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INFORMATION DIFFUSION IN TWO OVERLAPPING NETWORKS MODEL

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ABSTRACT

People's opinions published in social media became one of the crucial touch-points in the purchase cycle. Many analytical companies provide their customers with tools and models that allow them to evaluate their media campaigns in a single network. However, lack of marketing decision's supporting models that reflect the reality of a set of overlapping networks is a barrier to evaluate complex social media campaign effectiveness properly. The purpose of this paper is to present an algorithm allowing to generate a graph representing two combined scale-free networks that may overlap one another in order to simulate real-life example of social media networks coexistence. Using this model we investigate an example scenario calibrated using Polish Facebook and Twitter data. Experiment presents impact of company decisions regarding brand's profile placement and awareness building marketing campaign intensity on reach, that marketing information in a form of post can achieve.

1 INTRODUCTION

Social media networks' user base is growing every year. Their users currently do not look only for information regarding their peers, but also communicate and actively create content. It is predicted, that in 2017 2.55 billion people (increase by 29.4% in comparison to 2014), 34.5% of total global population, will be members of social media networks (eMarketer 2013a). Such growth will be possible, thanks to a surge of Facebook user base in India, Russia, Middle East and African countries.

Growth of number of social users proceeds, and will continue in the future, two-fold: new social media websites appear and old ones extend the user base. Social media websites are already placed among the biggest sites in the entire Internet. Three social networks can be found among top 20 websites in Alexa websites rating: Facebook on the 2nd place, Twitter on 8th and LinkedIn on 15th (Alexa 2015). It is estimated, that Facebook user base will account for 1.77 billion people worldwide in 2017, which is a 40% growth in comparison to 2014. In the US social media user base growth will be driven mostly by increasing number of mobile social media applications' users. It is expected, that in 2017 154.7 million of US citizens (up by 34.3% in comparison to 2014) will use Facebook mobile application and 43.3 million (up by 32.4% in comparison to 2014) will use Twitter mobile application (eMarketer 2013c).

Moreover, not only social media networks are growing, but the whole marketing industry still has not reached its saturation level. The example of that trend can be a case of marketing budgets of US companies, that have grown by 12.3% since 2011 and are expected to grow by additional 10.8% up to 2017 (eMarketer 2013b). The main factor behind such growth was increased budget for Digital communication that grew by 32% in 2013 compared to 2011. Increasing digital media budgets raise a discussion on how digital media actually work. The most popular approach to digital media is Paid, Owned and Earned Media structure, used primarily by Nokia (Goodall 2009). As general definition states,

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paid media are "paid placements that promote a product", owned media can be "any asset owned by the brand" (website, social media brand's profile etc.) and earned media are "brand-related consumer actions and conversations" (Burcher 2013).

Thanks to its unique features, social media fit an idea of owned and earned media perfectly. It is worth observing, that in 2011 83% of Fortune 500 list companies, used social media to communicate with their customers (Naylor, Lamberton and Patricia 2012). Moreover, uniqueness of social media is especially visible from an earned media perspective. Evans (2012) refers to it as Social Feedback Cycle – it is no longer only marketers' work to build awareness of a brand, which will affect consideration and at the end will lead to purchase. Social media's unique role starts after the product has already been purchased. Product's users tend to form their own opinions which then they can share easily with great number of other social media members. According to Stephen and Galak (2011) despite having relatively small impact on single purchase events, social media can build brand sales in a long-time perspective thanks to an enormous volume of comments and opinions, unparalleled in offline world. Moreover, authors perceive social media as a right tool for marketers "to generate social media activity and WOM" (Word-of-mouth) and drive incremental sales volume.

All arguments listed above, in particular those regarding growth of social media usage and advertising expenditure, are the main reasons for social media phenomenon importance from sociological and business points of view.

What is important, as the development of social media networks proceeds, Internet users are often members of more than one social network. Since previous models of network's structure are valid for modeling one network only, such approach may be insufficient.

Therefore, induced by rising business importance of social media marketing and lack of methods adequate for modeling complex, multi-network structures, we propose a new approach for network modeling.

Our main objective was to develop an algorithm that allows to simulate users of two overlapping social networks that own properties desired from scale-free network.

We show how to calibrate the proposed model with Polish data about Facebook and Twitter users. The modeled networks have different user-to-user mechanics. On Facebook a relation between two users is symmetric, while on Twitter it is more often asymmetric, as one user is a follower but may not be followed by another user.

As real-life social media networks tend to grow and have more and more common users, the second objective of this article is to verify, whether greater number of users that are members of multiple networks increases marketing information reach. In particular we want to verify, how placement of brand's profile affects marketing information reach.

2 MODEL SPECIFICATION

Research studies confirm, that social networks, such as Facebook or Twitter can be described as scale-free networks (Kunegis, Blattner, and Moser 2013). Therefore, degree distribution of their nodes (a number of connections every social media user has) follows a power law resulting from preferential attachment process that creates networks of such type. Degree of node y in this article will be denoted as $d(y)$ while in-degree (a number of neighbors that are connected to y, so that y is a head of a directed edge) as $d_{in}(y)$. One of network models that own desired properties is an Albert-Barabasi network (Albert and Barabasi 1999).

Facebook posts and Twitter mentions are assumed to follow preferential attachment rule. Kunegis, Blattner and Moser (2013), on a base of 47 investigated different online networks, analyzed preferential attachment function, which maps a node degree before a fixed moment in time to the number of newly connected to the node edges, added after it. They have used a preferential attachment function presented in (1) in which, parameter β is called a preferential attachment exponent: $f(d) = e^{\alpha+\beta ln(1+d)} - \lambda$.

According to their results, preferential attachment exponent can appear in several different variations that impact a network structure significantly:

1) $\beta = 0$: constant case that results in network similar to an output of Erdos-Renyi model;
2) $\beta < 1$: sublinear case that results in stretched exponential degree distribution;
3) $\beta = 1$: linear case, that results in scale-free network of Barabasi and Albert;
4) $\beta > 1$: superlinear case, that in exceptional circumstances may result in single node acquiring asymptotically all other edges.

According to Kunegis, Blattner, and Moser (2013), preferential attachment exponent is equal to 1.0379, 0.87210 and 0.51657 for Twitter mentions, Facebook wall posts and Facebook friendships network, respectively. Results show, that only Twitter mentions almost perfectly follow linear preferential attachment rule, while Facebook wall posts and friendship network fall into sublinear preferential attachment category. Due to the fact, that sublinearity of preferential attachment process is not the main objective of this article and to simplify calculations, all networks are assumed to follow linear preferential attachment process.

Modern literature presents numerous different approaches to creating networks that follow preferential attachment rule. In the following article the starting point is a procedure by Albert and Barabasi. However, one of crucial assumptions of their approach is stability of edges connecting nodes over time, which may be inappropriate in certain occasions. An example of such occasion in being presented by Ferligoj et al. (2015) who have investigated scientific collaboration dynamics and ended up with a conclusion that co-authorship between scientists may disappear with time. Therefore their network adjusts to changing situations and some nodes are deleted while others are created during time. Since a simulation length in our case is an equivalent of 50 days, which is a short enough period of time to assume that social relations between people are stable, a classic Albert-Barabasi approach is sufficient. On the other side, if one wanted to simulate social network behavior over longer timespan, a network changing over time would be a proper solution.

The algorithm responsible for creating network for this article is an extension of Albert-Barabasi procedure. Each newly created agent becomes randomly either the user of one of two social media networks Facebook and Twitter, or both. After being created, an agent becomes a neighbor of a random already existing agent according to the preferential attachment rule, but agents of each affiliation type (certain social media network user) can only become a neighbor with an agent with similar affiliation.

Table 1. Network users' connection logic for different social media services (affiliation matrix).

From\To	Facebook only user	Twitter only user	Both systems user
Facebook only user	Undirected edge	Connection not possible (other affiliation)	Undirected edge
Twitter only user	Connection not possible (other affiliation)	Undirected or directed edge in a random direction	Undirected or directed edge in a random direction
Both systems user	Undirected edge	Undirected or directed edge in a random direction	Undirected edge

The logic regarding specific type of connection between agents is presented in detail in Table 1. When Twitter user is involved in building of the connection it can be either directed or undirected. We assume that with probability equal to 1/3 the edge is undirected, and if it is directed (with probability equal to 2/3) each direction is equally probable.

In order to be in line with preferential attachment mechanics, probability that new agent j of some affiliation will be a neighbor of i-th existing agent is proportional to the number of i-th agent's neighbors compared to all agents of such affiliation in the system and is presented in detail in equation (1). Among agent neighbors we consider:

1) Agents connected to the i-th agent by the undirected link
2) Agents, that are connected to the i-th agent by a directed link, where i-th agent is a head and other agent is a tail:

$$P(i-j) = \frac{d(j)+d_{in}(j)}{\sum_{x=1}^{A} d(x)+ d_{in}(x)}. \tag{1}$$

For a user that is a member of two networks at the same time, a probability of connection is being calculated twice, for each affiliation separately. Thus, one and the same agent may be more likely to be connected in one network than in other one. Logic behind network creations is presented on Figure 1, where green dots represent Facebook users, blue dots represent Twitter users and red dots represent both systems' users. Line thickness represents probability, that a new agent will be connected to the certain agent.

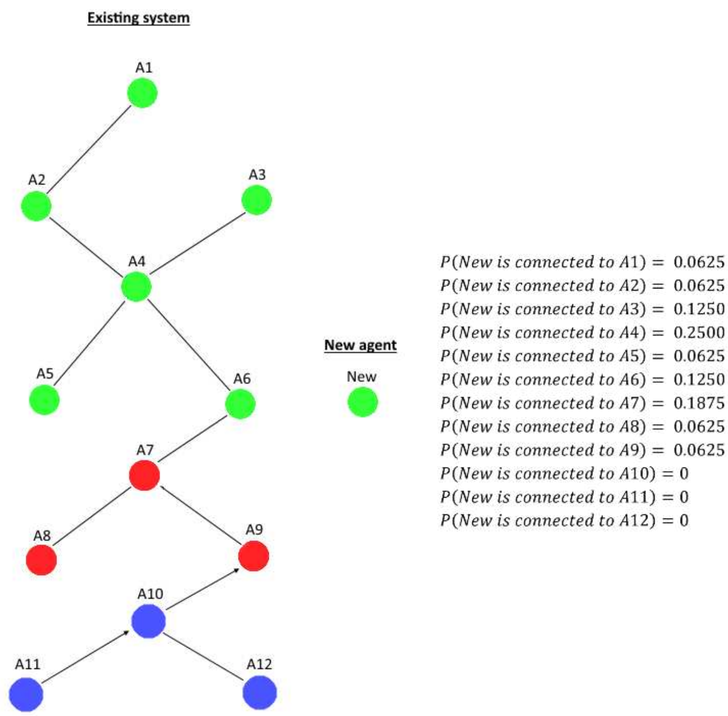

Figure 1: Network creation algorithm.

In equation (1) A term refers to a set of nodes that possess an affiliation similar to affiliation of node j. Affiliations that we consider as similar ones are presented in Table 1. After a connection is established, a certain connection type is drawn in line with Table 1.

It is worth emphasizing, that proposed algorithm from the field of social science is closely related to multiplex networks that are increasingly popular in a field of statistical physics (Lee, Min, and Goh 2015) and are often called as Network of Networks (NoN). Interestingly, similar algorithm creating duplex system of networks following linear preferential attachment was presented by Kim and Goh (2013). Its extension versus original Albert-Barabasi procedure was based on an assumption, that new agent incorporated into the system becomes a member of both layers and then is randomly connected to an agent in each layer. However, probability of being connected to an agent in one layer was not only driven by this agent's degree in this layer, but took into account also agent's degree in the second layer as well. Unlike Kim and Goh's algorithm, a method presented in the following paper is more flexible, as a new agent does not have to be a member of both layers in a multiplex network.

Our objective in this paper is to simulate diffusion of marketing information about some brand via Facebook and Twitter networks. Therefore we select a random agent with a number of neighbors belonging to the 98th percentile of agent connections' numbers to become a brand's profile in social media, which is the first source of information. With this logic we ensure that a brand's profile would not be placed in an isolated node that would have a crucial impact on the poor final campaign results. Information will spread across network and each agent in each period will be able to interact with it.

The most important for simulated diffusion are actions taking place in agent's neighborhood, while media communication and agent's attitude toward novelties in the model plays supporting role. High media frequency tends to create 'buzz' – agents are more likely to interact with brands they are familiar with and which they are aware of. Each time an online advertisement is being displayed to the Internet user is called in digital media planning practice as an 'impression'. Number of all available online advertisement impressions within simulated multi agent system, are being calculated directly from the following equation:

$$Frequency = \frac{Media\ budget}{Cost\ per\ Mille} * \frac{World\ Size}{18\,000\,000} * 1000 \tag{2}$$

Cost per Mille term reflects cost of displaying an advertisement to 1 000 digital users regardless of a fact if it was actually seen by user.

In equation (2) World Size term is used in order to recalculate Frequency from being valid for total Polish digital market (18 000 000 of active users) terms to the simulated system terms (1 000 agents). Obtained number reflects the on-going brand communication's intensity in digital media at the start of simulation. Then, at each step of a simulation a random number of remaining impressions is seen by agents increasing their own propensity to interact and lowering number of available impressions left in the system. The brand communication's support ends while there are no available impressions left in the system.

In the case of agent neighbors' actions, similarly to the algorithm that creates networks, propensity to interact with information is different for different systems and is calculated for Facebook and Twitter separately. I-th agent's propensity is assumed to be a function of media advertising campaign intensity mentioned above and the number of neighbors that have already liked, commented or shared information in a social network where agents are connected with. Moreover, probability that social media users would actually see information that their peer interacted with is based on, in case of Facebook, EdgeRank algorithm. Due to the lack of detailed information on its formulation and often changes that are applied to it, following model assumes constant values defining the probability that agent's action will be seen by its neighbor. Propensity for i-th agent in period tis defined by the equation:

$$Propensity_{i,t_k,N} = \frac{2}{1 - e^{-Z_{i,tk,N}}} - 1 + Att,$$

where Z function differs for Facebook and Twitter. For Facebook it is given in equation:

$$Z_{i,t_k,F} = \log\left(1 + Frequency_{i,t_k}\right) * \sum_{i=1}^{M} \sum_{a \in A_F} PF_a,$$

while for Twitter it is formulated as in equation:

$$Z_{i,t_k,T} = \log\left(1 + Frequency_{i,t_k}\right) * \sum_{i=1}^{M} \sum_{a \in A_T} PT_a.$$

In the above equations M indicates number of neighbors an i-th agent has, while A_F and A_R indicate sets of possible actions for Facebook and Twitter users respectively.

It is assumed, that i-th agent will interact with information if and only if his personal propensity exceeds threshold level specified for each action and presented in the following part of this section. Each agent that has not interacted with brand's communication has his propensity recalculated in the next period due to changing situation and his propensity is being compared to threshold levels once again.

Model presented in the article incorporates some of Little's guidelines for marketing models. His recommendations assume that marketing effect of communication is a nonlinear function (1) and agents tend to forget advertisements over time (2). Moreover, model should capture competitors' activities (3) and different customers' responsiveness to communication over time, due to copy changes and media environment development (4) (Little 1979). Model of diffusion, proposed in the article, includes features (1) and (2), while (3) and (4) tend to be unnecessary in the social media behavior simulation. Resulting from Little guidelines, agents in model, forget information seen in the previous periods, as per the relation: $Frequency_{i,t_k} = Frequency_{i,t_{k-1}} * e^{-0.001}$.

Single tick in the modeled system refers to 1 hour in the real world. Process does not have an explicit lifespan, but its stoppage time depends on the diffusion dynamics. Simulation stops if none of the monitored variables has grown in the last 200 periods (hours). Reaches of mentioned social media networks were calculated in line with Polish online usage report Megapanel for September 2014. Megapanel is a panel research conducted by Gemius company based on cookie data. Each panelist's cookie is being tracked down by Gemius in order to obtain viewership statistics on Internet websites. Data, that is presented on a monthly basis, is being weighted in order to ensure representativeness on the national level and delivers to marketers the most vital metrics regarding digital marketing, such as websites' reach and affinity, cross reach and many others. The research results are widely approved by marketing representatives in Poland. Their significance is additionally emphasized by the fact, that almost all settlements between media agencies and their clients are based on Megapanel outcomes.

According to this study, out of Facebook and Twitter users, 82% use only Facebook, 1% use only Twitter and 17% use both systems. Three types of social media network users can be interconnected with edges according to the guidelines mentioned in Table 1. The system consists of total 1000 agents, whose usage of particular social media networks is in line with the above numbers. Such world size ensures stability of results and is possible to be computed on an available machine.

According to the recent study, people, on average, use social media for 20 minutes a day (Smith 2014). To model daily activity routine, agents on each period can change their state: all agents active in the previous period turn inactive while inactive agents are being activated randomly with probability 1/72.

Probabilities of information being seen by peer users are, assumed for the purposes of the article, due to lack of satisfactory data. They are presented in Table 2.

Table 2: Probabilities of interactions in social media.

Social media network	Action	Probability, that neighbor agent would see an information	Indicator of probability
Facebook	Like	0.50	PFL
Facebook	Comment	0.65	PFC
Facebook	Share	0.75	PFS
Twitter	Comment	0.95	PTC
Twitter	Share	0.95	PTS

The parameter 'Att' term refers to agents' attitude towards novelties. According to Rogers (2003), people in general can be grouped into 5 types:

1) Innovators – 2.5% of total population;
2) Early adopters – 13.5%;
3) Early majority – 34%;
4) Late majority – 34%;
5) Laggards – 16%.

Based on the Rogers theory, 'Att' parameter follows equation:

$$Att = \begin{cases} 0.10 \; for \; Innovators \\ 0.05 \; for \; Early \; Adopters \\ 0 \; for \; Early \; and \; Late \; majorities \\ -0.75 \; for \; Laggards \end{cases}$$

Despite being developed in 1960s, Rodgers theory may be a proper, simple approach to modeling novelty adoption in modern societies as well. Shuen (2008) states that all five factors influencing adoption rate defined by Rodgers (Relative advantage, Compatibility, Complexity, Trialability, Observability) apply to Facebook adoption as a social network. Author also concludes, that Rodgers theory lacks of social influence or "peer-to-peer pressure" mechanics, that are hard to quantify.

Research on diffusion dynamics, conducted by Goet, Watts, and Goldstein (2012), as one of results confirm, that the most important factor conditioning final reach of novelty is the closest neighborhood of agents in network. Usually, diffusion covers only direct neighbors of information source, while global spread of information is rare and unusual. As 'Att' term is just an auxiliary factor in such simplistic approach to information diffusion, Rodgers theory regarding agents' attitude may be considered as valid.

In order to keep consistent naming convention, Twitter actions' names are similar to their Facebook equivalents. However, agreed names for Twitter actions are: reply for comment and retweet for share. Threshold levels are presented in equations below and are based on information from service Digital Marketing Ramblings that is monitoring Facebook statistics (Smith 2014). For Twitter, due to lack of satisfactory source of information, threshold levels are assumed to be 0.7 for reply and 0.65 for retweet.

$$P(i = like \; on \; Facebook \; |Propensity_{i,t_{k,F}} > 0.66) = 1$$
$$P(i = comment \; on \; Facebook \; |Propensity_{i,t_k,F} > 0.80) = 1$$
$$P(i = share \; on \; Facebook \; |Propensity_{i,t_k,F} > 0.73) = 1$$
$$P(i = comment \; on \; Twitter \; |Propensity_{i,t_k,T} > 0.70) = 1$$
$$P(i = share \; on \; Twitter \; |Propensity_{i,t_k,T} > 0.65) = 1$$

Unfortunately, Polish digital market is still not well investigated and because of that there is lack of research and insight focused on social media networks phenomena. Especially, a single source data would be needed, delivering complex information on social media users, their behavior and metrics that shall be helpful in describing agents within the model. A survey targeted on the social media users would in the best case scenario result in populating the models' environment with agents of different demographic profiles and therefore, different habits and behavior pattern. Insufficient data availability was the main cause of taking many assumptions, but regardless of that the core of proposed modeling framework remains valid. As soon as detailed data on the subject is gathered and analyzed, the model can be easily tuned-up to obtain stable results reflecting real state of world.

One of the methods used in order to ensure, that simulated model is as close to the reality as possible was usage of Megapanel research, that only provides fractions of users that are members of a certain network. However, from the other point of view such flexibility in selecting data sources used in model's calibration may be perceived as an advantage.

What is more, diffusion mechanics within the model are biased by lack of information on the exact algorithms that are being used by social media platforms to display information to their users, which are often being modificated. Obviously, it is one of the most vital elements of the whole system and therefore, from the company's point of view, it should remain confidential.

The whole model specification was also presented in line with ODD protocol (Grimm et al. 2010), that is a standard for summarizing agent based models. The ODD Protocol for the described model can be found here: http://www.bogumilkaminski.pl/pub/ond_model.pdf.

3 EXPERIMENT WITH THE MODEL

A company would like to simulate an expected reach of their social media campaign and the number of engaged users. Company understands reach as a fraction of social media users who have seen the information, while engaged fans are those, who interacted with the brand's content. A reach is being calculated regardless of a network an agent is a member of. That means an overall social media reach does not increase while an agent who already have seen information in one network sees it on another.

A company owns a brand's profile in social media, but wants to know what an optimal strategy of targeting consumers would be. Such strategy ought to maximize both: post's reach and generated engagement.

A company would also like to know, what an optimal level of advertising budget supporting social media is.

Model of two hypothetical overlapping small world networks with 331 elements is presented on Figure 1. Out of those 330 users, 152 use only Facebook (green nodes), 152 use only Twitter (blue nodes) and 26 use both systems (red nodes). Black star indicates brand profile position in the simulated world.

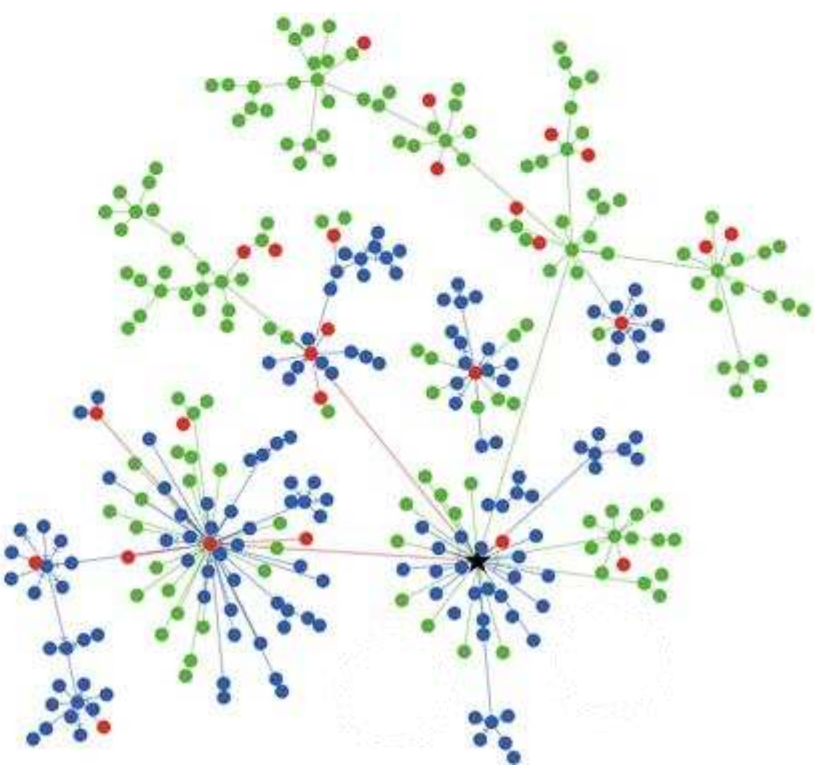

Figure 2: Two network model appearance.

In order to obtain valid results, model has been calibrated with parameters' values mentioned previously. The parameter range for conducted analysis is presented in Table 3. Parameter BPP (Brand profile's placement) can indicate its positions on Twitter only (TW), Facebook only (FB) or both systems (both). Budget for digital communication stays within range from 100k PLN to 2 million PLN, which seems to be a reasonable decision while taking into account Polish digital market specificity. All parameters regarding particular system's elements sizes (Facebook, Twitter and both systems size) are greater than 50 and lower than 950, as it is assumed that the whole system consists of 1 000 agents. Each input settings has been simulated 20 times to ensure results' stability.

Table 3: Parameter value range for simulation.

Parameter	Value (range)	Parameter	Value (range)
BPP	[TW, FB, both]	FB size	[25, 50, …, 950]
Budget in k PLN	[100, 200, … , 2000]	TW size	[25, 50, …, 950]
CPM in PLN	15	Both size	[25, 50, …, 950]

Model results confirm that higher number of both system users drives additional information reach and engagement, as more people interact with it. Moreover, it has a positive impact on engagement rate, defined as number of users who interacted with information to overall number of users who have seen it, which is presented on Figure 2.

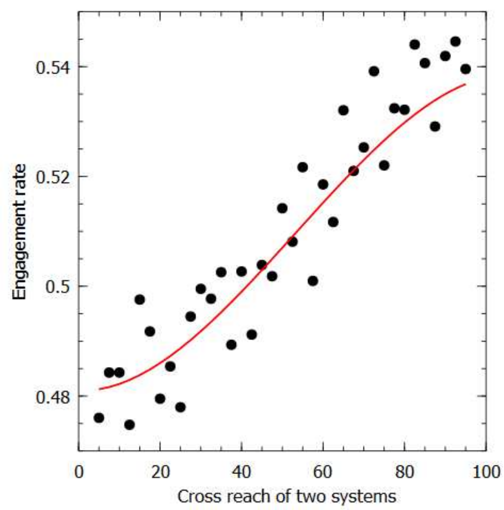

Figure 3: Reach of marketing information in social media dependent on level of both networks' overlapping.

Model results presented on Figures 3 and 4 state that information diffusion is more effective in networks characterized with lower distances between users. What's more, the second recommendation that can be formulated from such observation is, that communication in social media should be well targeted and its aim is to convert people with higher number of neighbors as that help to 'open' network hubs for future diffusion.

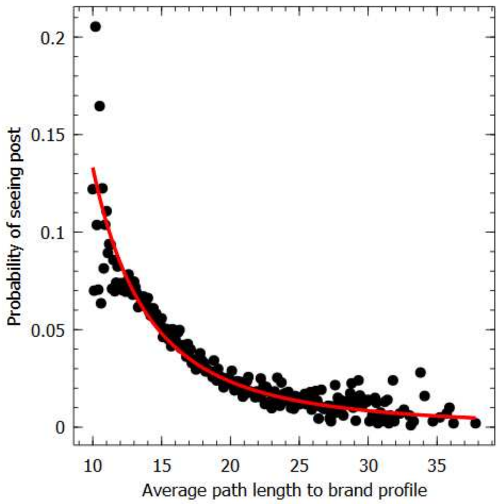

Figure 4: Information reach dependent on brand profile's position in network.

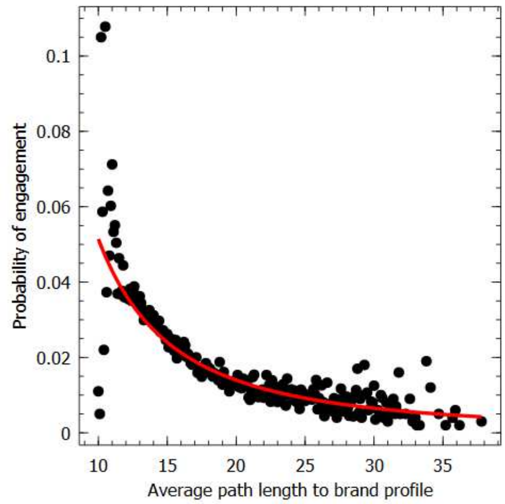

Figure 5: Information engagement dependent on brand profile's position in network.

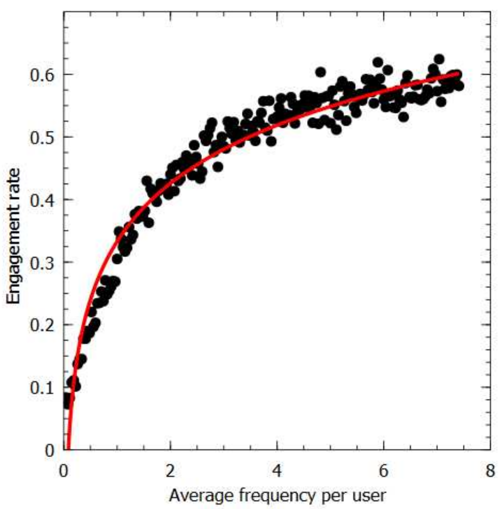

Figure 6: Reach of marketing information in social media dependent on expected frequency of awareness building marketing campaigns.

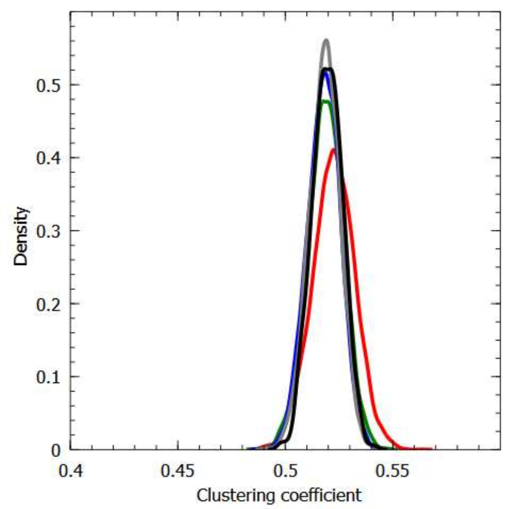

Figure 7: Density of estimated clustering coefficient for different levels of networks' overlap.

Brand's digital communication has a great impact on social media engagement rate. Being given a cost perspective of a digital campaign and a value that an engaged users for a company is, one can easily estimate the optimal budget expenditure in terms of social media engagement driving. Furthermore, it is possible to calculate bounds on intensity driving positive profit outside which an investment is being lost.

Average clustering coefficient of a small-world of Albert and Barabasi has been investigated by Klemm and Eguiluz (2001) and for different real-world networks it varies from 0.7 to 0.79. In case of simulated social media networks system, an obtained clustering coefficient's value, calculated in line with Watts and Strogatz (1998), is equal to 0.520 with standard deviation equal to 0.009. Calculation was made under assumption, that in case of network with mixed types of edges, all of them should be treated as undirected ones (Albrecht 2013). A Figure 7 presents distributions of estimated clustering coefficients for different levels of networks overlap. Colors indicate different levels of overlap red: <20%, green: 20-40%, blue: 40-60%, grey: 60-80%, black: over 80%.

4 CONCLUDING REMARKS

Despite sharp growth of Twitter user base, Facebook shall remain brand's first choice to place brand's profile in social media. Modeling results that were calculated for Polish social media system suggest that Twitter development will have a positive impact on information diffusion range in social media, as the networks' cross reach will increase, but Facebook will still be particularly responsible for driving reach. The main factor conditioning limited impact of Twitter on marketing information reach is unequal position between its members, as their relation is often asymmetric. Moreover, from the marketing point of view, despite possibility of uploading pictures and videos, Twitter posts are limited to 140 words of marketing announcement that may have an impact on users' responsiveness, but a detailed study on that issue is needed. First of all, it is the case particularly for weaker brands that are not strongly connected to easily recognizable advertisement copy.

It is essential for a brand that wishes to drive reach in social media, to ensure a sufficient digital media budget for awareness building campaign. Saturation level of intensity for such campaigns is placed at 5-6 contacts per user but it may vary depending on media campaign costs.

One of possible developments of the article should be generalization of network constructing algorithm for N overlapping networks. Social media are constantly in motion and once in a while new social media network gains popularity. Moreover, sharing information between networks is getting easier and now can be done in one click. Building model for more than two networks can be a useful tool to plan an integrated marketing campaign in the complex social media networks' environment.

Model parameters would predict real life social network if a dedicated research was conducted. Survey results would serve as initial parameters regarding user-specific measures such as thresholds' and probabilities' levels. Furthermore, in depth analysis of people's propensity determinants would be useful while constructing diffusion function. Due to the relatively new Facebook policy allowing its users to determine which friends' content is to be hidden in the results table, Facebook neighbor network should have an opportunity to switch some edges from being undirected to the directed state. An impact of that policy change on post's reach can be computed easily.

The level of overlap between networks has a crucial impact on system's metrics, such as clustering coefficient. What is important, the greater level of overlap results in lower estimated average clustering coefficient and it is less likely to obtain a network with hubs or clusters. Average values of clustering coefficient for different levels of overlapping vary from 0.518 to 0.523.

REFERENCES

Albrecht, M. 2013, Network Analyzer Online Help, Accessed June 19, 2015. http://med.bioinf.mpi-inf.mpg.de/netanalyzer/help/2.6.1/

Alexa. 2015. The Top 500 Sites on the Web. Accessed January 20, 2015. http://www.alexa.com/topsites

Barabasi, A.L, R. Albert, and H. Jeong. 1999, "The Diameter of the World Wide Web." Nature 401:130–131

Barabasi, A.L. and R. Albert. 1999, "Emergence of Scaling in Random Networks", Science 286:509–512

Burcher, N. 2012, Paid, Owned, Earned: Maximizing Marketing Returns in a Socially Connected World, London: Kogan Page Limited

eMarketer. 2013a. Worldwide Social Network Users: 2013 Forecast and Comparative Estimates. Accessed November 30, 2014. http://www.emarketer.com/Article/Social-Networking-Reaches-Nearly-One-Four-Around-World/1009976 eMarketer. 2013b. US Total Media Ad Spend Inches Up, Pushed by Digital. Accessed November 30, 2014.http://www.emarketer.com/Article/US-Total-Media-Ad-Spend-Inches-Up-Pushed-by-Digital/1010154

eMarketer. 2013c. Mobile User Bases at Facebook and Twitter Keep Growing. Accessed November 30, 2014.http://www.emarketer.com/Article/Mobile-User-Bases-Facebook-Twitter-Keep-Growing/1010135

Evans, D. 2010, Social Media Marketing: The Next Generation of Business Engagement, 2nd ed. New Jersey: Sybex

Ferligoj, A., L. Kronegger, F. Mali, T.A.B. Snijders, and P. Doreian. 2015, "Scientific Collaboration Dynamics in a National Scientific System", Scientometrics

Global Web Index. 2014. Digital vs. Traditional Media Consumption Summary. Accessed December 5 2014. http://insight.globalwebindex.net/hs-fs/hub/304927/file-1414878665-pdf/Reports/GWI_Media_Consumption_Summary_Q3_2014.pdf?submissionGuid=78487a74-bc3f-4dea-a5a3-7d9af08b229a

Goet, S., D.J. Watts and D.G. Goldstein. 2012, "The Structure of Online Diffusion Networks" In Proceedings of the 13th ACM Conference on Electronic Commerce, 623-638, New York: Association for Computing Machinery

Goodall, D. 2009, Owned, Bought and Earned Media. Accessed December 5, 2014 http://danielgoodall.com/2009/03/02/owned-bought-and-earned-media/

Grimm, V., U. Berger, and D.L. DeAngelis , J.G. Polhill, J. Giske and S.F. Railsback 2010, "The ODD Protocol: a Review and First Update", Ecological Modelling 221:2760-2768

Kim, J.Y. and K.I. Goh 2013, " Coevolution and Correlated Multiplexity in Multiplex Networks", Physical Review Letters 111: 058702

Klemm, K. and V.M. Eguiluz 2001, "Highly Clustered Scale-Free Networks", Physical Review 65:036123

Kunegis, J., M. Blattner, and C. Moser. 2013, "Preferential Attachment in Online Networks: Measurement and Explanations" In Proceedings of the 5th Annual ACM Web Science Conference, 205–214, New York: Association for Computing Machinery

Lee K.M., B. Min, and K.I. Goh. 2015, "Towards Real-World Complexity: An Introduction to Multiplex Networks", The European Physical Journal 88:1-20

Little, J. 1979, "Aggregate Advertising Model: The State of the Art.", Operations Research 27:629–667

Naylor, R.W., C. P. Lamberton, and M. W. Patricia. 2012, "Beyond the "Like" Button: The Impact of Mere Virtual Presence on Brand Evaluations and Purchase Intentions in Social Media Settings", Journal of Marketing 76:105–120

Rogers, E. 2003, Diffusion of Innovations, 5th ed. New York: Free Press

Shen, A. 2008, Web 2.0: A Strategy Guide, 1st ed. Sebastopol: O'Reilly Media

Stephen, A. and J. Galak. 2012, "The Effects of Traditional and Social Earned Media on Sales: A Study of a Microlending Marketplace", Journal of Marketing Research 49:624–639

Watts, D.J. and S. Strogatz. 1998, "Collective Dynamics of 'Small-World' Networks", Nature 393:440-442

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