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SIMULATION MODEL TO FORECAST GENDER PENSION WEALTH GAP IN THE LIGHT OF DEMOGRAPHIC CHANGES

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ABSTRACT

The ageing of the population has forced changes in many areas of social policy, including pension systems. Countries are reforming their retirement policies in such a way that the size of pension benefits depends on the total period of employment, contributions made, and life expectancy. Due to the fact that in these types of system, employment plays a significant role in the accumulation of pension capital, a gender pay gap translates into a gender pension gap. In this article, we propose a hybrid simulation model to analyze the impact of long-term economic and demographic changes on the level of pension benefits when a worker retires, with a special focus on gender wealth pension gaps. The model combines demographic simulation conducted using a systems dynamics approach with discrete stochastic simulation by means of which we model the employment history of men and women. The model uses data from Polish statistical databases.

1 INTRODUCTION

The progressive aging of the population is a major challenge for most countries. Although the Covid 19 pandemic disrupted this long-standing trend, causing a decrease in life expectancy, the basic trend of an increase in the number of old age groups (older than 65) relative to the number of people of working age (20–64) has not changed. The structure of the population (OECD 2021) is moving in the direction of an increasingly intense share in the proportion of aged people and a more pronounced decline in the size of working age groups (20–64). According to (OECD 2021), the number of people older than 65 per 100 working age people has increased from 20 in 1980 to 31 in 2020, and is expected to reach 53 in the next 30 years.

An aging population poses a significant challenge to many areas of public finance, but especially to pension systems. Therefore, for several years, governments in many countries around the world have been reforming pension systems to maintain their sustainability and at the same time make efforts to provide sufficient resources to ensure pensioners a secure life. The pension system that prevailed in the past was the defined-benefit (DB) system, in which the amount of pension depended on the number of years worked. This system worked well as long as both the population and the economy grew. During the last decade, we have witnessed a paradigm shift from DB to defined contribution (DC) systems. In DC schemes, the amount of pension benefit is closely related to the amount of earnings during the entire period of employment and life expectancy. Among European countries, the DC system has been introduced in ten countries, while six others have used various forms of automatic adjustment mechanism that automatically changes pension parameters according to specified indicators, such as life expectancy at a given age, wage, or funding balance. The Polish pension system was also radically changed in 1999 when the DB system was replaced by the DC system.

In DC systems, an increase in life expectancy automatically leads to a decrease in pension benefits. The timing of retirement is also important, as the longer a person accumulates pension contributions, the higher

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the accumulated capital, and the shorter the expected duration of pension payments. In many DC systems (including Poland's), the impact of demographics on subsequent retirement benefits is even more intense. This is because the pension capital accumulated during employment is indexed annually and the amount of indexation depends on the level of employment in the country and the average wages in the economy. Thus, the number of people of working age significantly affects the intensity of the accumulation of pension capital and the amount of future pensions. The more working-age people pay pension contributions and the higher wages they earn, the higher the annual indexation of accumulated capital. The shift in the size of the age groups toward the oldest groups further exacerbates the adverse impact of aging on pension benefits and adds another disadvantageous factor, besides extended longevity and a lowered retirement age.

The unfavorable demographic changes discussed above will be particularly painfully felt by women. Even a cursory analysis of the factors that determine the degree of retirement security clearly shows that the system favors men. Differences in earnings between men and women translate into differences in pension payments. In DC pension schemes, the main factor influencing the gender pension gap is participation in the labor market and earning levels (Cordova et al. 2022). The gender pay gap is caused by a number of factors, such as vertical and horizontal segregation between men and women, a motherhood penalty in wages (Cukrowska-Torzewska and Matysiak 2020), availability of childcare facilities, and cultural perceptions of roles in the family (Blau and Kahn 2017). All these factors work to the disadvantage of women by affecting their earnings and thus their accumulated retirement capital.

The primary goal of the study is to propose a framework, using a hybrid approach based on dynamic stochastic simulation, to forecast the financial implications of aging problems on the level of social security in DC pension systems, with a particular focus on differences in pension wealth between men and women. The article is a continuation of the research (Mielczarek 2022). The remainder of this paper is organized as follows. Section 2 provides background information on pension systems and methodological approaches used in the analysis of retirement systems. Section 3 briefly describes the Polish pension system. Section 4 outlines the general concept of a hybrid simulation model to forecast future pensions. Section 5 focusses on the simulation experiments and results of this study. Finally, Section 6 draws conclusions and summarizes the contributions of this research.

2 LITERATURE BACKGROUND

Jimeno et al. (2008) discussed basic methodological approaches to analyze the impact of demographic change on the sustainability of pension systems. These are aggregate accounting, general equilibrium models, and individual life-cycle profile models. Individual life-cycle profile models use simulation methods to trace the history of pension capital accumulation for many different individuals. Two simulation approaches are most commonly used here, namely Monte Carlo (MC) and dynamic microsimulation (MSM). Microsimulation works at the level of individual units. It simulates a virtual population by accessing detailed demographic data and pension payment records, and applying mathematical formulae to model individual behavior (van Sonsbeek and Alblas 2012; Halvorsen and Pedersen 2019). In contrast, MC simulation is based on one or more typical individuals and applies the resulting projections to a larger group of individuals (McFarland and Warshawsky 2010; Mielczarek 2013). Several input assumptions are made about the typical representative of the target occupational group and the conclusions can only be considered reliable for the defined group.

Population projection is essential when quantifying the impact of demographic effects on pension systems. Most studies assume that population ageing can be correctly captured by analyzing changes in population structure without a detailed analysis of the mechanisms driving the demographic process. However, population ageing is influenced not only by life expectancy, but also by fertility and migration, and the interrelationships between these parameters manifest themselves as interlocking, balancing, and reinforcing feedback loops. Moreover, in the case of pension systems, not only the size of the population of retirement age but also the size of the age-working group is of crucial importance throughout the professional life of the individual. Population projections may be performed within different simulation paradigms. Demographic phenomena have been successfully modeled using system dynamics (SD) (Eberlein and Thompson 2013), discrete event simulation (DES) (Olsson and Hössjer 2015), agent-based simulation (ABS) (Singh and Ahn 2017), and the Monte Carlo method (MC) (Tian and Zhao 2016), and microsimulation (MSM) (Davis et al. 2010).

Of the simulation methods, the SD method appears to be the most useful in modeling demographic change (Mielczarek and Zabawa 2018). The basic idea is to divide the population according to gender, and population ageing is modelled using input and output flows such as births and deaths. The precise modelling of the ageing population was comprehensively discussed by Eberlein and Thompson (2013). To accurately track the ageing population and to use data of varying levels of detail, Mielczarek and Zabawa (2020) developed a 'hierarchical cohorting' approach that introduces the concept of main and elementary cohorts.

3 CASE STUDY: POLISH DC PENSION SYSTEM

The DC pension system is based on the general concept, which assumes that savings accumulated throughout the entire period of paid work are recalculated at the time of retirement, taking into account the average life expectancy (Figure 1). The detailed solutions differ from country to country. The Polish pension system is based on three pillars. Two of them are mandatory. The first pillar is in the form of a notional (non-financial) account. The term 'notional' means that contributions (12.22% of an employee's salary) are built into individual accounts, and at retirement this capital is converted into a monthly pension using an algorithm based on life expectancy. Therefore, this capital can be treated as claims against the government (Kurach et al. 2019). The second pillar, to which 7.3% of salary is paid, is managed by private open pension funds or by a state company (Social Insurance Company, ZUS). The model presented here reproduces the first and second pillars (but only the one administered by the ZUS). Retirement age varies from country to country. In Poland, it is 60 for women and 65 for men.

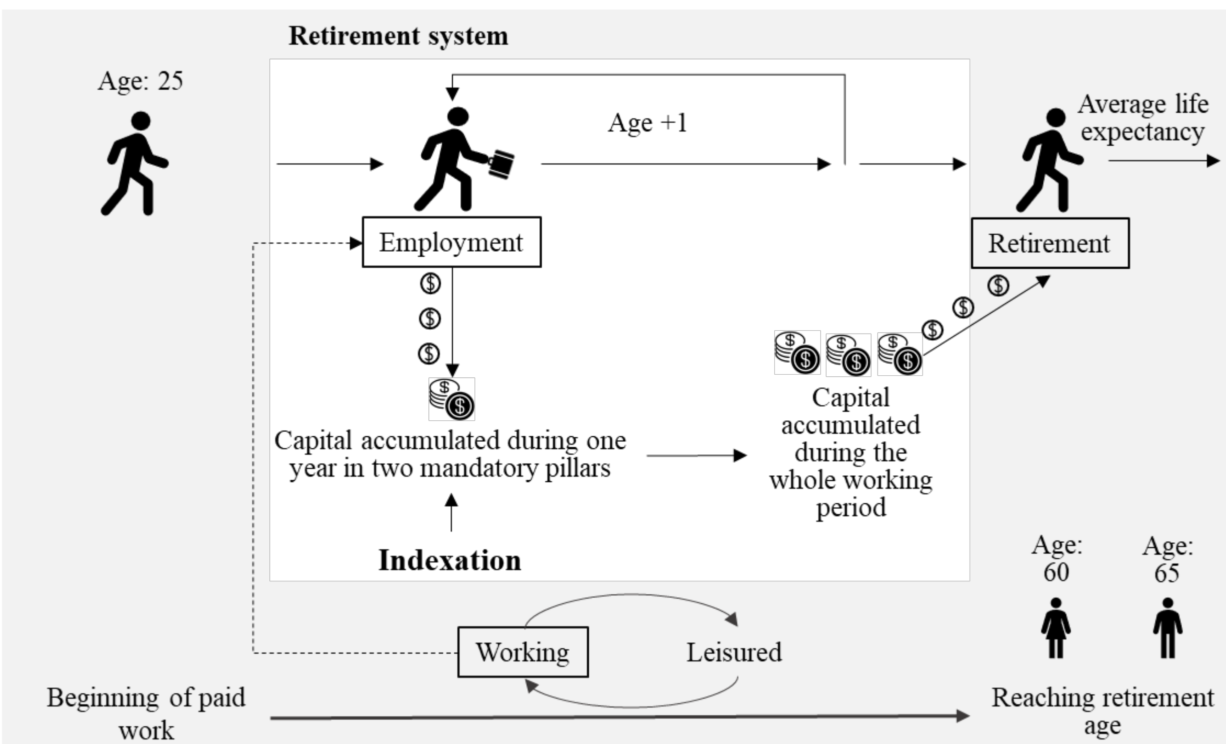

Figure 1: General process diagram for the DC pension system.

Indexation in both pillars follows the same algorithm, which relies on the annual multiplication of the total accumulated capital by the indexation factor. However, the macroeconomic indicators on the basis of which the indexation indices for the two pillars are calculated, are different. The indexation in the first pillar depends on inflation and the increase in pension premiums written in the previous year. The indexation cannot be negative. The indexation of the second pillar depends on the average nominal GDP growth in the last five years preceding the valorization date and cannot be negative.

The size of the annually collected contributions is limited by law and cannot exceed thirty times the projected average monthly salary in the national economy.

The state pension is calculated by dividing the capital accumulated in both pillars (*Pillar*1 and *Pillar*2) by the average life expectancy determined for persons of the same age as a person retiring (see Eq. 1).

$$Pension_k = \frac{Pillar1 + Pillar2}{Life\ expectancy_k} \tag{1}$$

where k is the retirement age.

Life expectancy is defined as the average number of months that people of a particular age could expect to live. It is determined jointly for men and women and is announced every year by the President of the Central Statistical Office. By 2045, life expectancy is expected to increase by approximately 12 months (OECD 2021).

4 METHODOLOGY

4.1 Overall Concept of Hybrid Simulation Model

The hybrid simulation model consists of two submodels (Figure 2). The Population submodel, based on the aging chain approach and system dynamics, captures the overall evolution of the population. This model was developed during earlier research (Mielczarek and Zabawa 2020) and has been used successfully to determine the impact of demographic changes on demand for inpatient hospital services (Mielczarek 2019). The Pension submodel is developed using discrete event simulation. This model traces individual life-cycle profiles for different professional groups. The model will also allow individual decisions to be taken into account relating to, for example, maternity leave (women), breaks for raising children (both genders) or those arising from emergency and unpredictable situations (long-term sick leave). This will allow us to deviate from the oversimplifying assumption that benefits for the average worker are the same as the average benefit across all workers.

This article presents the results of the first stage of the development of the pension submodel, which aims to verify the correctness of the discrete submodel by comparing it with another model developed in previous studies. A dynamic Monte Carlo (MC) simulation model was presented in (Mielczarek 2022). This model calculates the financial implications of the aging problem in conjunction with previously unanticipated demographic changes caused by the Covid-19 pandemic. The most important limitation of the MC model is the lack of correlation between demographic changes and capital indexation rates. The indexation parameters depend on the level of employment and the average wages in the economy. The size of the working-age population and the amount of contributions paid by employed people affect the annual indexation of accumulated capital. Therefore, we decided to develop a discrete model that, together with a population submodel, will make it possible to account for such dependencies.

The population model simulates demographic changes in the population starting in 2008. Historical data for the period 2008 to 2022 are entered into the model. This allows the model to be populated and calibrated accordingly. Starting in 2023, the demographic simulation is based on projected population scenarios. Each year, the pension simulation submodel is filled with the new cohort representing individuals entering the labor market. The DES model tracks the professional pathways of individuals throughout their careers until retirement. The number of individuals in every age cohort in DES submodel are adjusted annually based on data from the population submodel. Every year, the DES submodel is also filled with labor market data. The number of people of working age is a key factor that is taken into account when calculating the indexation parameters.

The main output measure are replacement rates of retirement benefits in relation to labor income for subsequent dynamically simulated cohorts, separately for both genders. This will make it possible to compare gender equality in terms of the level of pension income received. The sustainability of the system will be measured by the amount of capital contributed to the pension system by individuals from working-age groups.

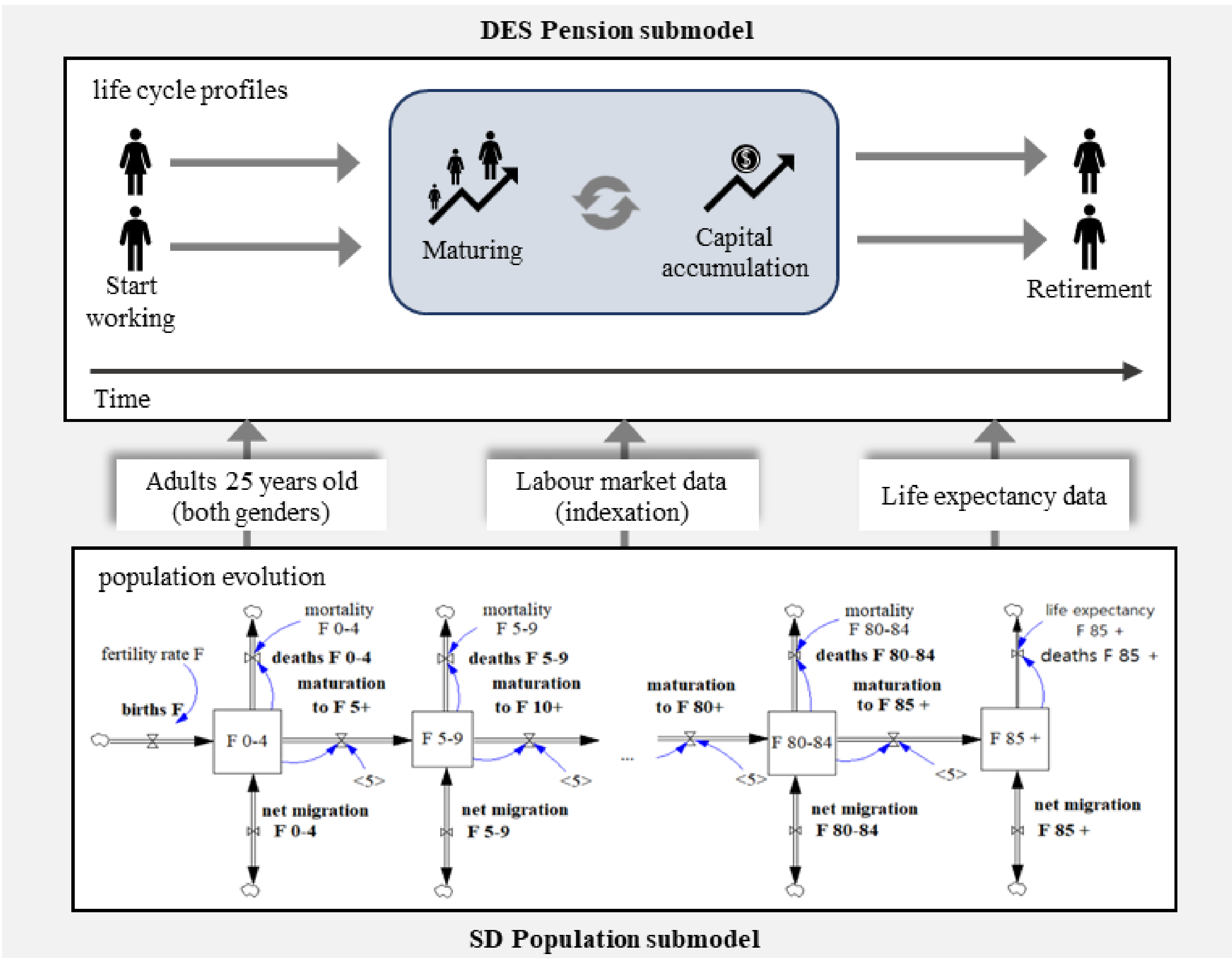

Figure 2: Overall concept of hybrid model.

4.2 Data

Historical population data by age and gender were obtained from the Central Statistical Office (GUS 2022) and were used to simulate the period from 2008 to 2022. Beyond 2022, forecasts that speculate on the behaviour of basic demographic parameters replaced historical data. The downloaded data included birth rates, mortality rates, migration parameters and average life expectancies for 18 main cohorts for each gender (36 in all). Each cohort covers five person years. Data for the pension submodel were extracted from historical data and entered using random distributions (Table 1). All available and published data were considered. We used data from the MC model; however, the most recent parameter values were also included, allowing the discrete model to be verified (Mielczarek 2022)

The indexation rates of the accumulated capital were forecasted based on the historical data from the period 2000–2022 (first pillar) and 2008–2022 (second pillar), published annually by the Minister of Family, Labour and Social Policy. These years cover the entire period that the current pension system has been in effect. Indexation means multiplying the sum of contributions by the index. The triangular distributions were fitted to the data. A change in the minimum and average wage is made once a year, taking into account the percentage rate of increase/decrease. Gamma distributions were fitted to the data.

In the Polish pension system, there is a limitation of the assessment basis for pension contributions. The limit is equal to 30 times the projected average monthly salary, which is announced annually by the Minister of Family, Labour and Social Policy and is effective for the following calendar year. The triangular distribution was fitted to the data.

Table 1: Input random distributions: fitted parameters and test statistics.

Input parameter	Random distribution	Statistic value
Indexation rates of the accumulated capital (%) – Pillar 1	Triangular: a = 101; b = 117; c = 105	Chi-square test *p–value* > 0.75 Kolmogorov–Smirnov test *p–value* > 0.15
Indexation rates of the accumulated capital (%) – Pillar 2	Triangular: a = 103; b = 105; c = 107	Kolmogorov–Smirnov test *p-value* > 0.15
Minimum wage – growth/decline in %	Gamma: alfa = 3.37; beta = 1.47; offset = 2	Kolmogorov–Smirnov test *p–value* > 0.15
Average wage – growth/decline in %	Gamma: alfa = 1.76; beta = 2.47; offset = 1	Kolmogorov–Smirnov test *p–value* > 0.15
Projected average monthly salary growth/decline in %	Triangular: a = -2; b = 5.4; c = 13	Kolmogorov–Smirnov test *p–value* > 0.15

4.3 Model Validation

The validation of the DES model was carried out by comparing the results with those of another model. The MC simulation model presented in (Mielczarek 2022) was used and the DES model was run for exactly the same values of the input parameters. Two experiments were conducted. Scenario 1 studied a group of people who start employment at the minimum wage level. Scenario 2 focused on individuals from average-wage working group. In each scenario, the ratio of pension to last salary was calculated for retirement ages of 60, 65 and 67. The validation results are shown in Table 2. The results obtained confirm the reliability of the DES model for different employment options.

Table 2: Validation of the DES submodel: Gross replacement rates measured as a ratio of Last salary/First pension. Comparative analysis with the MC retirement model presented in (Mielczarek 2022). Both models were replicated 500 times.

	Retirement age	Model MC		Model DES	
		Average	Half Width	Average	Half Width
Sc1. Minimum wage working group	60	0.3495	0.031	0.3431	0.004
	65	0.4781	0.042	0.4701	0.068
	67	0.5407	0.048	0.5330	0.079
	60	0.3479	0.031	0.3529	0.005

Sc2. Average wage working group	65	0.4741	0.042	0.4812	0.007
	67	0.5359	0.047	0.5309	0.008

5 RESULTS AND DISCUSSION

5.1 Assumptions for Simulation Experiments

The purpose of this phase of the still ongoing research project was to develop a hybrid simulation model framework to study the gender pension wealth gap. In particular, we wanted to look at how strongly differences in the level of social security in the DC pension systems are influenced by different career paths and career choices of men and women. We also wanted to see how strong the impact of gender differences in paid work and unpaid care work is on future retirement. The comparison of the gross replacement rates in Table 2 clearly shows how strong the impact of age of retirement on future pensions is. Even if we assume that women's lifetime benefits are comparable to men's (which they are not), the decision to retire at age 60 makes women's pensions nearly 50% lower than those of men retiring at age 65. This is because the design of the DC system is such that the higher the accumulated capital, the higher the annual valorization. Thus, each additional year of work significantly affects the size of the pension.

The simulation was run for the population of a region of Poland. Each year, the model is populated with successive age cohorts of young people aged 25 entering the labor force. Each year, the size of the age gender cohorts is adjusted according to the values predicted by the demographic sub-model. Depending on the experiment, the first salary was the minimum wage or the average wage. It is planned that the model will follow the life cycles of individuals taking into account the different course of gainful employment, different rates of wage growth, breaks in employment, and differences in earnings between men and women. The current version of the model takes into account the averaged characteristics of men's and women's careers taken from (GUS 2022). One replication was performed on the demographic model and demographic projections up to 2060 were formulated. 100 replications each were performed on the discrete submodel. Each replication lasts 50 years and takes into account successive cohorts of men and women retiring from age 60 (women) or 65 (men). In our experiments, we compared gross rates of replacement, reflecting the ratio of the first pension received to the last salary earned.

5.2 Experiment 1: the Postponed Time of Women Retirement

In the first experiment, we examined the change in output values when a woman decides to continue working for a living and not to retire despite reaching retirement age. Figure 3 shows how strongly each additional year of work after age 60 translates into a pension. This is because extending the period of gainful employment increases the pension capital set aside, but also shortens the life expectancy factor. Furthermore, annual indexation yields a significantly larger increase in pension capital in the final years of gainful employment. All these factors are key to determining the pension.

The simulation results are in accordance with projections (OECD 2022) that forecast a gross replacement rate for women retiring at age 60 of 32%.

5.3 Experiment 2: Career Breaks

Experiment 1 assumed that while employed, there has been no period of absence from the labor market. However, many people stop working for various reasons. This is especially true for women who decide to have children. During maternity leave, women receive credit for periods of maternity leave, but many women extend their periods of absence from the labor market and are involved in raising children for a longer period of time. Credits for children are less generous than paid employment; however, in our simulation we did not account for this difference. Instead, we examined how a break in paid employment just after maternity leave would affect women's pensions.

Figure 4 shows how the size of a woman's pension and earnings will decrease in cases where employment is interrupted for 5 and 10 years, respectively. A five–year break lowers future benefit entitlements on average by 28%, and this increases to 51% for a ten–year break.

Figure 3: A comparison of the pension amount when a woman is working longer. Despite reaching retirement age, a woman continues to work.

Figure 4: A decrease in the size of the pension and salary in case of breaks in employment. The woman starts working at the age of 25 and retires at the age of 60. the graph shows the ratio of the first pension (and, correspondingly, the last salary earned) for the discontinuous employment option to the non-discontinuous employment option.

5.4 Experiment 3: Lower Earnings of Women Compared to Earnings of Men

In Poland, women earn on average about 8% less than men. In a further experiment (Figure 4), we assumed that women and men work until the age of 65, with no breaks in professional career, but women's earnings are always lower than men's: on average by 1%, 2%, and 10%. It can be seen that differences in earnings do not translate on the same scale into pension size. In retirement, women's social security is significantly lower. The simulations show that if women's earnings are on average 1% lower than men's, then women's first pension is 83.16% of men's first pension. If the difference between men's and women's earnings is higher and is approximately 10% to the disadvantage of women, then women's first pension is only 42.47% of men's first pension.

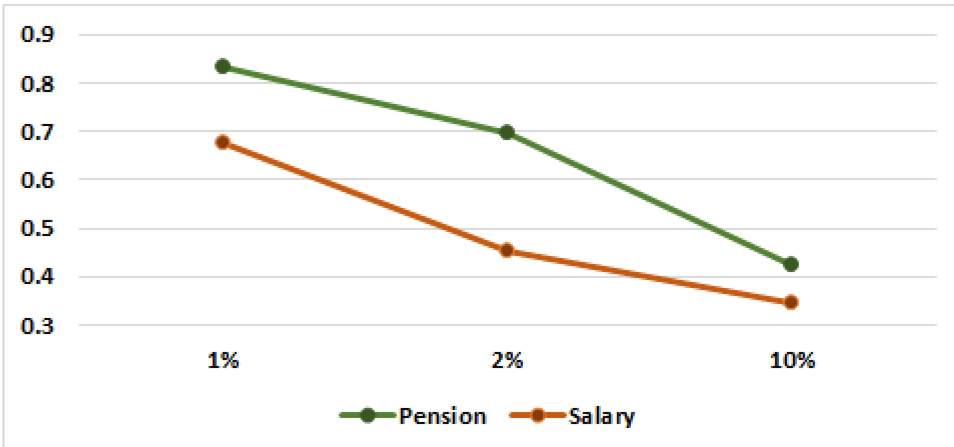

Figure 5: Comparison of women's and men's pensions assuming that women's average earnings are always lower (by a few per cent) than men's. The graph shows the amount of the first pension (and, correspondingly, the final salary) of women relative to the pension (salary) of men in the case where women's earnings are always lower.

6 CONCLUSIONS

Despite the importance and timeliness of the problem posed to pension systems by the growing trend of aging populations, there is a lack of research that would allow us to observe the changing demographic structure of the population in a comprehensive manner, with respect to the entire population, taking into account the diversity of career choices and individual decisions, over a long time horizon, and at the same time analyze the long-term effects of the observed demographic trends on the sustainability of pension systems and the level of individual security.

In this paper, we present a concept of a hybrid simulation model that will make it possible to track the projected changes in the population structure over a longer time horizon, taking into account different demographic scenarios (from the most optimistic to the most pessimistic) and, at the same time, simultaneously observe the impact of demographic changes on the labor market, the burden on the pension system, and the level of retirement security for people leaving the labor force.

The developed model has successfully passed the verification phase. The simulation experiments carried out focused on the extent to which the gender pay gap translates into a pension wealth gap. In the next stage, we plan to fully integrate the population model with the pension model and to use the data the the database Survey of Health, Ageing and Retirement in Europe (SHARE). SHARE is a research database of citizens aged 50 or older from 28 European countries and Israel and it is an ideal source of socio-economic and life-quality research.

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