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HOW TO COMBINE MODELS? PRINCIPLES AND MECHANISMS TO AGGREGATE FUZZY COGNITIVE MAPS

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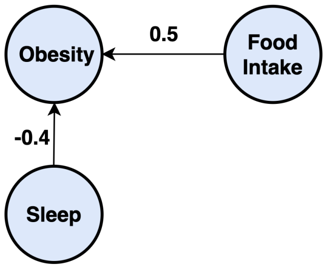
ABSTRACT

Fuzzy Cognitive Maps (FCMs) are graph-based simulation models commonly used to model complex systems. They are often built by participants and aggregated to compare the viewpoints of homogenous groups (e.g., anglers and ecologists) and increase the reliability of the FCM. However, the default approach for aggregation may propagate the errors of an individual participant, producing an aggregate FCM whose structure and simulation outcomes do not align with the system of interest. Alternative aggregation methods exist; however, there are no criteria to assess the quality of aggregation methods. We define nine desirable criteria for FCM aggregation algorithms and demonstrate how three existing aggregation procedures from social choice theory can aggregate FCMs and fulfill desirable criteria, enabling the assessment and comparison of FCM aggregation procedures to support modelers in selecting an aggregation algorithm. Moreover, we classify existing aggregation algorithms to provide structure to the growing body of aggregation approaches.

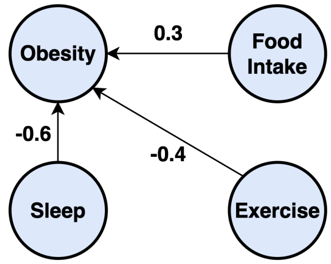
1 INTRODUCTION

Fuzzy Cognitive Maps (FCMs) are discrete simulation models capable of investigating the long-term behavior of a complex system and assessing the effects of interventions (Figures 1a to 1c). They capture a system as a directed weighted graph where labeled nodes represent concepts and edges depict causal relationships between concepts. Modelers can construct FCMs leveraging historical data via algorithms, through a participatory approach with facilitators, or a combination. Participatory modeling approaches develop a simplified representation of reality through the perspectives of participants, including experts, stakeholders, and community members (Edwards and Kok 2021). Constructing an FCM is a relatively simple process (Concepción et al. 2020) as algorithms in open libraries can quickly fit a model to data (Mkhitaryan et al. 2022) or facilitators can create a model with a participant within an hour (Giabbanelli et al. 2022). As a result of their transparency and ease of development, FCMs are used by over 20,000 studies (Kininmonth et al. 2021) that broadly cover two domains. Studies on *physical* systems (e.g., Papageorgiou and Salmeron 2012 for engineering) admit a ground truth measured from the target system, which is simplified by the model. In this context, participants' viewpoints are approximations whose correctness can be measured. Studies on *socio-environmental systems* (e.g., Mourhir 2021) cover the beliefs and values of human actors, so the participants' inputs are viewed as perspectives or preferences.

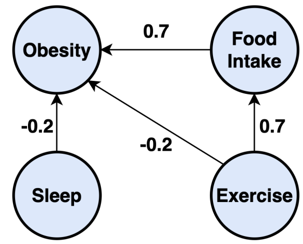
Creating an FCM that provides a relevant decision-support tool may require participants with diverse lived experiences or areas of expertise. For example, for a suicide model, participants may include individuals who have attempted, family, or academic experts. Given the interdisciplinary nature of modeling, experts may also have to represent multiple domains (e.g., psychology, epidemiology). In contrast with group-model building approaches, the need for multiple categories and domains often results in one-on-one model



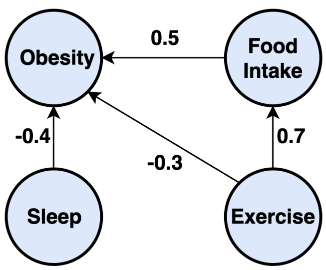
(a) Participant One's FCM



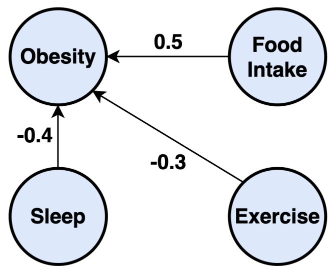
(b) Participant Two's FCM



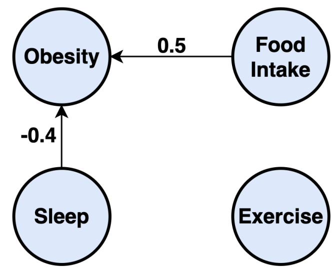
(c) Participant Three's FCM



(d) Union Aggregation



(e) Majority Rule Aggregation



(f) Intersection Aggregation

Figure 1: Three FCMs built by participants (Subfigures 1a to 1c). The structure of an aggregated map can be determined by one of three mechanisms: union (Subfigure 1d), majority rule (Subfigure 1e), or intersection (Subfigure 1f). Once the structure is obtained, the edges' weights are computed.

building where a trained facilitator creates an FCM with one participant. This approach also alleviates concerns about power dynamics, as participants no longer have to refrain from sharing views that may be judged by their peers. Individual FCMs are aggregated by modelers to produce a comprehensive and reliable group-level FCM or to compare the views of groups of participants (Aminpour et al. 2021). This article focuses on the goals of this aggregation and algorithms to satisfy these goals.

As summarized by Papageorgiou et al. (2020), "the weighted average method is considered as the benchmark method for FCM aggregation purposes, where the final FCM model is built by averaging numerical values for a given interconnection". It is a two-step process. First, modelers obtain the aggregate *structure* as the *union* of all nodes and edges in the set of FCMs (Figure 1d). Second, for each edge, modelers derive its *weight* as the average across all maps that contain it. Note an edge's weight is unaffected if the FCMs of some participants do not contain that it, instead of reducing the weight by counting missing occurrences as 0 (Figures 1a to 1d). Many real-world case studies use the weighted average method; for example, it has been used to model perceptions of water scarcity (Mehryar et al. 2019), fishery management (Lavin et al. 2018), and the Nigerian rice agri-food system (Edwards and Kok 2021). Despite its wide use, there are two potential shortcomings with this method. First, this default method of aggregation is reasonable if we assume that consensus is an informed average opinion that treats all participants as equal (Kosko 1988); however, participants have different knowledge and experience, so this assumption may not hold (Taber 1991). For example, one participant may have five years of experience and another 10 (Taber 1991), or they may express different levels of confidence on specific topics. Second, the method may propagate the errors of a participant, producing an aggregate FCM that fails to accurately represent the system. For example, one expert may believe exercising makes you eat more and that the increased consumption of food outweighs the benefits of exercise (Figure 1c), even if that is untrue (Bryner et al. 1997). The default aggregation would preserve this erroneous belief in the aggregate map, and this inaccurate representation (Figure 1d) can produce incorrect simulation outcomes.

Given these limitations, numerous FCM aggregation procedures have been proposed. However, there are currently no criteria to assess the quality of aggregation methods or guidance on the properties they should fulfill. This research gap hinders modelers from comparing aggregation algorithms and selecting an optimal one for their use case. Our paper contributes to addressing this gap through three contributions:

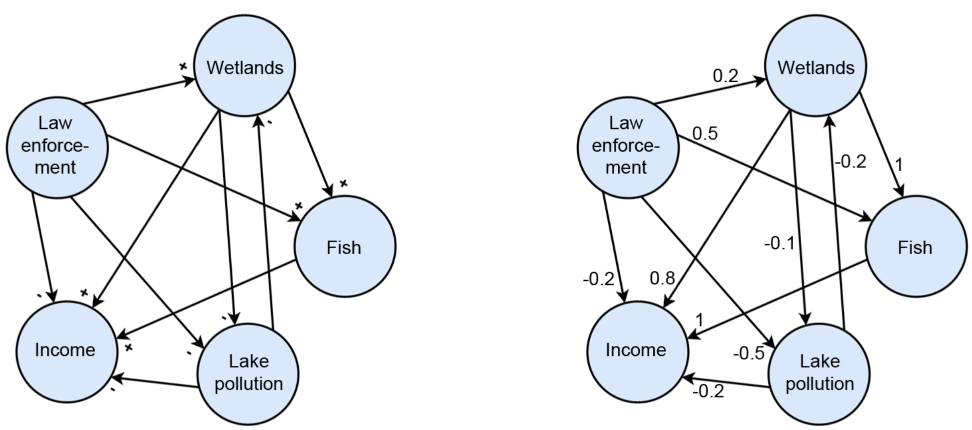
1. We identify three categories for aggregation procedures for FCMs and categorize existing procedures. This is the first *structured overview* for the growing body of FCM aggregation algorithms.
2. We define nine evaluation criteria for FCM aggregation procedures for social systems. These criteria can help modelers *assess the quality* of aggregation procedures and compare them.
3. We demonstrate how three existing aggregation procedures from *social choice theory* can be applied to FCM aggregation to fulfill desired criteria and use them on a real-world case study in our supplementary material available at https://doi.org/10.5281/zenodo.8104825. Although social choice theory has well-defined properties for aggregation and procedures that satisfy these properties, it has never been applied to FCMs.

The remainder of this paper is organized as follows. In section 2, we introduce foundational concepts of FCMs and social choice theory and examine and categorize existing aggregation procedures. We define desirable criteria for FCM aggregation algorithms in section 3 and demonstrate how existing aggregation procedures from social choice theory fulfill desired criteria in section 4. Finally, we discuss the importance of our results and opportunities for future work in section 5.

2 BACKGROUND

2.1 From Causal Maps to Fuzzy Cognitive Maps

Causal Maps depict systems as a graph, where concepts are labeled nodes and relationships are directed, typed edges (Figure 2a). Edges are either positive or negative. A positive edge signifies that an increase in the source directly causes an increase in the target concept. For example, as the fish population increases, income for fisheries increases (Figure 2a). Conversely, a negative edge means that an increase in the source leads to a decrease in the target. For instance, as lake pollution increases, income decreases (Figure 2a). Since relations are only typed and not quantified, there is no distinction between the strength of the two relationships, so it cannot be determined whether the impact of wetlands on lake pollution is stronger than the impact exerted by law enforcement (Figure 2a).



(a) Causal Loop Diagram (b) FCM

Figure 2: The causal map (a; left) is extended into a Fuzzy Cognitive Map (b; right) by adding a degree of influence to each causal relationship. This example is derived from Özesmi and Özesmi (2004).

Similar to how System Dynamics builds on Causal Loop Diagrams, Fuzzy Cognitive Maps (FCMs) expand on causal loop diagrams in three ways (Kosko 1986). First, they quantify the causal strengths of edges in the interval [-1, 1] (Figure 2b). An edge weight of -1 means the causal relationship is strongly negative, 0 is no causal relationship, and 1 is a strongly positive causal relationship. Second, FCMs allow concepts to take on values in the interval [0, 1] for a given case, where 0 means it is completely absent (e.g., no law enforcement) and 1 is entirely present (e.g., as much law enforcement as possible). Note that quantifying causal edges can be done with participants (i.e., participatory modeling) and/or via machine learning methods during model *development*. In contrast, the quantification of concepts is performed when *applying* the model, since the concepts form the input depicting a case of interest for model users.

Third, assigning numerical values to nodes and edges allows FCMs to perform *simulations* by computing the next values of each node until stabilization. In other words, a simulation is characterized by an *initial state* and the application of a *process* that can change this state until specific *halting criteria* are met. For an FCM, the *initial state* consists of the value of each node at step $t = 0$, the *process* is summarized by an update equation (detailed in the next paragraph) that uses the nodes' current values as well as the edges' permanent values, and the *halting criteria* are (primarily) to reach stabilization and (if needed) to perform up to a user-specified number of iterations. As other authors (Štula et al. 2017; Papageorgiou et al. 2011), we characterize an FCM as a discrete simulation model to emphasize that it proceeds in discrete iterations, while noting that the nodes' values are defined in the continuous interval [0, 1]. Since iterations do not represent time and the primary halting criterion is to stabilize, a simulation via an FCM is focused on system behavior rather than precise point estimates. Hence, they can compare interventions for desired outcomes. For example, Firmansyah et al. (2019) simulated different levels of funding cut in education (i.e., simulation scenarios) and showed the change in the social fabric of a city as compared to baseline. This approach showed that severe cuts would eventually be detrimental to the city, but it cannot say when and does not claim to predict a specific percentage.

The mathematics of FCMs have been presented at the Winter Simulation on several occasions (Mkhitaryan and Giabbanelli 2021; Lavin and Giabbanelli 2017). Thus, we provide a succinct overview here. An FCM performs simulations in discrete iterations that update node values synchronously. The current state of an FCM at iteration t is denoted by the tuple (V^t, W, f), where V^t is the vector of k concept nodes, W is the adjacency matrix with the edge weights between concepts, and f is a threshold function (e.g., a sigmoid) that keeps concept values in the interval [0, 1]. For each node j, its value V_j^{t+1} at iteration $t+1$ is obtained per Equation 1. There are variations of this equation depending on a modeler's needs. For example, the condition $i \neq j$ prevents the previous value of the concept from affecting itself. If a modeler requires *memory* to allow the previous value of the concept to affect itself, then they would instead use Equation 2 proposed by Stylios and Groumpos (1999). The equation is applied until either all or a specific subset of the nodes have stabilized (i.e., consecutive changes are less than a user-defined threshold). The resulting node values are not 'better' than the initial ones; rather, they reflect how an initial situation would gradually change under a given scenario.

$$V_j^{t+1} = f\,\underbrace{\Big(\sum_{\substack{i=1\\ i\neq j}}^{k} W_{ij} \times V_i^t\Big)}_{\text{total input to node}}, \qquad (1)$$

$$V_j^{t+1} = f\Big(\underbrace{\Big(\sum_{\substack{i=1\\ i\neq j}}^{k} W_{ij} \times V_i^t\Big)}_{\text{total input to node}} + \underbrace{V_j^t}_{\text{memory}}\Big) \qquad (2)$$

The choice of a function f affects the convergence of the FCM, which can lead to a (desirable) fixed-point, or become trapped in a limit cycle or chaotic trajectory. We refer the reader to section 2 by Felix et al. (2019) for a detailed summary of convergence and options for f.

2.2 Social Choice Theory

Social choice theory studies the aggregation of input from a set of individuals into a collective view. The field started in Economics and Political Science with the study of elections and measures of social welfare (Arrow et al. 2010). Recently, it was complemented by a lively community of computer scientists who focused on the algorithmic properties of social choice mechanisms and their adaptation to a digital environment (Brandt et al. 2016; Aziz et al. 2019). The mainstream methodology of the field is to evaluate aggregation functions and allocation mechanisms axiomatically. This requires the system designer or the population to collectively list a set of desirable properties, which, once formalized, restrict the set of possible collective choices. This method is well-adapted to purely preferential domains, such as the problem of aggregating FCMs in socio-environmental applications. A complementary stream of work views social choice methods, and voting rules in particular, as maximum likelihood estimators and assesses their adaptation to recovering a ground truth. The seminar work here is the jury theorem by De Condorcet (1785), but a significant body of work has been built since then, as discussed by Dietrich and Spiekermann (2019) alongside additional jury theorems.

The problem of aggregating FCMs is strongly related to two streams of work in social choice. First, the aggregation of graphs into a collective one has been extensively studied by Endriss and Grandi (2017), who focused on preserving properties of the individual graphs in the aggregate (a topic known as *collective rationality*). Second, the aggregation of argumentation frameworks and conditional-preference networks (CP-nets) both involve aggregating a set of graphs, with additional information in the form of a semantic that decides which arguments can be accepted and of the individual preferences over the variables. Coste-Marquis et al. (2007) were the first to consider the problem of aggregating argumentation frameworks, with further work by Dunne et al. (2012) and Delobelle et al. (2015). Chen and Endriss (2019) also used graph aggregation techniques to tackle this problem. In CP-nets, the first work considering input from multiple agents is by Rossi et al. (2004), with numerous follow-ups. We note that the definition of probabilistic CP-nets is particularly close to FCMs (Bigot et al. 2013), in particular when considering the aggregation of a set of CP-nets in a single probabilistic CP-net (Cornelio et al. 2021).

2.3 Previous Fuzzy Cognitive Map Aggregation Procedures

This subsection focuses on nine papers proposing FCM aggregation procedures identified by a Google Scholar search conducted in February 2023 and includes any publications released before that date. We assessed each paper in four ways: we categorized its approach, summarized it according to the features it offers, examined its additional needs, and inspected its evaluation (Table 1).

At a high level, we structure existing works using three categories: frequentist, set-theoretic, or participatory-based. Frequentist methods leverage the occurrence of items (e.g., nodes and edges) in the sample data to determine what should be retained by the aggregation procedure, whereas set-theoretic methods apply set and graph theory operations to produce the final map. In both cases, an algorithm directly generates the aggregate FCM. In contrast, participatory-based methods involve discussions with participants whose insight provides additional data for aggregation.

At a more detailed technical level, we note that methods share certain features. Structurally, a method may select nodes or edges quantitatively or incorporate an additional mechanism other than simply taking the union of all concepts used by experts. Numerically, an approach could average the edge weights to produce the final aggregated edge weight or use an alternative such as credibility weights.

Although modelers may already be interested in a specific method based on its technical design, we emphasize that two additional requirements apply to certain approaches. First, a method can require new parameters that the modeler must determine. For example, the Fuzzy C-Means adds a number of clusters and a fuzzification parameter (Obiedat et al. 2020). Second, a method may require further participant input, such as rating the credibility of other participants (Mazzuto et al. 2018). All but two of the existing aggregation approaches that offer more functionality require additional parameters or participant input, increasing the workload of the modelers or participants.

Finally, we examine four aspects of the evaluation for each method. First, we check whether an approach seeks to achieve specific criteria (e.g., axioms). In other words, do the authors define desirable properties and show their method guarantees their preservation during aggregation? For example, the majority rule for graph aggregation preserves symmetry (Endriss and Grandi 2017). Then, we checked whether the method was applied to an example or a real-case study. The example can be a small, hypothetical scenario or involve simulated data such as adding random noise to real FCMs (Taber 1991), whereas a case study must involve real-world data (e.g., an analysis of clinical risk in drug administration (Mazzuto et al. 2018)). Finally, we check if the proposed method is compared to the default approach of aggregation. Six of the eight additional aggregation procedures are applied to case studies, but only two compare their results to the default method. This provides descriptive statistics of practices in FCM aggregation; we do not suggest that certain practices are better or that a comparison to the default approach is necessary because there is no guidance on comparing the results.

Table 1 summarizes the methods and can help modelers choose an aggregation applicable to their situation. Three observations from Table 1 justify this paper. First, only two existing aggregation procedures address what nodes or edges to include. Deciding which nodes and edges to include in the aggregate FCM is a binary decision-making problem, and social choice theory has a rich existing body of literature and procedures for binary decision-making, which motivates the first examination of FCMs through social choice theory undertaken in our study. Second, all existing aggregation techniques are applied to an example, but none are supported through criteria. The lack of criteria supporting FCM aggregation highlights a central issue: *there are no agreed-upon evaluation criteria or notion of what a good FCM aggregation procedure is*. Third, we observe that none of the aggregation procedures are set-theoretic other than the default approach. Overall, we posit that social choice theory can benefit FCM aggregation research by setting evaluation criteria, defining a notion of a 'good aggregation' procedure, and guiding the development of set-theoretic aggregation methods that fulfill these evaluation criteria.

Table 1: Existing aggregation procedures. F refers to frequentist, T set-theoretic, and P participatory-based. The 'results compared' column for the default approach is not applicable as we cannot compare it to itself.

		Features		Needs		Validation			
Approach	Category	Nodes/Edges Selected	Edge Weights Averaged	Parameters Introduced	Additional Participant Input	Criteria	Example	Case Study	Results Compared
Default Approach (i.e., Union)	T	N	Y	N	N	N	Y	Y	N/A
Consensus Centrality Measure (Obiedat and Samarasinghe 2013)	F	N	N	Y	N	N	Y	Y	N
Fuzzy C-Means (Obiedat et al. 2020)	F	N	N	Y	N	N	Y	Y	N
Expertise Areas (Mazzuto et al. 2018)	P	N	N	N	Y	N	Y	Y	Y
Ordered Weighted Average (Papageorgiou et al. 2019)	F	N	N	Y	N	N	Y	Y	Y
Node Removal (Cunha et al. 2016)	P	Y	Y	N	Y	N	Y	Y	N
Consensus (De Maio et al. 2017)	P	N	N	Y	Y	N	Y	N	N
Simulation Weights (Taber 1987)	F	N	N	N	N	N	Y	N	N
Concept Quantiles (Blewett et al. 2022)	F	Y	Y	Y	N	N	Y	Y	N

3 DEFINING EVALUATION CRITERIA: WHAT SHOULD AN AGGREGATION ACHIEVE?

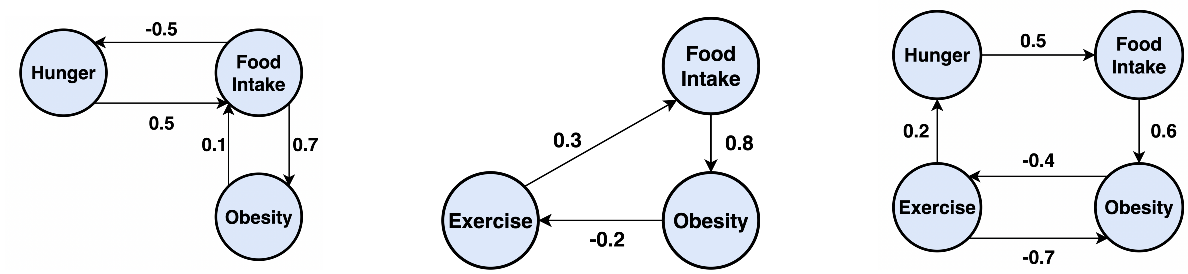
When defining desirable criteria, it is crucial to consider the application domain and the system of interest. We broadly distinguish between physical systems with a ground truth system and socio-environmental systems, which are (at least partly) value-based, and extensions of FCMs often make this distinction (Motlagh et al. 2014). In physical systems, participant input is a noisy approximation of the ground truth system, so the goal of the aggregation procedure is to minimize this noise function (e.g., as quantified by the mean squared error in regression problems), which signal processing has extensively studied (Vaseghi 2008). Alternatively, participant input can be viewed as preferences for socio-environmental systems since they lack ground truth and thus an error function to minimize. As a result, fulfilling desirable criteria (i.e., properties/axioms) is the primary goal of aggregation procedures for socio-environmental systems and has been widely studied in social choice theory (List 2022). Although the primary focus of aggregation when there is a ground truth is to minimize noise, fulfilling desirable criteria may still be applicable. In this section, we define desirable criteria for FCM aggregation.

We first formally define a graph and aggregation procedure, focusing on which edges to include, and we do not consider the edge weights because of social choice theory's strong history of addressing binary decision-making. We define a directed graph as $G = \langle V, E \rangle$ on a finite set of nodes V with a set of edges $E \subseteq V \times V$. Let $2^{V \times V}$ denote the set of all graphs. An *aggregation rule* is a function $F : (2^{V \times V})^n \to 2^{V \times V}$ that maps any set of individual graphs into a single graph. We refer to the collective graph produced by the aggregation procedure as an aggregated graph O and the set of all n input graphs as $I = (G_1, \ldots, G_n)$. Note the set of nodes V is the union of all nodes in the set of input graphs I, so an aggregation rule only decides which edges to include. If a node has no edge, it has effectively been eliminated from the aggregate since it influences no other part of the model per Eqs. 1–2. While such nodes may be eliminated (e.g., by requiring that nodes in the aggregate have at least one edge), tracking nodes that lack consensus may elicit fruitful discussions, potentially clarifying a node's role or lack thereof in the model.

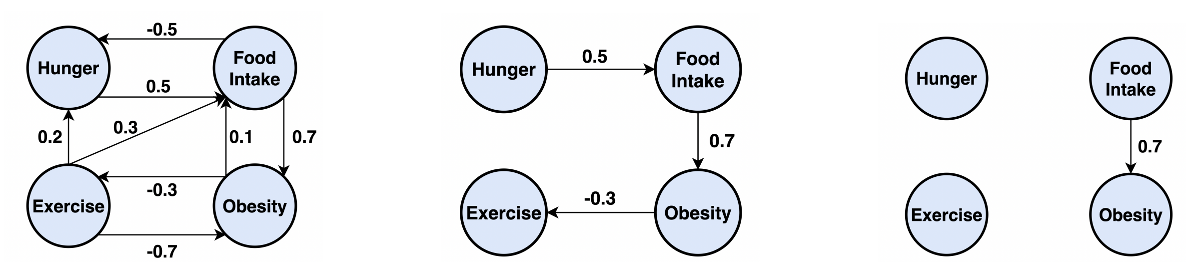
The first three of our nine criteria for FCM aggregation procedures originate from social choice theory (Endriss and Grandi 2017) and argumentation aggregation (Delobelle et al. 2015). First, an aggregation rule is *unanimous* if it accepts an edge present in all individual input graphs: $G_1 \cap \ldots \cap G_n \subseteq O$. A unanimous FCM aggregation procedure is highly desirable because if every participant agrees that a construct is relevant, it should be in the aggregate map. Second, an aggregation rule is *grounded* if it only includes edges that are part of at least one of the input graphs in the aggregated graph (i.e., the aggregation procedure does not create edges): $O \subseteq G_1 \cup \ldots \cup G_n$. In argumentation aggregation, this property is called *closure* (Delobelle et al. 2015). On the one hand, a grounded aggregation procedure can build trust in the modeling process by guaranteeing participants that the content only reflects their views. On the other hand, when integrating different views, it may be necessary to create new concepts and edges to bridge differences (Mkhitaryan and Giabbanelli 2021); this is an open problem as no aggregation technique currently automatically reconciles viewpoints. Finally, an *anonymous* aggregation rule treats all individuals the same, meaning $F(G_1, \ldots, G_n) = F(G_{\pi(1)}, \ldots, G_{\pi(n)})$ for any permutation π that maps set of n participants to itself. The equity of treatment afforded by an anonymous aggregation procedure is of interest when working with participants of a similar background. However, it may be undesirable when working with participants with different areas or levels of knowledge. For example, when scientists build FCMs, their impact on the model could be weighted based on a self-assessed level of experience on specific themes.

The next six criteria specify how properties of the aggregate may seek to *preserve* the properties expressed in all individual FCMs. We start by defining this notion of 'preserving' and then specify each property. In social choice theory, this preservation is called collective rationality and defined as follows: an aggregation rule is *collectively rational* for a property P if the aggregate graph O satisfies the property when all the individual inputs graphs in I do. First, a graph is *reflexive* if each node has a self loop: $\forall x.xEx$. Consequently, an aggregation method preserves reflexivity when all of its nodes still have a self-loop. Second, a graph is *irreflexive* if none of the nodes has a self loop: $\neg\exists x.xEx$; by extension, the aggregation method is also irreflexive when no node has a self loop. In the context of FCMs, reflexivity can be a way to provide some amount of memory by relaxing Equation 1 to account for self-loops (i.e., removing the term $i \neq j$) and setting the weight of the self-loop (W_{ii}) as desired. Third, a graph is *transitive* if whenever there is an edge from a node x to y and from y to z, then there is also an edge directly from x to z:

$\forall xyz.[\{xEy \wedge yEz\} \rightarrow xEz]$). Transitivity in an FCM means any node that has an indirect effect (via two edges) on another also has a direct effect. This is not generally recommended, as individuals should be selective in creating causal edges to capture the difference between indirect and direct effects (Giabbanelli et al. 2022). Consequently, it would be preferable if an FCM aggregation procedure did not actively seek to preserve transitivity, since it would be best to address misconceptions among individuals by removing edges that portray indirect effects as direct. Fourth, a graph is *connected* if there is a path between each pair of nodes (i.e., you can reach each node starting at every node). This is normally a requirement for causal maps (Wang and Giabbanelli 2023) as well as FCMs (Giabbanelli et al. 2022), hence this property should be maintained by aggregation procedures. Fifth, a graph is *cyclic* if it has at least one cycle (i.e., feedback loop) (Figures 3a to 3c). FCMs are often cyclic as they frequently represent complex systems where feedback loops either maintain concept values (negative feedback loops) or reinforce trends in the system (positive feedback loops). Finally, a graph is *acyclic* if there are no cycles. There are pros and cons between desiring an aggregation to have the cyclic or acyclic property. The quality of an FCM facilitation process is occasionally assessed by ensuring that cycles are present, since they are expected in complex problems (Firmansyah et al. 2019). However, cycles can make an aggregate map hard to understand; these cycles are thus removed (i.e., the graph is 'linearized') when explaining it to a wide audience (Shrestha et al. 2022). An acyclic aggregation yields a simpler structure, albeit less realistic.



(a) FCM One (b) FCM Two (c) FCM Three



(d) Union Aggregation (e) Majority Rule Aggregation (f) Intersection Aggregation

Figure 3: Three hypothetical *connected and cyclic FCMs* (3a to 3c) and their aggregation using the union (3d), majority rule (3e), and intersection (3f), demonstrating the union preserves cycles and connectedness and the majority rule and intersection do not.

4 FULFILLING EVALUATION CRITERIA: DESIGNING AGGREGATION ALGORITHMS

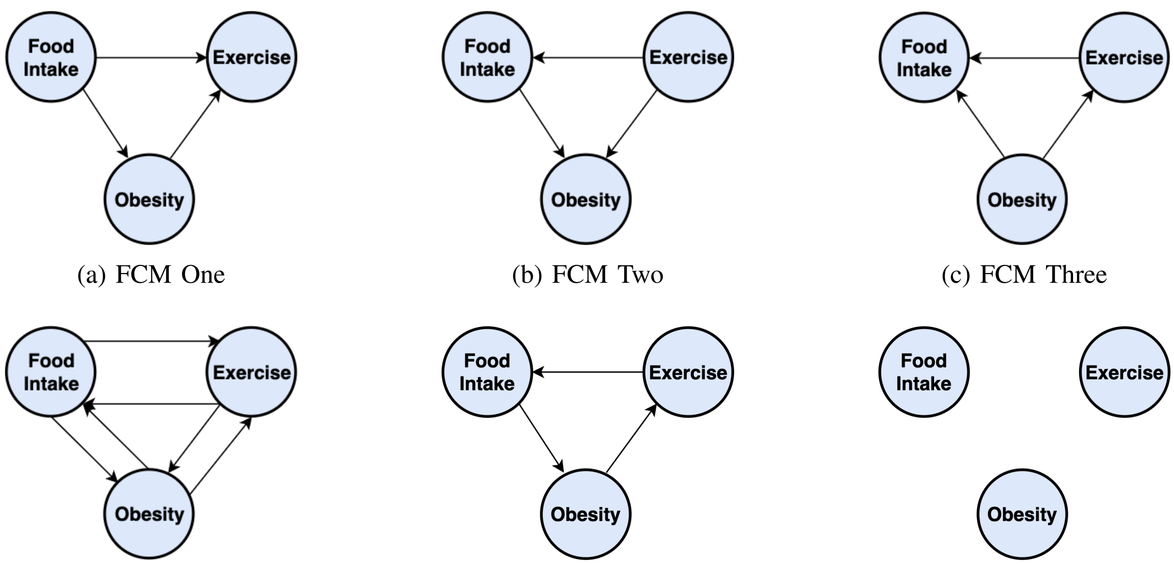
We formally define three set-theoretic aggregation procedures from social choice theory and then examine which criteria from section 3 they fulfill or preserve (Table 2). First, the *union rule* is the aggregation $F : I \rightarrow \langle V, E_1 \cup \ldots \cup E_n \rangle$ that includes every edge present in the input graphs (Figure 1d). Note the default approach for FCM aggregation uses the union rule to select nodes and edges. Second, the (strict) *majority rule* is the aggregation $F : I \rightarrow \langle V, e \in V \times V : |N_e^I| > \frac{n}{2} \rangle$, where N_e^I is the set of input graphs containing the

Table 2: The three aggregation mechanisms and the properties they fulfill and preserve from Section 3.

Aggregation Rule	Unanimous	Grounded	Anonymous	Reflexivity	Irreflexivity	Transitivity	Connectedness	Cyclic	Acyclic
Union	Y	Y	Y	Y	Y	N	Y	Y	N
Majority	Y	Y	Y	Y	Y	N	N	N	N
Intersection	Y	Y	Y	Y	Y	Y	N	N	Y

individual edge e, that admits an edge if and only if more than half of the individuals accept it (Figure 1e). Third, the *intersection rule* is the aggregation $F : I \rightarrow \langle V, E_1 \cap \ldots \cap E_n \rangle$ that only includes edges in every input graph (Figure 1f). The union rule yields the most edges followed by the majority rule, and lastly the intersection rule. Thus, the aggregation procedure affects the structure of the aggregate FCM.

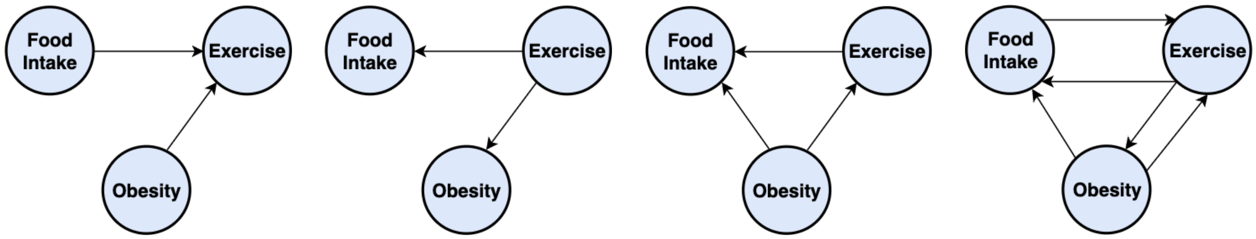
We provide the intuition of why each aggregation procedure fulfills or preserves each criterion and counterexamples when the property does not. Several of these properties are formally proven in (Endriss and Grandi 2017). The union, majority, and intersection aggregation procedures are unanimous, grounded, and anonymous as they depend on participant input (i.e., the set of input graphs I) and do not differentiate between participants. Similarly, they all preserve reflexivity because none of the procedures will eliminate self-loops if every node has a self-loop in all the input graphs and irreflexivity since none of the approaches will add self-loops not in the input graphs. Figure 3 shows that the union preserves connectedness and cycles, whereas the majority rule and intersection do not. Finally, Figures 4 and 5 depict that the majority rule and union are not collectively rational for transitivity and will not always produce acyclic graphs if all the input graphs are acyclic. In contrast, the intersection is collectively rational for transitivity and will generate an acyclic graph if all the input graphs are acyclic.



(a) FCM One (b) FCM Two (c) FCM Three

(d) Union Aggregation (e) Majority Rule Aggregation (f) Intersection Aggregation

Figure 4: Three hypothetical *acyclic and transitive* FCMs (4a to 4c) and their aggregation using the union (4d), majority rule (4e), and intersection (4f), demonstrating the majority rule is not collectively rational with respect to transitivity and will not always produce acyclic graphs if all the input graphs are acyclic. Edge weights are omitted for simplicity.



(a) FCM One (b) FCM Two (c) FCM Three (d) Union Aggregation

Figure 5: Three hypothetical acyclic and transitive FCMs (5a to 5c) and their aggregation using the union (5d). The union is not collectively rational with respect to transitivity because there is an edge from food intake to exercise and exercise to obesity but not edge from food intake to obesity. Additionally, the union will not always produce acyclic graphs if all the input graphs are acyclic (see 'food intake' ⇔ 'exercise').

5 DISCUSSION AND CONCLUSION

FCMs are widely used to model socio-environmental problems and physical systems. In participatory modeling, each participant may create their own FCM, and individual FCMs are aggregated to increase the reliability of the FCM. However, the default weighted average method of FCM aggregation may propagate the errors of individual FCMs, hence producing an aggregate FCM whose structure and simulation outcomes do not align with the system of interest. Numerous alternative aggregation procedures have been proposed, but we lacked criteria to determine which ones to choose given the modelers' needs.

We categorized nine existing aggregation procedures as frequentist, set-theoretic, or participatory-based to provide *structure* for the growing body of FCM aggregation algorithms, finding a lack of set-theoretic approaches. Additionally, we defined nine desirable criteria for FCM aggregation procedures with a focus on socio-environmental systems, providing criteria to *assess* the quality of and *compare* aggregation procedures and supporting modelers in selecting an optimal aggregation procedure for their use case. Finally, we demonstrated how the majority rule and intersection from social choice theory compare to the union used by the default approach and fulfill desirable criteria for FCM aggregation procedures.

Our paper suggests four areas of future work for FCM aggregation. First, new aggregation procedures could focus on physical systems and minimizing a noise function, leveraging existing literature from signal processing. Second, researchers could develop set-theoretic approaches that account for different expertise areas of participants because successful aggregation rules often rely on individual input where participants are informed, and their competence can vary (Dietrich and Spiekermann 2019). Third, future work may build upon this paper by proposing additional desirable criteria and developing methods that fulfill them. Finally, the combination of FCMs must take into account their execution (i.e., simulation), hence future studies will need to examine the impact of aggregation procedures both on the structure of FCMs (which affects the transparency of the model presented to stakeholders) and on the simulation results produced by the aggregate model.

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