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CHANCE-CONSTRAINED SCHEDULING WITH RECOURSE FOR MULTI-SKILL CALL CENTERS WITH ARRIVAL-RATE AND ABSENTEEISM UNCERTAINTY

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ABSTRACT

We consider a chance-constrained two-stage stochastic scheduling problem for multi-skill call centers with uncertainty on arrival rate and absenteeism. We first determine an initial schedule based on an imperfect forecast on arrival rate and absenteeism. Then, this schedule is corrected applying recourse actions when the forecast becomes more accurate in order to satisfy the service levels and average waiting times constraints with some predefined probabilities. We propose a method that combines simulation with integer programming and cut generation to solve the problem.

1 INTRODUCTION

We consider inbound multi-skill multi-period call centers. The objective of the scheduling problem is to minimize the total cost of agents to be assigned to a set of predefined shifts while ensuring that certain constraints on the quality of service (QoS) are met. Common measures of QoS are the service level (SL), defined as the fraction of calls answered within a given time target, and the average waiting time (AWT) before the call is answered. Over a given time period, these performance measures are random variables.

Many studies assume a known arrival rate and no absenteeism, but this is unrealistic. The following authors consider stochastic arrival rates, with either queueing or fluid approximations. Whitt (2006) optimizes the staffing for single-skill centers with (uncertain) absenteeism. Gurvich et al. (2010) use chance constraints for targets on long-term abandonment ratio for multi-skill centers. Gans et al. (2012) optimize work shifts with recourse in a single-skill center.

We propose a methodology to solve a two-stage stochastic scheduling problem with recourse for multi-skill call centers with arrival rate and absenteeism uncertainty. In contrast with typical problem formulations that use constraints on the average QoS in the long run (see, e.g., the above references and Avramidis et al. 2010), we use probabilistic constraints on the daily SLs and AWTs. This type of constraint formulation is appropriate because these daily QoS values are random and the tail of their distributions is relevant. The model and methodology can handle other types of performance measures as well.

2 MODELS AND SCHEDULING PROBLEMS

We consider a call center with K call types, P periods and I agent groups. Let $\{1,\ldots,Q\}$ be the set of all admissible shifts which are specified via a matrix $\boldsymbol{A}_0$ of size $P\times Q$ whose element (p,q) is $A_{p,q}=1$ if an agent with shift q works in period p, and 0 otherwise. In Stage 1 (e.g., days or weeks in advance), the manager selects $\boldsymbol{x}=(x_{1,1}\ldots,x_{1,Q},\ldots,x_{I,1},\ldots,x_{I,Q})^{\mathrm{t}}$ where $x_{i,q}$ is the number of agents of group i with shift q (type (i,q), for short), which have cost $c_{i,q}$. These are the initial schedules. Let $(\boldsymbol{\Lambda},\boldsymbol{\Gamma})$ be the random arrival rates and levels of absenteeism, defined as $\boldsymbol{\Lambda}=(\Lambda_{1,1},\ldots,\Lambda_{1,P},\ldots,\Lambda_{K,1},\ldots,\Lambda_{K,P})$ where $\lambda_{k,p}$ is the arrival rate for call type k in period p, and $\boldsymbol{\Gamma}=(\Gamma_{1,1}\ldots,\Gamma_{1,Q},\ldots,\Gamma_{I,1},\ldots,\Gamma_{I,Q})$ where $\gamma_{i,q}$ is the number of agents of type (i,q) that are not showing up. The distribution of $\boldsymbol{\Gamma}=\boldsymbol{\Gamma}(\boldsymbol{x})$ depends on $\boldsymbol{x}$. The choice of $\boldsymbol{x}$ is made given initial distributions for $\boldsymbol{\Lambda}$ and $\boldsymbol{\Gamma}=\boldsymbol{\Gamma}(\boldsymbol{x})$.

In Stage 2, e.g., in the morning of the targeted day, suppose the arrival rates λ and the absenteeism vector γ are known. (On the poster, this will be generalized to a situation where they are not perfectly known, but just known with less uncertainty. Here we simplify due to space constraints.) The number of agents of type (i,q) showing up is $w_{i,q} = x_{i,q} - \gamma_{i,q}$. Recourse actions can be applied based on this new information. Let $\boldsymbol{x}^+(\lambda,\gamma) = (x^+_{1,1}...,x^+_{1,Q},...,x^+_{I,1},...,x^+_{I,Q})^{\mathrm{t}}$ and $\boldsymbol{x}^-(\lambda,\gamma) = (x^-_{1,1}...,x^-_{1,Q},...,x^-_{I,1},...,x^-_{I,Q})^{\mathrm{t}}$, where $x^+_{i,q}$ and $x^-_{i,q}$ are the numbers of schedules of type (i,q) that we add and remove, respectively. Suppose that for type (i,q), adding an agent increases the cost by $c^+_{i,q} > c_{i,q}$ and removing an agent reduces the cost by $c^-_{i,q} < c_{i,q}$. After the recourse, the new number of agents of type (i,q) is $x_{i,q}(\lambda,\gamma) = w_{i,q} + x^+_{i,q} - x^-_{i,q}$. Let $\boldsymbol{c}, \boldsymbol{c}^+$, $\boldsymbol{c}^-$ and $\boldsymbol{x}(\lambda,\gamma)$ be the vectors with components $c_{i,q}, c^+_{i,q}$, $c^-_{i,q}$ and $x_{i,q}(\lambda,\gamma)$, respectively.

The staffing vector corresponding to $\boldsymbol{x}(\lambda,\gamma)$ is $\boldsymbol{y}(\lambda,\gamma) = \boldsymbol{A}\boldsymbol{x}(\lambda,\gamma)$, where $\boldsymbol{A}$ is a block-diagonal matrix with I identical blocks $\boldsymbol{A}_0$. Given $\boldsymbol{y}(\lambda,\gamma)$, the SL and AWT of calls of type k in period p are random variables $S_{k,p}(\boldsymbol{y}(\lambda,\gamma))$ and $W_{k,p}(\boldsymbol{y}(\lambda,\gamma))$. The SLs and AWTs aggregated for period p, call type k, and overall, are denoted $S_{0,p}$, $W_{0,p}$, $S_{k,0}$, $W_{k,0}$, $S_{0,0}$, $W_{0,0}$, respectively.

The only assumption we make on the stochastic model is that we can simulate the call center over one day to generate realizations of these random variables. Let $s_{k,p}$ and $w_{k,p}$ be the corresponding SL and AWT targets, respectively. The chance constraints require that these targets are reached with probabilities at least $r_{k,p}$ and $v_{k,p}$. The problem can be formulated as

$$\text{minimize} \quad \mathbb{E}_{\lambda,\gamma}\left\{\left[\boldsymbol{c}(\boldsymbol{x}-\gamma) + \boldsymbol{c}^+\boldsymbol{x}^+(\lambda,\gamma) - \boldsymbol{c}^-\boldsymbol{x}^-(\lambda,\gamma)\right]\right\}$$

subject to:

$$\begin{aligned}
\boldsymbol{A}\left(\boldsymbol{x}-\gamma+\boldsymbol{x}^+(\lambda,\gamma)-\boldsymbol{x}^-(\lambda,\gamma)\right) &\geq \boldsymbol{y}(\lambda,\gamma), \quad \forall(\lambda,\gamma);\\
\mathbb{P}[S_{k,p}(\boldsymbol{y}(\lambda,\gamma)) \geq s_{k,p}] &\geq r_{k,p}, \quad p=0,...,P;\ k=0,...,K;\ \forall(\lambda,\gamma);\\
\mathbb{P}[W_{k,p}(\boldsymbol{y}(\lambda,\gamma)) \leq w_{k,p}] &\geq v_{k,p}, \quad p=0,...,P;\ k=0,...,K;\ \forall(\lambda,\gamma);\\
\boldsymbol{x}-\gamma+\boldsymbol{x}^+(\lambda,\gamma) &\geq \boldsymbol{x}^-(\lambda,\gamma), \quad \forall(\lambda,\gamma);\\
\boldsymbol{x},\boldsymbol{y}(\lambda,\gamma),\boldsymbol{x}^+(\lambda,\gamma),\boldsymbol{x}^-(\lambda,\gamma) &\geq 0 \text{ and integer}, \quad \forall(\lambda,\gamma).
\end{aligned} \tag{P}$$

3 SIMULATION-BASED OPTIMIZATION ALGORITHMS

The objective in (P) is replaced by a sample average approximation (SAA) built by drawing m scenarios from the distribution of $(\boldsymbol{\Lambda},\boldsymbol{\Gamma}(\boldsymbol{x}))$, where $\gamma_{i,q}$ is a function of $x_{i,q}$. Each $(\lambda,\gamma(\boldsymbol{x}))$ scenario defines a second stage program, with corresponding nonlinear chance constraints on SL and AWT. The distributions of the SLs and AWTs are approximated by simulating the call center operations over n days, and the chance constraints are evaluated using SAA over the n days. The second-stage SAA problem is linearized using a cutting plane method as in Avramidis et al. (2010). More precisely, the non-linear constraints on the SLs and AWTs are replaced by a set of linear cuts. At each iteration, a new cut is added using the sub-gradients of the approximated probability functions. The algorithm stops when a feasible solution is found. We will show numerical examples on the poster.

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