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A TWO-STAGE STOCHASTIC MODEL FOR DRONE DELIVERY SYSTEM WITH UNCERTAINTY IN CUSTOMER DEMANDS

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ABSTRACT

Drone delivery is a popular logistics method for e-commerce businesses due to its efficiency and convenience, especially for last-mile delivery and emergency situations in areas with poor infrastructure. However, the uncertainty of customer demands can affect transportation costs in the long run, making it vital to design an effective delivery system. To tackle this issue, we propose a two-stage stochastic model that minimizes the sum of fixed and expected operating costs. The first stage minimizes the total cost of the delivery system, including the facilities fixed costs and expected operating costs, while the second stage arranges drones' routes according to simulated demands to estimate the minimal expected transportation cost and penalty cost. Since this stochastic programming has infinite scenarios, we deploy a sample average approximation method to estimate its bounds. Additionally, we use a heuristic simulation framework to find a satisfactory solution in an acceptable time.

1 INTRODUCTION

The use of drone delivery systems has grown rapidly in the past decade (Floreano and Wood 2015). Due to the composition of individual orders with different destinations, last-mile delivery generates disproportionately high costs and is often regarded as the most inefficient segment of the supply chain (Macioszek 2017). However, the emergence of drones, with their high speed and few traffic concerns, presents a potential solution that is both more efficient and cost-effective. This has led to a growing interest among logistics companies in establishing and maintaining drone delivery systems for last-mile delivery. Leading companies such as Amazon (CBS News 2013), Google (John Koetsier 2021), and FedEx (FedEx 2019), are investing heavily in the research and development of these systems to revolutionize last-mile delivery and improve the overall profitability and efficiency of the supply chain.

The drone delivery system has different features from traditional grounded delivery. Considering drones' limited capacity and flight range, researchers have approached the drone delivery problem from various perspectives. One of the most commonly studied scenarios is the drone-truck tandem delivery, where a drone is launched from a delivery vehicle, delivers a package, and returns for a new payload (Patchou et al. 2019). Other variations of this scenario include a larger drone, known as a mothership, that can carry smaller drones which then launch from the mothership to deliver packages and return for reassignments and charging (Wen and Wu 2022). This is also known as the mothership and drone routing problem (MDRP) (Poikonen and Golden 2020). Some systems also suggest using charging stations to extend the flight range and payload capacity, allowing drones to reach more customers, especially in sparsely populated areas such as rural regions (Huang and Savkin 2020).

Considering the uncertain drone performance, some researchers design drone delivery systems with uncertain flight distances and battery consumption. Chauhan et al. (2021) model the uncertain flight range by the availability and consumption of battery. Similarly, Kim et al. (2019) build a stochastic modeling framework to design a drone delivery system in humanitarian logistics. Besides, the customer demand is also unpredictable, which brings challenges to optimize facility locations. To address these issues, a two-stage approach was proposed by Cheng et al. (2021). They develop a column-and-constraint generation algorithm to solve the two-stage model, where the first-stage decides which locations are chosen, and the second-stage optimizes the worst case in uncertain customer demands and facility disruption. The assumption on the stochastic has two parts: the customer demand has a maximal deviation from the normal value and some facilities will be randomly disrupted.

In order to make good decisions with uncertainties in an acceptable time, simulation can be a useful tool (Kellner et al. 1999). Additionally, simulation can evaluate the performances of different optimization techniques and find better and faster solutions by being combined with other heuristic algorithms (Li et al. 2011). Considering NP-hardness, heuristic algorithms are proposed as solution methods. Shavarani et al. (2019) built a drone delivery network with fuel stations and warehouses by a Genetic Algorithm. They use fuzzy variables to model the nondeterministic factors and a multi-level facility location approach to deal with the different levels of the facilities. Besides, the solutions generated by heuristic algorithms are usually based on certain problem settings and assumptions, which may not be suitable if the environment is changed or some parameters are stochastic. For example, some routes decided in a determined environment may not be reachable due to the stochastic flight range of drones in reality (Kim et al. 2019).

Except for the heuristics, another wildly used method to deal with stochastic factors in the routing problem is sample average approximation (SAA), which is an approach for solving stochastic optimization problems by using Monte Carlo simulation (Verweij et al. 2003). Specifically, in the drone delivery problem, there are stochastic factors in both drone performance, such as speed, capacity, flight range, and the customer demand. Juan et al. (2019) provided a case study on a stochastic team orienteering problem. Considering random traveling time and reward, they compare SAA with simheuristics. A conclusion is made that SAA can efficiently solve small-scale instances while simheuristics performs better in large-scale instances. Besides, Panadero et al. (2023) hybridized biased-randomized heuristics with a variable neighborhood search and Monte Carlo simulation to solve the stochastic team orienteering problem which performs better within a shorter time than SAA.

According to the previous literature, drone-aided delivery and its variant problems are widely studied (Macrina et al. 2020). Additionally, considering uncertain factors in the real world, some papers provide more robust drone routing plans (Chauhan et al. 2021; Cheng et al. 2021). However, few papers focus on how to design a drone delivery system with drone features, especially those uncertain factors, which will have a huge effect on the performance of the delivery system. In this paper, we are going to build a two-stage stochastic mathematical model to optimize the drone delivery system design and use a simulation-heuristic solution approach to provide a satisfactory solution for the drone delivery system design in a short time with consideration of the expected cost in the unpredictable delivery process.

This paper is organized as follows: Section 1 presents an introduction to drone delivery systems and a literature review of various drone delivery problems, including the features of different types of drones, facility location problems, and vehicle routing problems. In Section 2, the problem is described, and several assumptions are illustrated. In Section 3, a two-stage stochastic mathematical model is developed, and its notations and constraints are explained. Section 4 presents a simulation-heuristic solution approach. Section 5 provides a numeric experiment and sensitivity analysis. Finally, a summary of the findings is presented in Section 6.

2 PROBLEM DESCRIPTION

In order to establish a drone delivery system in a specific location, it is important to take into account the coverage area for each potential customer as well as the associated costs. A common application for this type of system is for delivery services to local supermarkets, such as Walmart and Publix, which typically have multiple locations in a given area and offer similar products (Walmart 2021). When a customer places an order, the delivery system will assign a particular supplier based on factors such as proximity and availability, even though all local suppliers are capable of fulfilling the order.

However, the integration of drones into the existing delivery system may require significant changes to the supply chain and logistics operations, including the need for new technology and personnel training, which brings extra costs (Sudbury and Hutchinson 2016). Another reason to consider when implementing drone delivery into the existing supermarket delivery system is that it may not be necessary to establish a drone depot at every single supermarket location (Shavarani et al. 2019). Additionally, varied ordering frequencies among customers can create challenges in predicting demand and organizing the daily delivery route of drones. This uncertainty can impact the decision-making process when selecting locations for drone depots, as some locations may be more suitable for accommodating demand fluctuations. By strategically selecting drone depot locations and optimally routing for delivery, the drone delivery system could be more cost-effective and efficient, while still ensuring that all customers in the area are adequately served.

Our objective is to create a system that can efficiently serve customers with uncertain and real-time demands while also considering the limitations of drones. As previously mentioned, we have identified potential drone depot locations, but we must also consider the distribution of potential customers and their likelihood of making an order. While we cannot make precise predictions for all customers, it is possible to obtain the locations of potential customers and estimate their probability of placing an order, which are stochastic parameters in the model.

Figure 1 displays a map illustrating four drone depot candidates. However, only two of them will be selected as the drone depots while the drone depots with a red cross inside are not chosen. As the figure shows, the two selected drone depots are able to cover all the potential customers represented by the green hollow circles. The red points denote customers with varying demands, which may appear differently sized in the figure. While this selection approach can ensure the coverage of all potential customers, it may not be the optimal design for the drone delivery system. An alternative solution is to replace the left drone depot with the middle one, which has a shorter distance to its customers and therefore, results in lower operating costs in daily delivery tasks. In summary, it is important to consider both fixed and operating costs in designing such a drone delivery system.

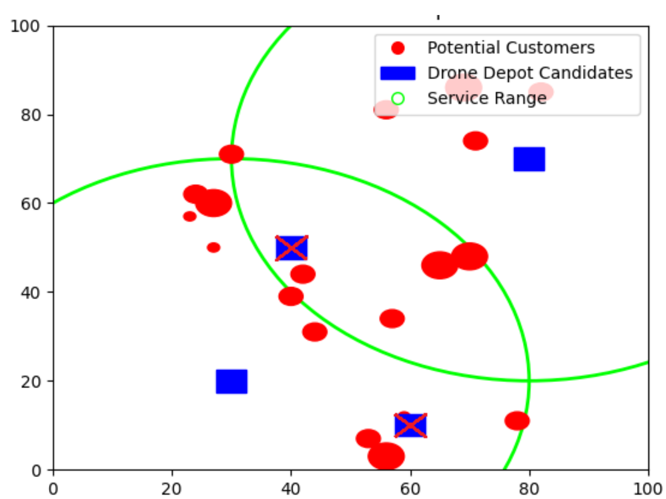

Figure 1: Selection of drone depots.

To achieve this goal, we propose a two-stage model. The objective function is to minimize the total cost of the delivery system, which includes both fixed costs of facilities and operating costs. The fixed cost contains the cost for upgrading or building a drone depot, including equipment setting and personnel training. The operation cost contains the estimated delivery cost and estimated penalty cost for the unserved orders in daily delivery. Both types of operating costs will have an effect on the total cost by a given multiplier. In the first stage, we will choose the optimal locations to serve as drone depots. Then, in the second stage, we will use these depots to allocate drones to fulfill real-time orders while minimizing operating costs, including travel costs for the drones and penalty costs for unserved orders.

Here are several key assumptions for the mathematical model:

- Each potential customer can be covered by at least one drone depot candidate.
- The heaviest order does not exceed the capacity limitation of drones.
- The number of drones in each depot is given.
- Each drone has to return to its original depot.
- The time for customers receiving packages and drones changing batteries is ignored.
- In a certain scenario, the order's weight, location, and time window, are known before delivery.
- The outside conditions, like weather and temperature, are assumed to not affect drone operations.

3 MATHEMATICAL MODEL

Many literature reviews on drone delivery systems have overlooked the uncertain nature of customer demand, leading to less efficient designs in reality application (Cheng et al. 2021). To address this gap, we propose a two-stage mathematical model that considers real-time demand from customers when determining suitable locations for drone depots. We also incorporate constraints such as limited capacity and flight range, which are often overlooked in similar studies. Furthermore, we recognize that customers may have specific time preferences for package delivery, such as Walmart's delivery service. To study the impact of real-time order fluctuations on delivery decisions, we build a time-window drone delivery model. This model should be expressed as a two-stage stochastic model. The notations used in the model are shown in Table 1.

The objective function aims to minimize the total cost of this drone delivery system, which includes the fixed cost of building the drone depots and the estimated operating cost of daily delivery:

$$\text{minimize} \quad \sum_{l \in O} C_l y_l + E_\varepsilon[F(y,\varepsilon)] \tag{1}$$

Subject to:

$$\sum_{l \in O} z_{il} \geq 1, \forall i \in N \tag{2}$$

$$z_{il} \leq y_l, \forall i \in N, l \in O \tag{3}$$

$$z_{il} d_{il} \leq R/2, \forall i \in N, l \in O \tag{4}$$

In the second stage, certain locations have been selected as drone depots, and customers may have stochastic orders with weights and time windows, which brings multiple different cases. To simplify the representation, we use notations i and j for all the selected drone depots and existing customer locations in a particular scenario. It is important to note that these notations may refer to only a subset of these locations which is denoted by the constraints. The set of all selected drone depots is denoted by D, which is predetermined in each certain scenario ε. The set of all locations in scenario ε, including customers and drone depots, is represented by G^ε. M is a very big constant number. The objective function for this stage aims to minimize the total cost of the drone delivery system and is represented as follows:

$$F(y,\varepsilon) = \text{minimize} \sum_{i \in N} \sum_{j \in N} \sum_{k \in K} \sum_{l \in D^\varepsilon} \sum_{m \in Q} x^\varepsilon_{ijklm} d_{ij} \alpha + \sum_{i \in N} \sum_{k \in K} \sum_{l \in D^\varepsilon} (1 - p^\varepsilon_{ikl}) \beta \tag{5}$$

Table 1: Notations in the model.

Index	Definition	Type
N	The set of all potential locations for customers	
O	The set of all potential locations for the drone depots	
D^ε	The set of all selected drone depots in scenario ε	
G^ε	The set of all locations in scenario ε	
K	The maximal number of drones in each drone depot	
Q	The maximal number of trips for each drone in a scenario	
Variables		
x^ε_{ijklm}	If the drone k in the depot l will visit location j once after visiting location i in its mth trip in the scenario ε, x^ε_{ijklm}=1; Otherwise, x^ε_{ijklm}=0	Binary
y_l	If location l is chosen as a drone depot, y_l=1; Otherwise, y_l=0	Binary
z_{il}	If customer i is covered by drone depot l, z_{il}=1; Otherwise, z_{il}=0	Binary
w^ε_{ikl}	The weight of total load of drone k in depot l before it visiting location i in the scenario ε	Continuous
u^ε_{ikl}	The left flight range of drone k in depot l after it visiting location i in the scenario ε	Continuous
t^ε_{iklm}	The time for drone k in depot l to arrive the location i in its mth trip in the scenario ε	Continuous
p^ε_{ikl}	If location i is visited on time by the drone k in depot l in the scenario ε, p^ε_{ikl}=1; Otherwise, p^ε_{ikl}=0	Binary
Parameters		
C_l	The fixed cost for drone depot candidate l	
c^ε_i	The weight of demand in location i in the scenario ε	
a^ε_i	The earliest time for location i to receive the package in the scenario ε	
b^ε_i	The latest time for location i to receive the package in the scenario ε	
d_{ij}	The distance between location i and j	
V	The speed for each drone	
U	The capacity for each drone	
R	The maximal flight range for each drone	
α	The cost for each drone to travel per unit distance	
β	The penalty cost for each order not on time	

Subject to:

$$M\sum_{j\in G^\varepsilon}\sum_{k\in K}\sum_{l\in D^\varepsilon}\sum_{m\in Q}x^\varepsilon_{ijklm}\geq c^\varepsilon_i,\forall i\in G^\varepsilon-D^\varepsilon \tag{6}$$

$$\sum_{i_1\in G^\varepsilon}x^\varepsilon_{i_1jklm}=\sum_{i_2\in G^\varepsilon}x^\varepsilon_{ji_2klm},\forall j\in G^\varepsilon,k\in K,l\in D^\varepsilon,m\in Q \tag{7}$$

$$x^\varepsilon_{ijklm}=0,\forall i\in D^\varepsilon,j\in D^\varepsilon,k\in K,l\in D^\varepsilon,m\in Q,i!=j \tag{8}$$

$$x^\varepsilon_{ijklm}=0,\forall i\in G^\varepsilon,j\in G^\varepsilon,k\in K,l\in D^\varepsilon,m\in Q,i!=l \tag{9}$$

$$x^\varepsilon_{ijklm}=0,\forall i\in G^\varepsilon,j\in G^\varepsilon,k\in K,l\in D^\varepsilon,m\in Q,j!=l \tag{10}$$

$$w^\varepsilon_{ikl}=0,\forall i\in D^\varepsilon,k\in K,l\in D^\varepsilon \tag{11}$$

$$w^\varepsilon_{ikl}\leq U,\forall i\in G^\varepsilon,k\in K,l\in D^\varepsilon \tag{12}$$

$$w^\varepsilon_{ikl}-w^\varepsilon_{jkl}\geq(\sum_{m\in Q}x^\varepsilon_{ijklm}-1)U+c^\varepsilon_i,\forall i\in G^\varepsilon-D^\varepsilon,j\in G^\varepsilon,k\in K,l\in D^\varepsilon \tag{13}$$

$$u^\varepsilon_{ikl}=0,\forall i\in D^\varepsilon,k\in K,l\in D^\varepsilon \tag{14}$$

$$u^\varepsilon_{ikl}\leq R,\forall i\in G^\varepsilon,k\in K,l\in D^\varepsilon \tag{15}$$

$$u^\varepsilon_{ikl}-u^\varepsilon_{jkl}\geq(\sum_{m\in Q}x^\varepsilon_{ijklm}-1)R+d_{ij},\forall i\in G^\varepsilon-D^\varepsilon,j\in G^\varepsilon,k\in K,l\in D^\varepsilon \tag{16}$$

$$a_i\leq\sum_{k\in K}\sum_{l\in D^\varepsilon}t^\varepsilon_{ikl}+M(1-p^\varepsilon_{ikl}),\forall i\in G^\varepsilon-D^\varepsilon \tag{17}$$

$$b_i \geq \sum_{k\in K}\sum_{l\in D^{\varepsilon}} t^{\varepsilon}_{ikl} - M(1-p^{\varepsilon}_{ikl}), \forall i \in G^{\varepsilon} - D^{\varepsilon} \tag{18}$$

$$t^{\varepsilon}_{iklm} + d_{ij}/v - M(1-x^{\varepsilon}_{ijklm}) \leq t^{\varepsilon}_{jklm}, \forall i \in G^{\varepsilon}, j \in G^{\varepsilon}, k \in K, l \in D^{\varepsilon}, m \in Q \tag{19}$$

$$t^{\varepsilon}_{jklm} \leq M * \sum_{i\in G^{\varepsilon}} x^{\varepsilon}_{ijklm}, \forall j \in G^{\varepsilon}, k \in K, l \in D^{\varepsilon}, m \in Q \tag{20}$$

$$t^{\varepsilon}_{jklm} \geq l * \sum_{i\in G^{\varepsilon}} x^{\varepsilon}_{jiklm}, \forall j \in G^{\varepsilon}, k \in K, l \in D^{\varepsilon}, m \in Q \tag{21}$$

$$t^{\varepsilon}_{iklm_1} \leq t^{\varepsilon}_{jklm_2} + T * (1 - \sum_{i\in G^{\varepsilon}} x^{\varepsilon}_{ijklm_2}), \forall i \in G^{\varepsilon}, j \in G^{\varepsilon}, k \in K, l \in D^{\varepsilon}, m_1 \in Q, m_2 \in Q \tag{22}$$

The first stage of the model represents a facility location problem, with the main objective minimizing the overall cost of the drone delivery system. This cost is composed of a fixed cost and an estimated operating cost in the second stage. The objective function 1, aims to minimize this total cost.

To ensure that each potential customer is covered by at least one drone depot, we impose the constraint given by Equation 2. Equation 3 ensures that if customer i is covered by a specific drone depot j, then there must be a drone depot located at j. Finally, to guarantee that the distance between a customer i and its corresponding drone depot j does not exceed the maximum service range, which is half of the drone's flight range considering the returning trip, we require the inequalities expressed in Equation 4.

In the second stage, the model assumes that certain locations have been chosen as drone depots, denoted by the set N, while customers have stochastic orders with non-determined weights and time windows. For simplification, we use the variables i and j to refer to all considered locations, including chosen drone depots and customers with orders in a particular scenario. The objective function in 5 aims to minimize both the transportation cost and penalty cost in the given scenario ε.

This model extends the classical vehicle routing problem by incorporating multiple drone depots, limited flight range, and capacity, as well as time window constraints and associated penalty costs for late deliveries. To ensure that all customers receive their orders, the Equation 7 confirms the trip of every drone is round. Constraints 8 – 10 are used to force every drone to return to its original depot rather than change batteries in other depots. Due to the limitation on the drone's capacity, Constraints 11 – 13 ensure that the load never exceeds the capacity during delivery and eliminate subtours at the same time. Similarly, Constraints 14 – 16 express the limitation of the flight range of drones. Considering the time window of each customer, we use a pair of inequalities, Equations 17 and 18, to impose a penalty cost for each package delivered late. Constraint 19 deals with the traveling time of each drone, which includes travel distance and speed. For Constraint 20, it forces the time for drone k to arrive at location j on its mth trip to be 0 if there is no such trip. Constraint 21 ensures that each trip should not start before the time episode, and the last Constraint 22 limits that the later trip will start no earlier than the previous trips, ensuring that the drones work in the correct sequence in different time episodes.

The whole model contains a facility location problem and a vehicle routing problems, which both are NP-hard. Additionally, there are an infinite number of scenarios based on the customers' demands in the second stage, which makes it impossible to obtain the precise expected operating cost using an exact algorithm. Due to the combination of these difficulties, finding an exact solution to this model is not possible. Nonetheless, we employ the sample average approximation method to obtain a good confidence interval for the solution of this problem on a small scale. For large-scale problems, we propose a method that combines heuristic algorithms with simulation to quickly obtain a satisfactory solution.

4 SOLUTION METHOD

4.1 Sample Average Approximation

SAA is a popular approach for solving stochastic programming problems that generates random samples of uncertain parameters and then approximates the expected value of the objective function using these samples.

This method can help to estimate the optimal solution in cases where the true probability distribution of the uncertain parameters is unknown or difficult to model (Birge and Louveaux 1997).

In this problem, since it aims to find the minimal objective function, we start with S_1 different scenarios in the second stage to estimate the upper bound of the original problem. We generate S_1 scenarios repeatedly and calculate the objective value M times, resulting in M different solutions for the selection of drone depot locations, denoted by $F_i, i = 1,2,...,M$, and M different estimated total costs of the drone delivery system, denoted by $R_i, i = 1,2,...,M$. The estimated upper bound is then obtained as the average of the M solutions, which is $\frac{\sum_{i=1}^{M} R_i}{M}$.

Next, the estimated lower bound is calculated based on the solution F_i from the first stage. We generate S_2 different scenarios of customers' demand and calculate the total cost with a given F_i. This generates M different solutions, denoted by r_i, corresponding to S_2 different scenarios. The estimated lower bound is then obtained as the average of all r_i (Santoso et al. 2005).

As this method is grounded in statistical theory, we can obtain a certain confidence interval for both the estimated lower bound and upper bound by repeating this process multiple times and recording their average and variance.

Algorithm 1 shows how SAA works in this problem. The constant k represents the 95 % percentile of the Student's t-distribution with $M-1$ degrees of freedom.

Algorithm 1 Sample average approximation in the two-stage model.

Initialize M, S_1, S_2
for $i \leftarrow 1$ to M **do**
    Randomly generate S_1 scenarios in the second stage
    Solve the two-stage model as a determined model
    Record the objective value R_i and solution of the first stage F_i
end for
The estimated upper bound is $U = \frac{\sum_{i=1}^{M} R_i}{M}$
The 95 % confidence interval is $[U - \frac{k\sigma_R}{\sqrt{M}}, U + \frac{k\sigma_R}{\sqrt{M}}]$
for $i \leftarrow 1$ to M **do**
    Randomly generate S_2 scenarios in the second stage
    Solve the S_2 different second stage models with F_i
    Add the average operating cost with the fixed cost of F_i
    Record the total r_i
end for
The estimated lower bound is $L = \frac{\sum_{i=1}^{M} r_i}{M}$
The 95 % confidence interval is $[L - \frac{k\sigma_r}{\sqrt{M}}, L + \frac{k\sigma_r}{\sqrt{M}}]$

4.2 Heuristic Simulation

Heuristic simulation is a problem-solving approach that uses a simulation model to test different scenarios. It involves creating a simplified model of a complex system or process and then using that model to test different hypothetical scenarios or decisions in order to determine the likely outcomes.

In this problem, we have infinite scenarios in the second stage model, which can be seen as a drone routing problem with uncertain customer demands. So we can get an estimated operating cost if given a set of certain drone depots. Besides, since all the scenarios have the same logic and parameters, excluding the initial settings of selected drone depots and the stochastic customer orders, we can create an environment with drone depots and customers as its input and simulate how it works in a time period to have an estimated operating cost.

Here we use a Genetic Algorithm (GA) to optimize the locations of the drone delivery system with minimum total cost. Each feasible solution generated by GA, which is called "chromosome", is encoded as a set of binary variables to represent the selections of all the drone depot candidates. And this solution will be used as input data in the simulation model, and we can get a determined fixed cost from the solution. Due to the uncertainty of customers, we randomly generate N scenarios, in which there are different customers making orders, and customers have different demands and time-windows. Every simulation scenario will treat the customers' information as determined data, and allocate the drones based on certain depots. When all simulation scenarios are done, the average operating cost in all scenarios is the estimated operating cost of this set of drone depots. Then we can get the estimated total cost of this solution by adding fixed cost and operating cost, which is the "fitness value" of this "chromosome" in the GA framework. Since usually the larger "fitness value" is better, here we set the opposite value of the estimated total cost as is the "fitness value".

The GA will have multiple "generations". Each "generation" has a "population" which contains multiple "chromosomes", which are also called "individuals" in the "population". Based on the "fitness value" of each "chromosome" in the "population", we will do "selection" in the "population" and let them "crossover" and "mutate" to generate a new "population" in the next "generation". This step will be repeated until reaching the maximal generation, and the output is the "chromosome" with the largest "fitness value" in the last "generation". The framework with input and output of this heuristic simulation method is shown in Figure 2.

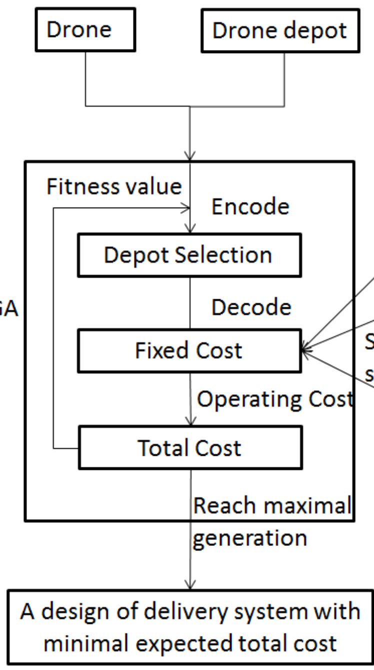

Figure 2: The framework of this heuristic simulation.

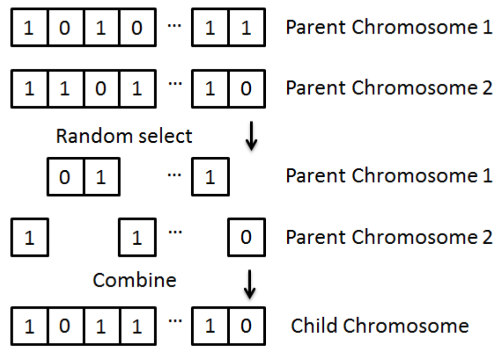

Figure 3: Crossover in this GA.

In this particular problem, each "chromosome" is a set of binary variables. To perform "crossover", we randomly select a crossover point in the chromosomes of the parents and exchange the genetic material beyond that point, resulting in two new offspring solutions that are a combination of genetic material from the parents. This process is illustrated in Figure 3. Since "chromosomes" have varying performances in this environment, as indicated by their "fitness values", the probability of a "chromosome" i being chosen to have offspring is $\frac{f_i}{\sum_i f_i}$.

However, since this search method depends on the first "generation", it has a limited region in the feasible solution. To extend the search area of this GA, we also include a "mutation" step in the process. In this problem, the "mutation" step involves randomly exchanging some values in the "chromosome".

Each "chromosome" can be decoded as a selection of drone depots. So we can randomly generate different customers to simulate how the selected drone depots work in delivery. When doing simulation, we use a greedy method to calculate the estimated operating cost. The following is a description of the drone delivery system process based on given customer and facility locations:

- Customer Allocation: Each customer is allocated to its nearest drone depot based on distance.
- Sorting: The drone depot sorts all customers based on their respective time windows.
- Drone Allocation: Drones are allocated to customers in sequence until the current drone is fully loaded or reaches the maximum limit of the flight distance.
- Operating Cost: The drones carry out their allocated tasks and update the traveling cost in delivery and penalty cost for the orders not on time.

It is important to note that the penalty cost is incurred due to the operational constraints of the drone delivery system. These constraints may include factors such as drone capacity, flight range, and time windows. Algorithm 2 shows how the heuristic simulation works.

Algorithm 2 Pseudo code for the heuristic simulation method.

```
Set maximum iteration M and size of population S
Initialize the crossover rate p and mutation rate q
Set the number of scenarios N for each chromosome
Generate S random chromosomes as the first population P_1
while Generation g ≤ M do
   for Chromosome i ∈ P_g do
      Decode the chromosome into the selection of drone depots D_i
      Calculate the fixed cost of i
      for Simulation scenario n ∈ N do
         Randomly generate a set of customers c_n
         Allocate every customer to its nearest drone depot
         for drone depot j ∈ D_i do
            while There are customers to be served do
               Update the drones' availability
               if The first available drone can serve the next customer then
                  Allocate the next customer to the first available drone
               else
                  The first available drone becomes unavailable and prepare to start delivery
                  Record the drone's operating cost and update the total operating cost
               end if
            end while
         end for
         Calculate the estimated operating cost by the average
      end for
      Compute the fitness value
   end for
   Generate the next population P_{g+1} by crossover and mutation
end while
Output the chromosome with the largest fitness value in the last generation
```

5 NUMERICAL EXPERIMENTS

To validate the heuristic simulation method, we conducted five scenarios with different problem scales. These scenarios are designed with the same drone settings. Specifically, all the drones used in this section have a maximum carrying capacity of 10 kg and can travel up to 15 km at a speed of 50 km/h in a single trip. The fixed cost for each drone depot is set to 1,000, and each depot can store a maximum of 3 drones, with the cost of each drone included in the fixed cost of the depot. Additionally, we assume that the average transportation cost for a drone to travel 1 km is 1, and there will be a penalty cost of 100 for an undelivered order that exceeds its time window.

The five scenarios generated have differences only in the scale of the problem, with varying numbers of drone depot candidates and potential customers, which is: 5 and 10; 10 and 50; 20 and 100; 50 and 500; and 100 and 1,000, respectively.

In the SAA method, we use the same algorithm provided by (Santoso et al. 2005). Since it is a minimizing problem, the feasible solutions obtained in the first phase can be treated as an estimated upper bound. While the lower bound is obtained in the second phase which is the expected value based on the solutions from the first phase with a larger sample. We set the number of repetitions for each customer demand data to be $M = 10$. For the first phase of the method, we set the number of scenarios considered for the model to be $S_1 = 10$, and for the second phase, we set $S_2 = 50$. These parameters represent the number of scenarios considered for the model in each phase. However, due to the limited time available, it may not be possible to obtain the optimal solution for those models. Therefore, we choose the best solution obtained within 3,600 seconds as our result.

In the heuristic simulation method, we use GA and set the maximum number of iterations to be $M = 100$, and the size of the population to be $S = 100$. We also set the crossover rate to 0.9 and the mutation rate to 0.1.

The result is shown in Table 2. We use D to represent the number of potential drone depots, C to represent the number of potential customers. LB and UB mean the estimated mean of the lower bound and upper bound obtained by SSA.

Table 2: Numerical results in 5 scenarios.

D	C	SAA				Heuristic simulation
		LB	95 % CI of lower bound	UB	95 % CI of upper bound	Obj.
5	20	2,008.21	[2,007.91, 2,008.52]	2,078.41	[2,073.10, 2,083.72]	2,049.53
10	50	2,091.25	[2,077.87, 2,104.64]	2,102.15	[2,044.60, 2,159.70]	2,069.33
20	100	-	-	-	-	2,078.07
50	500	-	-	-	-	7,082.06
100	1,000	-	-	-	-	20,087.37

Table 2 reveals that we can obtain a relatively accurate lower bound for the two smallest scenarios. For instance, when there are five drone depot candidates and 20 potential customers, the estimated lower bound for the total cost of the drone delivery system is [2,008.21, 2,078.41]. The heuristic simulation method provides a solution of 2049.53, which lies between the estimated bounds of SAA. Although the solution of the heuristic simulation method in the second scenario is not in the estimated bound of SAA [2,091.25, 2,102.15], it still falls within the 95 % confidence interval of the estimated upper bound [2,044.60, 2,159.70]. In this case, our problem scale is much larger than the first experiment, which means it could obtain very different estimated bounds based on the random selection of scenarios. Compared with the estimated bounds of the first experiment, the intervals of both the estimated lower bound and upper bound are much larger. Besides, the estimated upper bound interval has covered the estimated lower bound, which makes the estimated bound not very precise. However, since the solution of the heuristic simulation is very close to the SAA interval, and it is in the 95 % confidence interval of the upper bound, we can say that it is still a satisfactory solution.

Based on the obtained results, the first three experiments get similar results in the heuristic algorithm. This can be attributed to the relatively small number of customers involved. However, despite their proximity, these customers may be dispersed over considerable distances, allowing for the possibility of at least two depots being capable of serving them adequately, albeit with two depots proving sufficient. In contrast, the last two experiments entail a significantly larger customer base of 500 and 1,000 individuals, respectively. Given the substantial increase in the area that needs to be covered, it is reasonable to expect a higher number of depots to fulfill the demand, resulting in notable differences in the outcomes when compared to the first three experiments.

However, due to the complexity of this model, SAA cannot find a solution when the number of potential customers exceeds 100 or when there are more than 20 drone depot candidates. In such cases, the heuristic simulation can still deliver a reasonable and satisfactory solution within a limited time. In reality, most drone delivery systems with multiple drone depots serve far more than 100 customers, making it impossible for SAA to provide accurate solutions and estimated bounds. Thus, it is more practical to use the heuristic simulation method to aid in decision-making when constructing a drone delivery system.

6 CONCLUSIONS

In this study, we aim to develop a delivery system solely based on drones. To account for the uncertainty in customer demands, we propose a two-stage stochastic mathematical model that minimizes the fixed and operating costs of the drone delivery system. Due to the problem's complexity, we present two approaches to solving it. The first method utilizes a sample average approximation technique to address the infinite-scenario two-stage problem. The second method combines a heuristic algorithm with simulation, where the genetic algorithm generates a solution that is used as input for the simulation. The fitness value of the solution is obtained through multiple simulations and then fed back to GA. After iterations and evolution in GA, we will get a satisfactory solution.

To validate the effectiveness of our proposed methods, we conducted numerical experiments on problems of varying scales. Specifically, we found that the SAA method can get a relatively good bound when the problem scale is small, such as when there are fewer than 50 potential customers and 10 drone depot candidates. However, in real-world scenarios, there are often hundreds or even thousands of potential customers in a given area. The results show that SAA will not be able to even a feasible solution for the models in the first phase when the potential customers are more than 100. In such cases, the heuristic simulation method is able to provide a solution within a reasonable amount of time.

In future research, an exact solution can be attained by dividing the infinite customer demand scenarios into limited cases. By leveraging historical data, machine learning algorithms can provide a more precise prediction of customers' demands, facilitating better decision-making in designing the drone delivery system and scheduling scheme. Additionally, since this problem is a mixed-integer two-stage problem, it may be possible to find more efficient methods to obtain the optimal solution based on the problem's unique structure.

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