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HAFI—HIGHEST AUTOCORRELATED FIRST: A NEW PRIORITY RULE TO CONTROL AUTOCORRELATED INPUT PROCESSES AT MERGES

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ABSTRACT

Many intralogistics systems expose autocorrelated arrival processes with significant influence on the systems' performance. Unfortunately there are no control strategies available which take this into account. Instead standard strategies like First Come First Served are applied which lead to systems tending to exhibit long queues and high volatility, even though these strategies perform well in the case of uncorrelated processes. So, there is a strong need for control strategies managing autocorrelated arrival processes. Accordingly this paper introduces *HAFI* (Highest Autocorrelated First), a new strategy which determines the processes' priority in accordance to their autocorrelation. The paper focuses on controlling autocorrelated arrival processes at a merge. The strategies First Come First Served and Longest Queue First will serve as reference. As a result and in respect to properly designed facilities, HAFI leads to comparatively short queues and waiting times as well as balanced 95$^{\text{th}}$ percentile values of the queue lengths of autocorrelated input processes.

1 INTRODUCTION AND MOTIVATION

Autocorrelated arrival processes can be found in many domains, e.g. in telecommunications (Taylor 2007, Ibrahim et al. 2012) or in server-client-systems in the IT-domain (Leland et al. 1994, Baryshnikov et al. 2005, Paul et al. 2011). A discussion of autocorrelated streams in traffic networks and relevant literature is given by Cheng et al. (2011). Regarding autocorrelation in the field of logistics and intralogistics, comprehensive literature surveys are provided by Altiok and Melamed (2001), Civelek et al. (2009), Nielsen (2004), and Rank et al. (2012). Based on a survey the latter states that about 95 % of all input processes in intralogistics exhibit significant autocorrelation. However, when autocorrelation was found, the systems' behavior differed from the case of uncorrelated processes. Usually the performance decreases which is expressed by e.g. longer queues and cycle times. On a theoretical and analytical basis these effects are well understood—see for example Runnenburg (1962) or more recently Livny et al. (1993), Jagerman et al. (2004), Balcioğlu et al. (2008). From a more practical point of view Pereira et al. (2012) and Rank et al. (2013) provide case studies indicating and quantifying the remarkable influence of autocorrelated streams in logistics systems. For example, in case of autocorrelated inter-arrival times an order picking system consisting of several picking stations, an automated storage and retrieval system, and some conveyors shows up to 45 % higher cycle times. Additionally, an considerable increase of the systems' volatility can be observed. So, the occurrence and influence of autocorrelated streams in queuing systems seems beyond question. Therefore, there can only be one conclusion: The application of rules and control strategies, respectively, mitigating the negative effects of correlated arrivals. Unfortunately, all of the mentioned sources above concentrate on pointing out the effects but lack of finding or applying some alternative strategies to First Come First Served (FCFS)—which is in fact not the best choice. So this paper introduces a novel strategy called *HAFI*—Highest Autocorrelated First. As the name suggests, the approach decides about priorities dependent on the processes' autocorrelation. Even though this idea can be adopted to other situations and domains, the paper concentrates on intralogistics systems and the specific case of controlling the priority of autocorrelated streams at a merge. This is because merges are part of virtually any material handling system and are frequently depicted as bottleneck elements which noticeably effect facilities' performance (see e.g. Johnstone et al. 2015). In order to asses and compare HAFI with alternative strategies, a discrete event simulation study is conducted.

The paper is structured as follows: Section 2 will provide some basics and an overview of related work. This includes the definition of autocorrelation and the generation of autocorrelated random variates, a short introduction into relevant control strategies, and a literature review of how autocorrelated arrival processes are currently handled. Section 3 explains the principle of HAFI and presents the results of a simulation study evaluating the impact of HAFI. Additionally heuristics to parametrize HAFI are given. This is followed by a discussion of the results in Section 4. This last section also gives a brief summary as well as an outlook.

2 BASICS AND RELATED WORK

This section provides a short introduction to autocorrelation and corresponding random number generators as well as an overview of relevant control strategies in intralogistics systems.

2.1 Autocorrelation and Random Number Generators

For the statistical background, see Box et al. (2008). Autocorrelation is a statistical figure quantifying the degree of linear dependency a given time series shows with a lagged version of itself. Taking the observations $(x_t, x_{t+\tau})$ of the time series X with length N and $t = 1 \ldots N$ for a lag τ, the sample autocorrelation coefficients r_τ are defined by

$$r_\tau = \frac{\sum_{t=1}^{N-\tau} (x_t - \bar{x})(x_{t+\tau} - \bar{x})}{\sum_{t=1}^{N} (x_t - \bar{x})^2}, \quad \bar{x} = \sum_i \frac{x_i}{N} \quad \text{for} \quad \tau = 0, 1, 2, \ldots, N-1. \tag{1}$$

In Section 3 a simulation study is conducted. Corresponding events of the input processes are modeled on the basis of pseudo random numbers. In this context the generation of autocorrelated variates is not as straightforward as for uncorrelated ones. The latter usually can be obtained from congruence generators and the application of transformation techniques like the inverse method (L'Ecuyer 2006, Law and Kelton 2000). The most common approaches to generate autocorrelated random numbers are Markovian Arrival Processes (Kriege 2012), Copulae (Nelsen 2006), distortion methods like Transform-Expand-Sample (Melamed 1991) or Minification and Maxification (Lewis and McKenzie 1991), and autoregressive models like ARMA introduced in Box et al. (2008). Most of them require iterative user interaction and a high level of expertise in order to get variates with a desired marginal distribution and autocorrelation function. Therefore JARTA (Uhlig et al. 2013) was developed. This is a JAVA library and implements ARTA (Autoregressive To Anything), a generalization of the ARMA approach. JARTA uses an autoregressive base process Z_t, applies the standard normal cumulative distribution function Φ and the inverse transformation method so that $Y_t = F_Y^{-1}[\Phi(Z_t)]$ where Y stands for the desired marginal distribution. The numerical search procedure to determine the correct initial values of Z_t is adopted from Cario and Nelson (1996). JARTA requires no user interaction apart from a definition of the desired target marginal distribution and the variates' autocorrelation function (for some deeper discussion, see Uhlig et al. 2016).

2.2 General and Autocorrelation-based Control Strategies

An introduction into control strategies in logistics systems and a taxonomy is given by Gudehus (2012). Accordingly, there are strategies to accomplish the tasks of dispatching, scheduling, routing and to determine the right of way. Each category holds a vast number of priority rules which in some cases are specific for a particular system. Some well known 'standard' strategies are e.g. First Come First Served, Last Come First Served or Random-Order-Of-Service.

In practice a common rule to control priorities at merges is First Come First Served (Furmans et al. 2012). Based upon experience, Arnold et al. (2004) consider FCFS as the best strategy because of its simplicity, transparency, and fairness to equally allocate the work load. Therefore FCFS will serve as a reference to HAFI.

To the best of our knowledge there are no publications about approaches to control autocorrelated input processes at merges. There neither exists literature generally considering the control of autocorrelated processes in intralogistics systems. We are only aware of Mi et al. (2009) and earlier publications of the authors. On the fields of informatics they describe algorithms to decide whether server requests should be delayed or even rejected. The decision is made on the basis of the autocorrelation in service times. Some similar research can be found in Zhang et al. (2008). However, in the context of this paper the approaches are not applicable because autocorrelation is assumed to occur at the arrival process in non-closed systems and, even more importantly, in intralogistics it is hardly possible to delay or reject jobs.

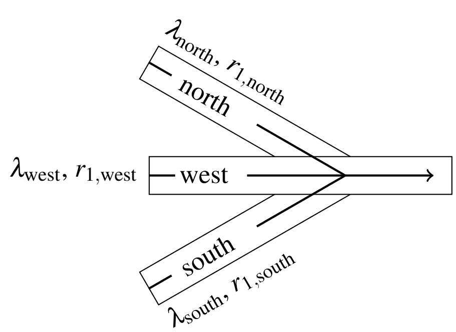
Furthermore, a literature review reveals no publications providing an analytical approach adequately tracking queuing behavior at merges (nevertheless, the interested reader is referred to Balcioğlu et al. 2008 who investigate the superposition of Markovian arrival processes or Boon and van der Mei 2011 who provide an excellent literature review about polling models). This holds for uncorrelated as well as autocorrelated input processes (see also Table 2 in Furmans et al. 2012). So for assessment of the control strategies a simulation study is conducted (see paragraphs subsequently).

3 HAFI—HIGHEST AUTOCORRELATED FIRST

This section introduces HAFI and evaluates its performance. Prior, remarks and basic thoughts are given.

3.1 Preliminary Remarks

The simulator AutoMod$^{\text{TM}}$ is applied. A sketch of the model and annotations are shown in Figure 1. The conveyor is assumed to be an accumulating one. The marginal distribution of the arrivals processes is exponential with identical mean and variance. The utilization of the merge Λ and its streams' arrival rate λ_d of direction d are denoted as $\Lambda = (\lambda_{\text{north}}, \lambda_{\text{south}}, \lambda_{\text{west}})$. It holds $\Lambda = \sum_d \lambda_d < 1$. For random number generation JARTA is applied (see Section 2.1). To conform to most of the sources stated in Section 1, only the autocorrelation of lag 1 $r_{1,d}$ are specified so that $R = (r_{1,\text{north}}, r_{1,\text{south}}, r_{1,\text{west}})$ simplifies annotation.



legend:
— d —: arrival process of direction d
λ_d : arrival rate of direction d
$r_{1,d}$: autocorrelation coefficient of lag 1 of direction d

with: $d \in \{\text{north}, \text{south}, \text{west}\}$

Figure 1: Model of merge with annotations.

In order to evaluate the performance of a strategy queue lengths and dwelling times are used. Doing so two different perspectives are covered: Usually an operator cares about cycling times rather than dimensioning a facility and its buffer spaces as small as possible whereas planners think vice versa. In this respect it is common to design buffer spaces in accordance to the queue lengths' expected 95$^{\text{th}}$ percentile of their particular cumulative distribution function (Sunarjo et al. 2007). So, for the following studies a strategy performs well when it leads to

- small queue lengths,
- small dwelling times (expressed by delay), and
- equal values of the queue lengths' expected 95$^{\text{th}}$ percentile.

The last mentioned item is important because autocorrelated arrivals tend to show longer queues than uncorrelated ones when FCFS is applied (see Section 1 and subsequent paragraphs). In case the buffer spaces of the lanes have to be dimensioned equally but their autocorrelation function significantly differ, it will lead to the issue of deciding to potentially under- or oversize the system in respect to the particular arrivals processes.

In order to prove and quantify the effect of different queue lengths mentioned above, Table 1 is used. It shows the queue lengths' 95$^{\text{th}}$ percentile of the streams when the northern arrival process shows varying autocorrelation. The autocorrelation coefficient is chosen in accordance to earlier studies' observations (see sources in Section 1). Each stream of direction d is parametrized with an arrival rate $\lambda_d = 0.3$. The figures are mean values of about 50 seeds each.

Table 1: 95 % quantile of input processes' queue lengths when FCFS applies.

direction d	autocorrelation of the northern input process—$r_{1,\text{north}}$						
	0.0	0.2	0.3	0.4	0.5	0.6	0.8
north	8	10	12	15	18	22	50
south	8	8	8	8	8	8	9
west	8	8	8	8	8	8	9

experiments' parameters:
strategy = FCFS
$R = (\{0,\ldots,0.8\},0,0)$
$\Lambda = (0.3,0.3,0.3)$

As expected, when autocorrelation gets higher the 95 % quantile of the northern queue length increases whereas the uncorrelated southern and western stream constantly show a length of 8 and 9 for $r_{1,\text{north}} = 0.8$, respectively. Even with moderate autocorrelation of $r_{1,\text{north}} = 0.2$, the difference between north and south/west already amounts to 25 %. Further, approximately from $r_{1,\text{north}} = 0.4$ on, the northern buffer space has to be designed at least twice as big as the southern and western one.

A first conclusion to get balanced queue lengths would be to give priority to the stream which currently shows the longest queue. This *Longest Queue First* (LQF) strategy and its optimality in queuing behavior is discussed e.g. in Gail et al. (1993). Table 2 shows the results in case same experiments of Table 1 are done with LQF instead of FCFS.

Table 2: 95 % quantile of input processes' queue lengths when LQF applies.

direction d	autocorrelation of the northern input process—$r_{1,\text{north}}$						
	0.0	0.2	0.3	0.4	0.5	0.6	0.8
north	7	8	9	10	11	12	21
south	7	8	9	10	11	12	20
west	8	9	9	10	11	12	21

experiments' parameters:
strategy = LQF
$R = (\{0,\ldots,0.8\},0,0)$
$\Lambda = (0.3,0.3,0.3)$

According to Table 2, for autocorrelated input processes LQF indeed leads to perfectly balanced values of all queue lengths' 95 % quantiles—apart from some minor differences caused by rounding. However, this advantage is at the expense of dwelling time in queues. Table 3 allows to compare the mean time a job is delayed caused by queuing.

Table 3: Mean delay caused by queuing.

strategy	autocorrelation of the northern input process—$r_{1,\mathrm{north}}$						
	0.0	0.2	0.3	0.4	0.5	0.6	0.8
FCFS	22	25	28	31	34	41	74
LQF	34	37	38	41	45	50	83

experiments' parameters:
strategy = {FCFS, LQF}
$R = (\{0,\ldots,0.8\},0,0)$
$\Lambda = (0.3,0.3,0.3)$

Taking FCFS as a basis, LQF performs up to 50 % worse in mean delay when no or only minor ($r_{1,\mathrm{north}} = 0.2$) autocorrelation in the northern input process is observed. The relative difference decreases from about 35 % to 10 % as the autocorrelation coefficient $r_{1,\mathrm{north}}$ increases.

In short: For the task of controlling autocorrelated streams at merges, FCFS leads to different queue lengths and short dwelling times—for LQF it is vice versa. To resolve the discrepancy, HAFI is suggested (see subsequent sections).

3.2 Basic Principle of HAFI

The main concept of HAFI intends to give input processes priority in accordance to their (first lag) autocorrelation. That is because the higher the autocorrelation, the higher is the likelihood of batch arrivals which results in temporarily long queues when no right of way is given. HAFI bases on FCFS and gets active if direction-wise a specific queue length is exceeded. The particular threshold t_d of the direction d depends on the processes' autocorrelation. In short, the higher the autocorrelation the lower the threshold (a more detailed discussion is given in Section 3.3). If more than one queue exceeds its threshold, then the stream with the highest offset gets priority. As long as no threshold is exceeded, FCFS applies. Subsequently the vector of the thresholds is denoted as $T = (t_{\mathrm{north}}, t_{\mathrm{south}}, t_{\mathrm{west}})$ and corresponding queue lengths as $q_{\mathrm{north}}, q_{\mathrm{south}}, q_{\mathrm{west}}$.

Suppose T is known then for HAFI following procedure applies. Each time an entity enters a queue q_d its direction d is saved in an ordered list A (remark: lists are subsequently denoted with capital letters and their indexing starts at 1):

Pseudo Code 1: Keeping order of arrivals.

```
add d to A at end              # chronological order of arrivals depending on their directions
```

Each time a entity wants to enter the merge zone, following pseudo code applies to decide which direction gets right of way:

Pseudo Code 2: HAFI algorithm.

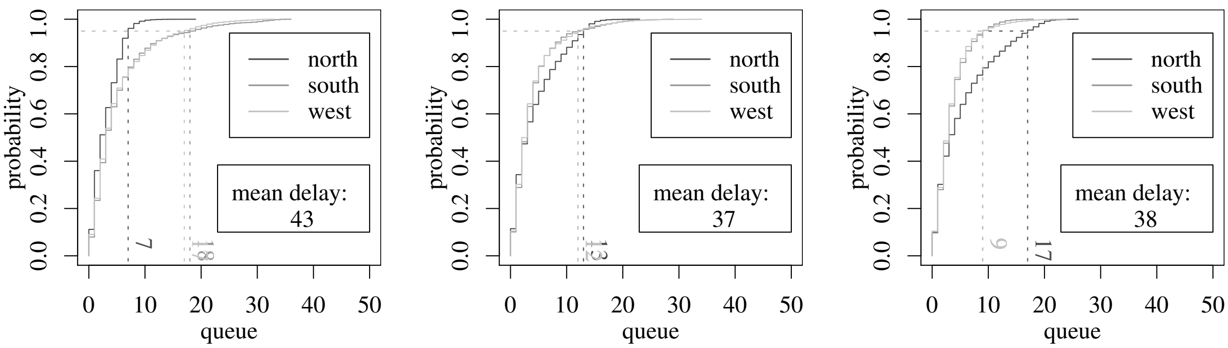
```
# D: list of directions with arrival rate > 0
for each d in D {
 set x_d = q_d - t_d

 if (x_d > t_d){
  add x_d to X
 }
}
```

```
if(length(X) > 0){
 set X = sortDescending(X)
 set p = getDirection(X[1]) # corresp. d of x_d
 set X = NULL               # clear X
}
else{
 set p = A[1]        # First Come First Served
}

remove p from A at beginning
give p right of way
```

It becomes obvious that the goodness of HAFI depends on the determination of T. In this connection, Figure 2 gives a general impression how the determination of T influences the results. For simplification, only the northern arrival process shows autocorrelation so that $t_{\text{south}} = t_{\text{west}} = \infty$ which means the southern and western stream will never get priority because their queue lengths exceed a specific value (see Pseudo Code 2). For the experiment it holds $\Lambda = (0.3, 0.3, 0.3)$.



(a) $t_{\text{north}} = 3$ (b) $t_{\text{north}} = 7$ (c) $t_{\text{north}} = 10$

Figure 2: The impact of varying thresholds t_{north} on mean delay and the cumulative distribution function of the queue length.

As it can be expected, the lower the HAFI parameter t_{north} the higher the prioritization of the northern stream and vice versa. Both, an underestimation (Figure 2a) as well as an overestimation (Figure 2c) of t_{north} leads to suboptimal results. In either case the 95 % queue length quantiles of the uncorrelated streams significantly differ from the autocorrelated northern one. In addition, the values of the mean delay are higher for $t_{\text{north}} = 3$ and $t_{\text{north}} = 10$ compared to $t_{\text{north}} = 7$. With overvaluing t_{north} the results start to get similar with FCFS (compare also to Table 1). Undervaluing ($t_{\text{north}} = 7$) leads to bad values at all.

Recapitulating the last paragraphs, the goodness of HAFI essentially hinges on T. Unfortunately we are currently not able to give a closed solution to exactly determine T. Nevertheless, in subsequent Section 3.3 a heuristic will be given.

3.3 Determination of HAFI's Parameters and Numerical Results of a Case Study

This sections presents a heuristic to dynamically determine the thresholds in T. This is followed by numerical results of applying DyDeT for HAFI.

3.3.1 Dynamic Heuristic to Determine Thresholds

The acronym DyDeT stands for Dynamic Heuristic to Determine Thresholds T. The specific thresholds t_d are parameters of HAFI. DyDeT is divided into three phases. It holds for the case input processes show stationarity and there are three input processes as a maximum. Additionally, only positively autocorrelated arrival processes are considered (see also Section 4). For better explanation of DyDeT, a new rank index $i \in \{1, 2, 3\}$ replaces the index d. It refers to the order of the stream's autocorrelation lag one coefficient values.

In phase *one* an initial T is determined. During conducting the simulation study in preparation of this paper, most of the T have been found by trial and error. When trying to find a connection between T and the systems' parameter it turned out that with only minor error the highest autocorrelated stream's threshold t_1 can be estimated from the median of the aggregated queue length of all streams in case the merge is controlled by FCFS and LQF, respectively. Table 4 lists optimal t_1 got by trial and error and median queue length values of different model parametrizations. For simplification, $r_{1,2} = r_{1,3} = 0$ is assumed.

Table 4: Approximate the optimal t_1 from the median of the aggregated queue length of all streams.

utilization of merge	autocorrelation of the highest autocorrelated input process—$r_{1,1}$											
	0.2			0.4			0.6			0.8		
	t_1	median queue length FCFS	median queue length LQF	t_1	median queue length FCFS	median queue length LQF	t_1	median queue length FCFS	median queue length LQF	t_1	median queue length FCFS	median queue length LQF
0.40	2	2	2	2	2	2	2	2	2	2	2	2
0.60	2	3	4	2	3	4	2	3	4	2	3	4
0.80	4	5	7	4	5	7	4	5	7	5	6	8
0.90	6	7	10	7	8	10	9	9	12	12	12	15
0.96	15	15	19	17	20	22	20	23	29	30	45	52

experiments' parameters:
strategy = {HAFI, FCFS, LQF}
$R = (\{0.2,\ldots,0.8\},0,0)$
$\Lambda = (\{0.13,\ldots,0.32\}, \{0.13,\ldots,0.32\}, \{0.13,\ldots,0.32\})$
$T = (\{2,\ldots,30\},\infty,\infty)$

According to Table 4, in the given range of autocorrelation coefficients and for utilization levels ≤ 0.9 the optimal t_1 for HAFI differs only by 1 at maximum from the median queue length of a corresponding model where FCFS is applied. When a merge utilization of 0.96 and autocorrelation coefficients > 0.4 are observed, this simple approximation starts to get faulty. In that case a reduction of the median value of about 10 % will lead to acceptable results. For an utilization of 0.96 and $r_{1,1} = 0.8$ the correction factor is 50 %. In a similar manner, reducing the median queue length of LQF by one third also seems to be a good estimator for the optimal t_1.

In order to estimate the initial t_1 in practice, the first lag autocorrelation coefficients $r_{1,i}$ and their ranks i, respectively, as well as the aggregated queue length have to be determined empirically by either running the system with FCFS or LQF strategy for a certain period of time (for a detailed discussion how to estimated figures statistically correct, see Law and Kelton 2000, chapter 9—in the present case we recommend a time series with a minimum length of 5000). The initial thresholds t_2 and t_3 can be derived from t_1. As already mentioned, if an input process does not show positive autocorrelation, its threshold is set to infinity and does not change anymore. Pseudo Code 3 summarizes phase one of DyDeT.

Pseudo Code 3: DyDeT phase one.

```
set t1 = q50%, LQF/1.3 # see table 4

if(r1,3 = r1,1){
 for each i in (2, 3) {
  set ti = t1
 }
}
else if(r1,2 = r1,3){
 for each i in (2, 3) {
  set ti = t1 * 2
 }
}
else if(r1,1 = r1,2){
 set t2 = t1
 set t3 = t1 * 2
}
else{
 set t2 = t1 * 1.5
 set t3 = t1 * 2
}

for each i in (1, 2, 3) {
 if(r1,i <= 0 )
  set ti = ∞
}
```

Remember, in phase one of DyDeT the merge was controlled via FCFS or LQF in order to set initial values t_i. From phase *two* on the merge is controlled via HAFI and the adjustment of the correct relationship between the thresholds t_1 and t_2 starts. Every period of events—we suggest at least 1000 passes of the merge—the 95$^{\text{th}}$ percentiles of the input processes' queue lengths $q_{i,95\,\%}, i \in \{1, 2\}$ are checked for equality. We propose a deviation of 20 % in order to admit inequality (subsequently denoted as $\gg$ or $\ll$). If they differ, then the corresponding t_i will be decremented. This is iteratively done as long as $q_{95\,\%,\,1} \approx q_{95\,\%,2}$ is found—see Pseudo Code 4.

Pseudo Code 4: DyDeT phase two.

```
if(q95%, 1 >> q95%, 2){
 decrement t1 by 1
}
else if(q95%, 1 << q95%, 2){
 for each i in (2, 3) {
  decrement ti by 1
 }
}
```

In phase *three*, if $r_{1,2} \neq r_{1,3}$, the thresholds t_1 and t_2 are stepwise adjusted so that $q_{95\,\%,1} \approx q_{95\,\%,2} \approx q_{95\,\%,3}$. For this purpose it is regularly checked if the 95$^{\text{th}}$ percentiles of the input processes' queue lengths equal each other. Again, we suggest period of at least 1000 passing elements. Pseudo Code 5 shows the algorithm.

Pseudo Code 5: DyDeT phase three.

```
if(q95%, 1 >> q95%, 3){
 for each i in (1, 2) {
  decrement ti by 10 %
  floor ti                # round down
 }
}
else if(q95%, 1 << q95%, 3){
 for each i in (1, 2) {
  set viTmp = ti
  increment ti by 10 %
  floor ti                # round down

# ensure threshold is at least inc. by 1
  if(viTmp = ti){
   increment ti by 1
  }
 }
}
```

After each iteration it is checked whether $t_i \geq t_{i+1}, i \in \{1,2\}$ holds. If so and if the initial $t_i \neq t_{i+1}$ then t_i is decremented by 1 until the mentioned condition is not true any more. This ensures the compliance of the ranks of the streams' lag 1 autocorrelation coefficients and their corresponding thresholds. In addition and eventually it is ensured that t_1 lies within an interval $\pm 25\,\%$ of its initial value by stepwise increasing or decreasing it—the same amount of increment and decrement, respectively, it is applied to t_2 and t_3.

As described above, the adjustment steps do only apply if the 95 % queue length quantiles of the streams differ significantly. In Section 1 it is mentioned that systems with autocorrelated arrival processes tend to show higher volatility. In order to smoothing this effect and to ensure an adjustment does not base on a single outlier, "goodness/badness counters" $c_{1,2}$ and $c_{1,2,3}$ are introduced which are checked and set after each period. $c_{1,2}$ controls the relationship between $q_{95\,\%,1}$ and $q_{95\,\%,2}$. So, after each period, if $q_{95\,\%,1} \gg q_{95\,\%,2}$ holds, $c_{1,2}$ is incremented—$c_{1,2}$ is decremented in the opposite case of $q_{95\,\%,1} \ll q_{95\,\%,2}$. Only if $c_{1,2} \notin [-2,2]$ then phase two described above is applied and $c_{1,2}$ is set to 0 as well as $c_{1,2,3}$ (see paragraph below) is decremented by 1.

$c_{1,2,3}$ is used to control the application of the adjustments described in phase three. The counter gets incremented if $q_{95\,\%,1} \approx q_{95\,\%,2} \approx q_{95\,\%,3}$ and decremented in any other case. As soon as $c_{1,2,3}$ gets below zero, thresholds are redefined as mentioned and $c_{1,2,3}$ is set to 0.

The following Figure 3 exemplarily shows the development of the HAFI thresholds in T by applying the heuristic DyDeT described before. The utilizations are given by $\lambda_{\text{north}} = \lambda_{\text{south}} = \lambda_{\text{west}} = 0.3$ and the autocorrelation coefficients are $r_{1,\text{north}} = 0.4$, $r_{1,\text{south}} = 0.2$, and $r_{1,\text{west}} = 0.0$. The corresponding optimal thresholds T_{TaE} were found by trial and error and serve as reference. They are $t_{\text{north}} = 6$, $t_{\text{south}} = 9$, $t_{\text{west}} = \infty$.

In respect to Figure 3 the given heuristic DyDeT determines and dynamically adjusts the particular thresholds necessary to apply HAFI quite well. After the initial phase till period 5 the heuristic values only differ minorly from the optimal ones (compare the dashed with the solid lines). This discrepancy is caused by temporarily slightly different system parameters in a period and the fact that the thresholds may get adjusted accordingly. However, when it comes down to a direct comparison of applying T_{TaE} and T_{DyDeT}, the differences are negligible: The mean delay of HAFI$_{\text{TaE}}$ is 37.6 and the 95 % queue length quantiles of the three streams are 12, 12, and 13 whereas HAFI$_{\text{DyDeT}}$ leads to a mean delay of 38.6 and queue lengths of 12, 13, and 12. Further numerical results are given in the next paragraph.

3.3.2 Numerical Results

In the following numerical examples of applying HAFI are given. Table 5 shows results of the same simulation study as before but applying HAFI and the corresponding dynamic heuristic DyDeT—it is

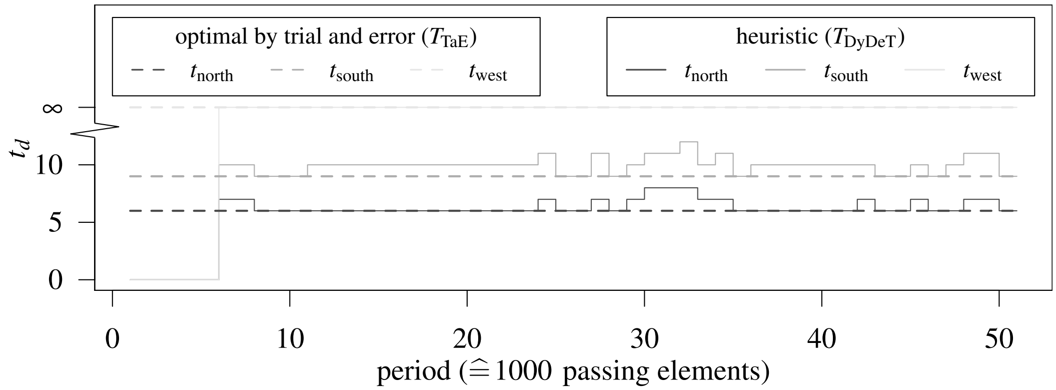


Figure 3: Application of DyDeT: Heuristic to dynamically determine and adjust thresholds for HAFI.

denoted as $\text{HAFI}_{\text{DyDeT}}$. Furthermore the table lists HAFI_{TaE} which is HAFI with the best static thresholds which could be found by trial and error. For direct comparison with FCFS and LQF, see Table 1, 2 and 3.

Table 5: 95 % quantile of input processes' queue lengths and mean delay when HAFI applies.

strategy	autocorrelation of the northern input process—$r_{1,\text{north}}$						
	0.0	0.2	0.3	0.4	0.5	0.6	0.8
	95 % quantile input processes queue lengths: north | south | west						
HAFI_{TaE}	8 | 8 | 8	9 | 8 | 8	10 | 9 | 9	11 | 9 | 10	13 | 12 | 11	15 | 13 | 14	24 | 25 | 22
$\text{HAFI}_{\text{DyDeT}}$	8 | 8 | 8	9 | 9 | 9	10 | 9 | 9	11 | 10 | 10	12 | 12 | 12	13 | 13 | 14	18 | 21 | 23
	mean delay						
HAFI_{TaE}	22	25	28	31	37	42	75
$\text{HAFI}_{\text{DyDeT}}$	22	26	28	31	36	42	68

experiments' parameters:
strategy = $\{\text{HAFI}_{\text{TaE}}, \text{HAFI}_{\text{DyDeT}}\}$
$R = (\{0,\ldots,0.8\},0,0)$
$\Lambda = (0.3,0.3,0.3)$
$T = (\{7,\ldots,\infty\},\infty,\infty)$

Obviously, HAFI combines the advantages of LQF and FCFS which results in well balanced 95$^{\text{th}}$ percentile values of the streams' queue lengths and a mean delay on the level of FCFS. With taking rounding errors into account, the difference between the 95 % quantile values for the particular autocorrelation levels is about 10 % at maximum. That is a little higher than observed with LQF (compare Table 5 with Table 2) even though LQF sometimes also shows some minor differences in 95 % quantile values. The deviations of the mean delay values between HAFI and FCFS are negligible (compare Table 5 with Table 3). Comparing HAFI_{TaE} with $\text{HAFI}_{\text{DyDeT}}$, the results indicate that DyDeT determines the thresholds as good as trial and error can. From a practical point of view there are only insignificant differences.

HAFI also does a good job when more than one stream shows autocorrelated arrivals. Table 6 shows the results of three different, randomly chosen sets of R for a given set of utilization $\Lambda = (0.3,0.3,0.3)$ when HAFI, FCFS and LQF are applied.

According to Table 6, applying HAFI leads to well balanced 95$^{\text{th}}$ percentiles of the input processes' queue lengths. With only minor discrepancy the values are on the level of LQF. For $R = (0.8,0.6,0.2)$ HAFI tends to show longer particular queues then LQF. Compared to FCFS, HAFI performs better in any respect. The mean waiting times of HAFI are on a similar level or even slightly shorter. More in detail, HAFI_{TaE} and its static thresholds shows less inequalities in queue length then $\text{HAFI}_{\text{DyDeT}}$. The maximum discrepancies are 2 and 4 units. On the other hand, $\text{HAFI}_{\text{DyDeT}}$ performs about 2 time units better in mean delay. In any case, the survey presented above shows that HAFI leads to remarkable results in order to control a merge with autocorrelated arrivals. The Highest Autocorrelated First strategy combines the advantages of FCFS (small delays) and of LQF (balanced queue lengths).

Table 6: 95 % quantile of input processes' queue lengths and mean delay.

strategy	autocorrelation input processes: $(r_{1,\text{north}}, r_{1,\text{south}}, r_{1,\text{west}})$		
	$(0.6, 0.4, 0.0)$	$(0.8, 0.6, 0.2)$	$(0.6, 0.4, 0.4)$
	95 % quantile input processes queue lengths: north | south | west		
HAFI_{TaE}	16 | 15 | 16	29 | 31 | 29	18 | 19 | 19
$\text{HAFI}_{\text{DyDeT}}$	16 | 16 | 14	27 | 31 | 30	18 | 20 | 19
FCFS	25 | 15 | 9	54 | 29 | 13	26 | 17 | 17
LQF	15 | 15 | 15	26 | 25 | 25	17 | 17 | 17
		mean delay	
HAFI_{TaE}	50	100	62
$\text{HAFI}_{\text{DyDeT}}$	48	97	60
FCFS	52	103	62
LQF	61	108	72

experiments' parameters:
strategy = $\{\text{HAFI}_{\text{TaE}}, \text{HAFI}_{\text{DyDeT}}$, FCFS, LQF$\}$
$R = (\{0.6, 0.8\}, \{0.4, 0.6\}, \{0, 0.2, 0.4\})$
$\Lambda = (0.3, 0.3, 0.3)$
$T = (\{8, \ldots, 12\}, \{10, \ldots, 24\}, \{24, \ldots, \infty\})$

4 SUMMARY, DISCUSSION, AND OUTLOOK

Autocorrelated input processes can be observed in nearly any queuing system and hence in intralogistics as well. Up to now, for some inexplicable reasons, there have been no strategy to appropriately control these processes. 'Traditional', popular/established control strategies like First Come First Served or Longest Queue First are rather unable to properly manage autocorrelated input processes which evidently lead to bad system performance like long queues and delays. So *HAFI*—Highest Autocorrelated First—and a corresponding heuristic *DyDeT* to dynamically determine its parameters were introduced in this work. The main principle of HAFI intends to determine the input processes' priority in accordance to their autocorrelation function. It therefore combines the strengths of First Come First Served and Longest Queue First which serve as reference strategies. By applying HAFI to control merges, well balanced queue lengths of the streams and minimal delays can be achieved. This is only true, if direction-wise the thresholds T to activate HAFI are appropriately chosen. These depend on the system's utilization and the input processes' autocorrelation functions which results in some effort determining them. In this regard, if considerably wrong thresholds are applied the system behavior can even get worse compared to the application of FCFS or LQF. Unfortunately we currently can not derive T analytically. Nevertheless, we give a heuristic to determine T. A comprehensive simulation study revealed strong evidence that for each merge system with a specific utilization and autocorrelated streams there exists a set of thresholds which lead to good/optimal results.

Furthermore, HAFI and DyDeT have only been tested for stationary processes. Due to an upper and lower border which hinders our heuristic DyDeT to extensively adjust the thresholds, the algorithm is restricted to stationary systems. For some facilities in practice this constraint may be too hard because of e.g. diurnal variations in utilization. However, this procedure helps to spot outliers and deduct HAFI parameters from empirical time series. Further, if the initial T is supposed to be derived solely from the median queue length of a system driven by FCFS or LQF the given approximations start to get error-prone for merge utilization levels close to 1 and autocorrelation coefficients > 0.4. Nevertheless, from our experience this should not be a big deal for the vast majority of facilities. This also applies for the restriction of DyDeT to determine the particular thresholds of a merge with three input processes as a maximum.

Additionally, as already mentioned, in the context of HAFI only positively autocorrelated arrival processes are considered. This is because negatively correlated streams usually do not have an negative effect on systems' performance—quite to the contrary, they lead to better performance—and from a quantitative point of view their influence is low. Future work will concentrate on adopting the main idea of HAFI—to prioritize autocorrelated processes—to other situations like e.g. dispatching tasks. Additionally the determination of optimal thresholds will get attention to find something more robust than a heuristic.

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