $Q^B(t)$ has the following representation

$$Q^B(t) = \sum_{j=1}^{N^{(b)}(t)} \{a_j < t < a_j + s_j\} \cdot \{M^B(a_j-) \geq b_j\}. \quad (2)$$

This model differs from many other queueing models as it experiences blocking only when an auxiliary process fails to fulfill a bail request. In most other blocking models, the queue length is a direct determinant of the blocking mechanism. However, in this bail fund application, the blocking mechanism depends indirectly on the queue length. One can observe in the sequel that relaxing the blocking mechanism can provide insight into approximating the dynamics of the blocking model.

2.2 Bail Out Everyone Model

Thus, this introduces a simpler model in which all defendants can be bailed out regardless of how much money is available in the bail fund. This is called the "bail everyone out" model. Thus, it is assumed that there are unlimited resources or perhaps the bail fund has an unlimited credit limit. It is known that in reality, this may not be the case, however, as with many stochastic process models, it is often convenient to view the system as if it had all the resources it would need to operate effectively. This is especially true when the ideal system would rarely run out of resources for the community it is serving. From a queueing perspective, a system with unlimited resources is referred to as the offered load, and in the context of bail funds, this means that the bail fund will serve anyone it possibly can. A visual picture of this setup is given in Figure 2.

From our perspective, we will define $M^\infty(t)$ as the money remaining that the bail fund has when no blocking occurs. Since it serves all defendants regardless of money level, it is allowed to go negative, in which the bail fund is borrowing to serve the community. Since this model is more tractable, we provide the mean and variance of $M^\infty(t)$ as this will be useful for developing new approximations for our original blocking bail fund model.

$$\begin{aligned} M^{\infty}(t) &= M_0 + \int_0^t c(s)ds + \sum_{i=1}^{N^{(d)}(t)} d_i - \sum_{j=1}^{N^{(b)}(t)} b_j + \sum_{j=1}^{N^{(b)}(t)} (1-p_j)\cdot b_j \cdot \{t > a_j + s_j\} \\ Q^{\infty}(t) &= \sum_{j=1}^{N^{(b)}(t)} \{a_j < t < a_j + s_j\}. \end{aligned}$$

Theorem 1 The mean and variance of the amount of money left in the bail fund with no blocking, $M^{\infty}(t)$, is given by the following formulas

$$\begin{aligned} \mathbb{E}[M^{\infty}(t)] &= M_0 + \int_0^t c(s)ds + \lambda^{(d)} t \mathbb{E}[d] - \lambda^{(b)} t \mathbb{E}[b] + \lambda^{(b)} \cdot \mathbb{E}[b] \cdot (1-\mathbb{E}[p]) \cdot \int_0^t G(t-u)du \\ \mathrm{Var}[M^{\infty}(t)] &= \left(\lambda^{(d)} E[d^2] + \lambda^{(b)} \mathbb{E}[b^2]\right) \cdot t + \lambda^{(b)} \mathbb{E}[b^2] \cdot \mathbb{E}[p^2 - 1] \int_0^t G(t-u)du. \end{aligned}$$

Proof. For the mean we have

$$\begin{aligned} \mathbb{E}[M^{\infty}(t)] &= \mathbb{E}\left[M_0 + \int_0^t c(s)ds + \sum_{i=1}^{N^{(d)}(t)} d_i - \sum_{j=1}^{N^{(b)}(t)} b_j\right] + \mathbb{E}\left[\sum_{j=1}^{N^{(b)}(t)} (1-p_j)\cdot b_j \cdot \{t > a_j + s_j\}\right] \\ &= M_0 + \int_0^t c(s)ds + \lambda^{(d)} t \mathbb{E}[d] - \lambda^{(b)} t \mathbb{E}[b] + \lambda^{(b)} \cdot \mathbb{E}[b] \cdot (1-\mathbb{E}[p]) \cdot \int_0^t G(t-u)du \end{aligned}$$

and for the variance we have

$$\begin{aligned} \mathrm{Var}[M^{\infty}(t)] &= \mathrm{Var}\left[M_0 + \int_0^t c(s)ds + \sum_{i=1}^{N^{(d)}(t)} d_i - \sum_{j=1}^{N^{(b)}(t)} b_j + \sum_{j=1}^{N^{(b)}(t)} (1-p_j)\cdot b_j \cdot \{t > a_j + s_j\}\right] \\ &= \mathrm{Var}\left[\sum_{i=1}^{N^{(d)}(t)} d_i\right] + \mathrm{Var}\left[\sum_{j=1}^{N^{(b)}(t)} b_j + \sum_{j=1}^{N^{(b)}(t)} (1-p_j)\cdot b_j \cdot \{t > a_j + s_j\}\right] \\ &= \lambda^{(d)} \cdot \mathbb{E}[d^2] \cdot t \\ &+ \mathrm{Var}\left[\sum_{j=1}^{N^{(b)}(t)} b_j(\{t \le a_j + s_j\} + \{t > a_j + s_j\}) + \sum_{j=1}^{N^{(b)}(t)} (1-p_j)\cdot b_j \cdot \{t > a_j + s_j\}\right] \\ &= \lambda^{(d)} \cdot \mathbb{E}[d^2] \cdot t + \lambda^{(b)} \mathbb{E}[b^2] \mathbb{E}[p^2] \int_0^t G(t-u)du + \lambda^{(b)} \mathbb{E}[b^2] \int_0^t \overline{G}(t-u)du. \end{aligned}$$

This completes the proof. □

Figure 3 provides an example of the bail fund under the bail everyone out model. It is assumed that the parameters of the model are given by Example 1 of Table 2. It is observed in Figure 3 that our expressions for the mean and variance given in Theorem 1 match the simulated results extremely well. The mean and variance are also plotted in Figure 3 using the parameters from Example 2 and it is easy to observe that the formulas are accurate at describing the mean and variance dynamics, but as accurate as Figure 3. This is because the blocking mechanism is having more of an impact in the parameters of Example 2.

Theorem 2 The moment generating function for the amount of money left in the bail fund, $M(t)$, is given by the following expression

$$\mathbb{E}[e^{\theta M^{\infty}(t)}] = e^{\theta M_0 + \theta \int_0^t c(s)ds + (\psi(\theta)-1)\lambda^{(d)} t + \lambda^{(b)} \cdot (\varphi(\theta)-1) \int_0^t \overline{G}(t-u)du + (\gamma_{p,b}(\theta)-1)\lambda^{(b)} \int_0^t G(t-u)du}.$$

Table 2: Model Parameters of Examples.

Examples/Parameters	$\lambda^{(b)}$	$\lambda^{(d)}$	b_i	d_i	p_i	s_i	M_0
Example 1	**1**	**1**	**1**	**1**	**0**	**10**	**10**
Example 2	**1**	**1**	**2**	**1**	**0**	**10**	**10**

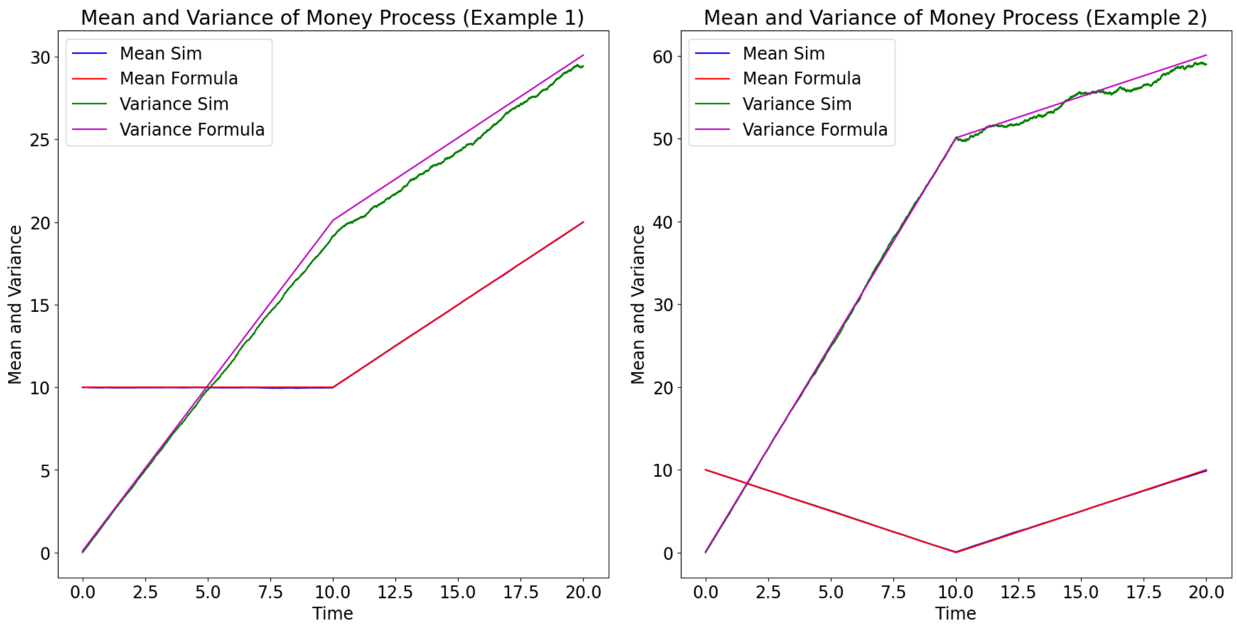

Figure 3: Mean and Variance of the Bail Everyone Out Money Process (Example 1, left) and (Example 2, right).

Proof. The proof follows from recognizing that M_0 and $\int_0^t c(s)ds$ are deterministic, the donation process is an independent compound Poisson process, and the processes for the money being lent to defendants and the money returned back to the bail funds are independent processes by the property of Poisson random measures, see for example Eick et al. (1993) and Daw et al. (2020). □

Now that the results for the model with unlimited resources have been demonstrated, it is important to show how to exploit the central limit theorem to develop new approximations and algorithms for stabilizing the performance of the bail fund with limited resources.

2.3 Approximating the Blocking Probability

After defining the new model of the bail fund where requests can be blocked due to a lack of funds, it is crucial to determine the probability of blocking in this model. However, unlike the scenario where everyone is funded in the infinite server setting, it is not evident how to analyze the blocking probability in this setting. To address this challenge, we can utilize the bail out everyone exceedance levels to approximate a blocking probability. This is a common approach in loss queues, see for instance Massey and Whitt (1994), Hampshire et al. (2003), and Pender (2015a). Our approach is to use a conditional probability argument introduced in Hampshire et al. (2003) and Massey and Pender (2018) for staffing Erlang loss queues. Thus, we can approximate the blocking probability using the following conditional probability expression.

Proposition 3 Suppose that a bail request is equal to b at time t, then we can approximate the blocking probability (the bail fund does not have enough money for the bail request) by the following conditional

Gaussian probability expression

$$\mathbb{P}\left(M^B(t) < b\right) \approx \frac{\Phi\left(\frac{b-\mathbb{E}[M^\infty(t)]}{\sqrt{\mathrm{Var}[M^\infty(t)]}}\right) - \Phi\left(\frac{-\mathbb{E}[M^\infty(t)]}{\sqrt{\mathrm{Var}[M^\infty(t)]}}\right)}{\overline{\Phi}\left(\frac{-\mathbb{E}[M^\infty(t)]}{\sqrt{\mathrm{Var}[M^\infty(t)]}}\right)}. \tag{3}$$

Proof.

$$\begin{aligned}
\mathbb{P}\left(M^B(t) < b\right) &\approx \mathbb{P}\left(M^\infty(t) < b \mid M^\infty(t) > 0\right) \\
&\approx \mathbb{P}\left(\mathbb{E}[M^\infty(t)] + \sqrt{\mathrm{Var}[M^\infty(t)]}\cdot \mathscr{X} < b \mid \mathbb{E}[M^\infty(t)] + \sqrt{\mathrm{Var}[M^\infty(t)]}\cdot \mathscr{X} > 0\right) \\
&= \frac{\mathbb{P}\left(\mathbb{E}[M^\infty(t)] + \sqrt{\mathrm{Var}[M^\infty(t)]}\cdot \mathscr{X} < b \ , \ \mathbb{E}[M^\infty(t)] + \sqrt{\mathrm{Var}[M^\infty(t)]}\cdot \mathscr{X} > 0\right)}{\mathbb{P}\left(\ \mathbb{E}[M^\infty(t)] + \sqrt{\mathrm{Var}[M^\infty(t)]}\cdot \mathscr{X} > 0\right)} \\
&= \frac{\Phi\left(\frac{b-\mathbb{E}[M^\infty(t)]}{\sqrt{\mathrm{Var}[M^\infty(t)]}}\right) - \Phi\left(\frac{-\mathbb{E}[M^\infty(t)]}{\sqrt{\mathrm{Var}[M^\infty(t)]}}\right)}{\overline{\Phi}\left(\frac{-\mathbb{E}[M^\infty(t)]}{\sqrt{\mathrm{Var}[M^\infty(t)]}}\right)}.
\end{aligned}$$

□

In addition to estimating exceedance probabilities using conditional Gaussian approximations like in Equation 2.3, it is also possible to use these same approximation techniques to approximate the mean and variance of the money process. In the blocking model, an approximation for the mean and variance of the remaining money process is given by

$$\begin{aligned}
\mathbb{E}[M^B(t)] &\approx \mathbb{E}[M^\infty(t) | M^\infty(t) > 0] \\
&\approx \mathbb{E}[\mathbb{E}[M^\infty(t)] + \sqrt{\mathrm{Var}[M^\infty(t)]}\cdot \mathscr{G} \mid \mathbb{E}[M^\infty(t)] + \sqrt{\mathrm{Var}[M^\infty(t)]}\cdot \mathscr{G} > 0] \\
&= \mathbb{E}[M^\infty(t)] + \sqrt{\mathrm{Var}[M^\infty(t)]}\cdot \frac{\varphi\left(\frac{-\mathbb{E}[M^\infty(t)]}{\sqrt{\mathrm{Var}[M^\infty(t)]}}\right)}{\overline{\Phi}\left(\frac{-\mathbb{E}[M^\infty(t)]}{\sqrt{\mathrm{Var}[M^\infty(t)]}}\right)} \qquad (4) \\
\mathrm{Var}[M^B(t)] &\approx \mathrm{Var}[M^\infty(t) | M^\infty(t) > 0] \\
&\approx \mathrm{Var}[\mathbb{E}[M^\infty(t)] + \sqrt{\mathrm{Var}[M^\infty(t)]}\cdot \mathscr{G} \mid \mathbb{E}[M^\infty(t)] + \sqrt{\mathrm{Var}[M^\infty(t)]}\cdot \mathscr{G} > 0] \\
&= \mathrm{Var}[M^\infty(t)]\cdot \left(1 + \left(\frac{-\mathbb{E}[M^\infty(t)]}{\sqrt{\mathrm{Var}[M^\infty(t)]}}\right)\cdot \frac{\varphi\left(\frac{-\mathbb{E}[M^\infty(t)]}{\sqrt{\mathrm{Var}[M^\infty(t)]}}\right)}{\overline{\Phi}\left(\frac{-\mathbb{E}[M^\infty(t)]}{\sqrt{\mathrm{Var}[M^\infty(t)]}}\right)} - \left(\frac{\varphi\left(\frac{-\mathbb{E}[M^\infty(t)]}{\sqrt{\mathrm{Var}[M^\infty(t)]}}\right)}{\overline{\Phi}\left(\frac{-\mathbb{E}[M^\infty(t)]}{\sqrt{\mathrm{Var}[M^\infty(t)]}}\right)}\right)^2\right). \qquad (5)
\end{aligned}$$

These approximations for the mean and variance of the remaining money process in the blocking model are given in Equation 4 and Equation 5 respectively. The approximations are constructed by combining an infinite server approximation while also maintaining that the infinite server money process remains positive. Specifically, the mean and variance expressions leverage the analysis of Pender (2015b) for the central moments of the truncated normal distribution. This approach provides a more accurate approximation of the remaining money process and allows us to better understand the dynamics of the bail fund under a limited resource environment.

Below, the parameters of Example 2 are applied to simulate the blocking model in order to calculate the mean and variance of the money process and the blocking probability.

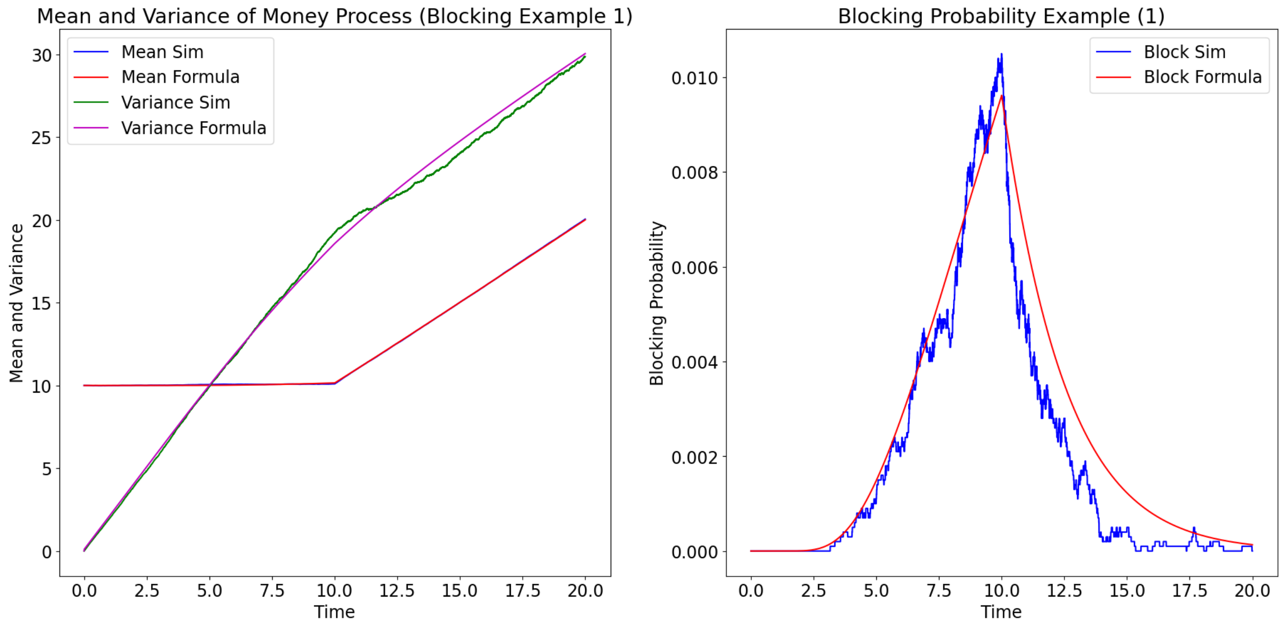

Figure 4: Mean and Variance of the Blocking Money Process (left) and blocking probability (right) for Example 1.

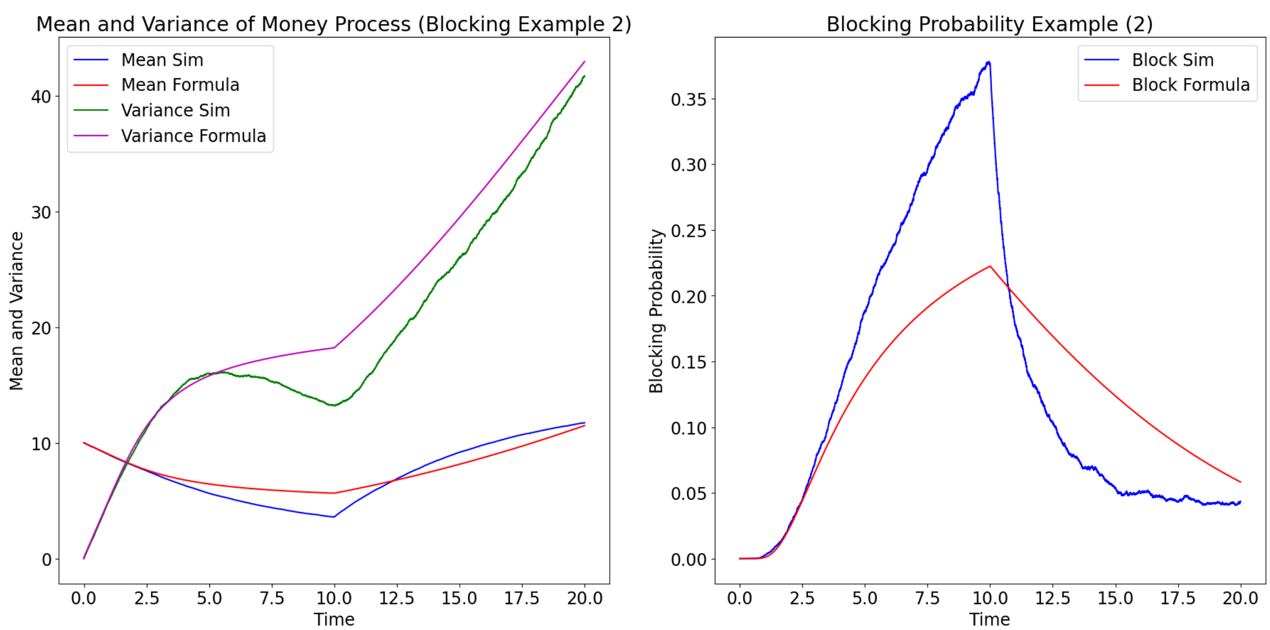

Figure 5: Mean and Variance of the Blocking Money Process (left) and blocking probability (right) for Example 2.

On the left of Figures 4 and 5, the mean and variance of Example 1 and Example 2 of the blocking model are plotted. It is observed that the conditional Gaussian approximations are quite good at describing the behavior of the mean and variance. On the right of Figures 4 and 5, the probability of being blocked from the bail fund is plotted. It is observed that the probability is modeled quite well with a Gaussian approximation with one notable exception. When the mean queue length is near zero, the approximations break down. In fact, the difference between Example 1 and Example 2 is a factor of 35 times more blocking experienced in Example 2. The gap in the performance of the two examples possibly hints at using higher order moments to correct for this behavior like in Massey and Pender (2013) and Pender (2014). We believe that by adding novel skewness and kurtosis corrections, we might be able to better approximate the blocking probability with better accuracy around this point of being near the boundary.

3 STABILIZING THE PERFORMANCE OF THE BAIL FUND USING SIMULATION

The success of a bail fund relies heavily on the community's generosity in making donations. However, this reliance raises questions about how much funding is needed to ensure the bail fund's operational stability. To determine this, one crucial performance metric is the probability of the bail fund resorting to credit to fulfill a bail request. Therefore, the objective is to stabilize this probability by ensuring that the bail fund has sufficient funds available at all times. Achieving this efficiently requires an understanding of the interplay between the bail request process, the donation process, and the limited resources available to the bail fund. As a result, finding the optimal balance between available funds and incoming requests is essential to minimize the likelihood of the bail fund being exhausted and having to resort to credit usage. Thus, our goal is to stabilize the following probability of blocking

$$\mathbb{P}\left(M^B(t) < b\right) \approx \varepsilon \quad \forall\, t \geq 0. \tag{6}$$

In this section, the performance of the bail fund is stabilized using a simulation approach. The simulation approach is similar to Jennings et al. (1996) and Feldman et al. (2008). Below, the steps of the simulation algorithm for stabilizing the probability of blocking in the bail fund are outlined.

Algorithm 1 Simulation Algorithm for Stabilizing Blocking Probability

Step 1: Simulate the queue roughly 10,000 times with an initial constant capital amount c and record the blocking probability.

Step 2: If the blocking probability at specific time t is greater than the desired value ε, then reduce previous value by 2. Then repeat Step 1 again.

Step 3: If the blocking probability at a specific time t is less than the desired value ε, then perform a standard bisection search for the stabilizing capital amount until the system achieves near the desired probability of $1-\varepsilon$.

In Figure 6, the probability of blocking in the blocking model by using the above stabilization algorithm is plotted. It is observed that the algorithm is quite good a stabilizing the blocking probabilities at their respective values of ε. However, in some cases, we need to take away money from the bail fund to do this since there are some cases where the bail fund is too well funded. Thus, it is possible to stabilize the performance of bail funds with a deterministic inflow or outflow of cash throughout all time.

4 CONCLUSION & FUTURE DIRECTIONS

This paper proposes a novel stochastic model for analyzing the effectiveness of community bail funds in the context of the bail setting. By applying infinite server queueing theory, it is possible to derive several key results that shed light on the dynamics of the bail fund process. It is observed that the Gaussian based approximations are excellent at estimating the dynamics of blocking models where there is very little blocking. As more blocking occurs, the Gaussian based approximations break down and it is suspected that higher-order methods are needed to improve the Gaussian based approximations.

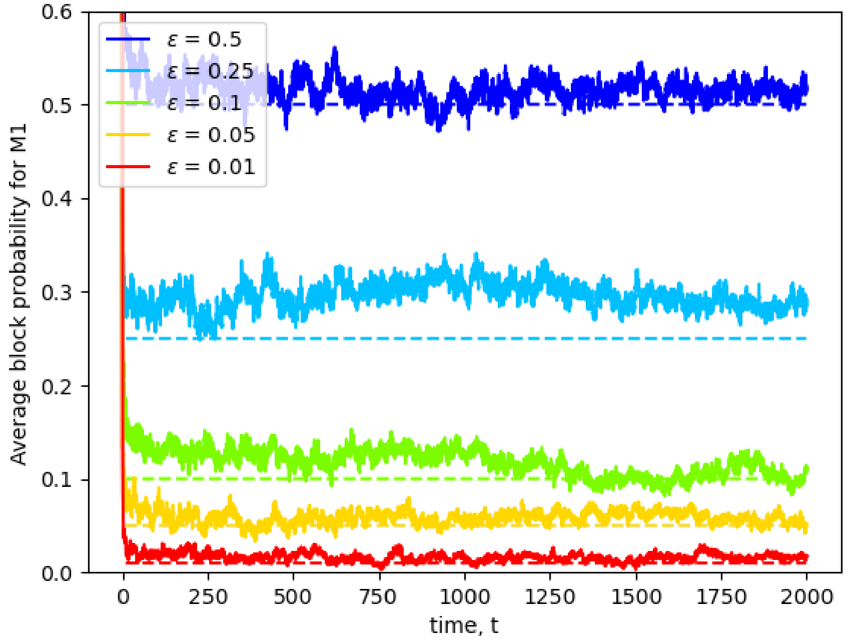

Figure 6: Stabilizing the probability of blocking for $\varepsilon \in \{.5, .25, .1, .05, .01\}$.

There are several important considerations that remain unknown and require further investigation. For instance, in Tompkins County, criminal charges tend to increase during the winter months due to the lack of housing for the homeless population, highlighting the need to account for time-varying crime patterns and larger social issues. Additionally, it would be interesting to explore how many theoretical results can be derived for the limited funds case and to determine the amount of funding required for a nationwide bail system to ensure the release of all individuals charged with minor crimes or offenses. Finally, given that many bail requests come from repeat offenders, future work could explore retrial models to gain a deeper understanding of the bail fund process.

This paper also proposes a limited credit model where bail requests can be denied due to insufficient funds in the bail fund. This type of blocking has been observed in real-world scenarios, such as during the George Floyd protests where many bail funds were unable to accommodate the large number of requests Deflem (2022) and Rosen (2021). To gain a better understanding of this phenomenon, there are several important questions that need to be addressed within the context of this model. Building upon the work of Daw et al. (2021), it is possible to use a bivariate Hawkes process to model the interdependence between the arrival of bail requests and donations. By doing so, one can explore how the dynamics of these processes affect the availability of funds in the bail fund and the probability of bail request denial. Overall, this limited credit model provides a framework for analyzing the performance of community bail funds and identifying areas for improvement.

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AUTHOR BIOGRAPHIES

SOPHIA GUNLUK is a first-year Research Master's student at Mila, and previously studied at Cornell University. Her research interests are in applying machine learning to fairness and policy-making. Her e-mail address is sg764@cornell.edu.

YIDAN ZHANG is a second-year PhD student in Systems Engineering at Cornell University. Her research interests are in stochastic processes, public policy, and queueing theory. Her e-mail address is yz123@cornell.edu.

JAMOL PENDER is an Associate Professor in Operations Research and Information Engineering (ORIE) at Cornell University. He earned his PhD in the Department of Operations Research and Financial Engineering (ORFE) at Princeton University. His research interests include queueing theory, stochastic simulation, dynamical systems and applied probability. His e-mail address is jjp274@cornell.edu. His website is https://blogs.cornell.edu/jamolpender/.