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A MULTI-ASSET MONTE CARLO SIMULATION MODEL FOR THE VALUATION OF VARIABLE ANNUITIES

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ABSTRACT

A variable annuity is an insurance contract that contains financial guarantees. Due to the complexity of guarantees, there are no closed-form formulas to calculate the value of these guarantees. Insurance companies rely heavily on Monte Carlo simulation to calculate the value of the guarantees. However, almost all simulation software for variable annuity is proprietary, posing a substantial barrier to academic researchers who want to study the computational problems related to variable annuity. In this paper, we present a multi-asset Monte Carlo simulation model by using fund mappings and economic scenario generators. The simulation model is realistic in the sense that it reflects the current practice in industry. We also implement the simulation model in Java as open source software, which can be used by students and researchers to investigate the computational problems arising from the variable annuity area.

1 INTRODUCTION

A variable annuity is an insurance contract between a policyholder and an insurance company. A variable annuity is very similar to a mutual fund in the sense that the policyholder's money is invested in a set of equity/bond funds. Unlike a mutual fund, however, a variable annuity includes certain guarantees or riders, namely guaranteed minimum death benefits (GMDB) and guaranteed minimum living benefits (GMLB). There are several types of GMLB: guaranteed minimum accumulation benefits (GMAB), guaranteed minimum maturity benefits (GMMB), guaranteed minimum income benefits (GMIB), and guaranteed minimum withdrawal benefits (GMWB).

Since the guarantees embedded in variable annuities are complex and path-dependent, there are no closed-form formulas to calculate the fair market value of the guarantees except for simple cases (Gerber, Shiu, and Yang 2013). In practice, insurance companies rely on Monte Carlo simulation to calculate the fair market value of the guarantees. However, Monte Carlo simulation is computationally intensive for large portfolios of variable annuities (Gan 2013, Gan and Lin 2015). To address the computational problem, Gan (2013) proposed a metamodeling approach with a clustering algorithm (Gan 2011) and a kriging method (Isaaks and Srivastava 1990). Gan and Lin (2015) proposed another metamodeling approach with a universal kriging method (Caballero, Giraldo, and Mateu 2013) to address the problem under a stochastic-on-stochastic framework. However, the simulation model used in (Gan 2013, Gan and Lin 2015) is based on a single asset and assumes that all variable annuity policies are issued on the same day. In this paper, we present a more general Monte Carlo simulation model than the those used in previous studies. The objective of this work is to develop a piece of open source software that can be used by students and researchers in academic institutes to investigate the aforementioned computational problem arising from the variable annuity area.

2 A MULTI-ASSET MONTE CARLO SIMULATION MODEL

The multi-asset Monte Carlo simulation model consists of the following components: a fund mapping, a yield curve, a risk-neutral scenario generator, and a variable annuity cash flow projection engine. A fund mapping is used to map a mutual fund to a combination of tradable and liquid indices such as the S&P500 index. Economic scenario generators are used to simulate movement scenarios of the indices according to an asset model. There are two types of scenarios: risk-neutral and real-world. Risk-neutral scenarios are simulated under the risk-neutral measure; while real-world scenarios are simulated under the real-world measure. Risk-neutral scenarios are usually used to calculate the fair market values of financial derivatives such as the guarantees embedded in variable annuities. Real-world scenarios are usually used to calculate solvency capitals or evaluate hedging strategies.

The asset model is a multivariate Black-Scholes model with k indices $S^{(1)}$, $S^{(2)}$, ..., $S^{(k)}$. The risk-neutral dynamics are given by (Carmona and Durrelman 2006):

$$\frac{\mathrm{d}\,S_t^{(h)}}{S_t^{(h)}} = r_t\,\mathrm{d}t + \sum_{l=1}^{k} \sigma_{hl}\,\mathrm{d}B_t^{(l)}, \quad S_0^{(h)} = 1,\, h = 1, 2, \ldots, k \tag{1}$$

where $B_t^{(1)}$, $B_t^{(2)}$, ..., $B_t^{(k)}$ are independent standard Brownian motions, r_t is the short rate of interest, and the matrix (σ_{hl}) is used to capture the correlation among the indices. The risk-neutral scenarios of the mutual funds can be blended from those of the tradable indices.

Once we have the risk-neutral scenarios for all the mutual funds, we can project the cash flows of guarantees embedded in a variable contract along all paths according to contract specifications and the purpose of valuation. The value of the guarantees is the average present value of the cash flows.

3 SOFTWARE IMPLEMENTATION

The Monte Carlo simulation model is implemented in Java in an object-oriented way. The source code of this software can be downloaded from https://github.com/ganml/va. The software package also includes a variable annuity policy generator, a yield curve generator, and a risk-neutral scenario generator.

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