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A FAST APPROXIMATION OF AN INHOMOGENEOUS STOCHASTIC LANCHESTER MODEL

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ABSTRACT

Combat modeling is a key area of military science and related research. Here, we propose a moment matching scheme with a modified stochastic Lanchester-type model. An experiment shows that the proposed scheme makes approximations more rapidly while maintaining a high level of accuracy compare to the Markovian model.

1 Introduction

Combat modeling, as first proposed by Lanchester (1916), is a fundamental area in military research. The Lanchester equation describes a homogeneous battle simply and clearly. However, since it does not consider randomness, it has limited expressiveness of battles. Therefore, many researchers have studied the stochastic Lanchester-type combat model. Amacher and Mandallaz (1986) attempted to represent stochasticity using a random kill rate parameter which contained brownian motion. The most commonly used model is the Markov model by Taylor, but this model is hard to expand into inhomogeneously armed case because of the curse of dimensionality, and inevitably the computation cost gets higher.

In the present paper, we suggest a difference equation modeled from the Markov model and propose a moment matching scheme which can be widely use for any stochastic Lanchester-type model.

2 Stochastic Formulation

Taylor (1980) developed a stochastic type of combat model with a Markov model. In this article, we propose a new but familiar model which is modeled after the Chapman-Kolmogorov equation, a common approach to express stochasticity. The modified difference equation considered here is shown below.

$$\begin{aligned} B_{t+\Delta t} - B_t &= -\mathbb{1}_{aR_t\Delta t} \\ R_{t+\Delta t} - R_t &= -\mathbb{1}_{bB_t\Delta t} \end{aligned} \tag{1}$$

Here, $\mathbb{1}_p$ is a Bernoulli random variable with a probability p, and a and b are positive constants.

The main reason behind the use of a stochastic model is to determine not only the average behavior, i.e. the mean of each side, but also the covariance structure. This involves much more information than that used in a deterministic model. From (1), we compute first and second moments to obtain the mean and covariance. We denote the moment vector $M_t = (E[B_t], E[R_t], E[B_t^2], E[R_t^2], E[B_tR_t])$. The moment vector at time $t+\Delta t$, $M_{t+\Delta t}$ can be calculated by M_t. Therefore, (1) can be rewritten in the following form with the appropriate value of V, which is derived from (1).

$$M_{t+\Delta t} = (1+V\Delta t)M_t + o(\Delta t^2) \tag{2}$$

As $\Delta t \to 0$, $o(\Delta t^2)$ becomes 0 and the following holds.

$$M_t = expm(V)M_0 \tag{3}$$

where *expm* is an exponential operator for the matrix. The moments at time t can be evaluated by the initial moments, and the mean and covariance at time t can easily be computed by this moment vector.

The difference equation (1) represents a homogeneously armed case, and it is simple to expand this into an inhomogeneously armed case. The mean and covariance in this case as well can be calculated by the moment vector in (3) with a suitable value of V.

$$\begin{aligned} B_{i,t+\Delta t} &= B_{i,t} - \mathbb{1}_{\sum_{k=1}^{N} a_{ki} R_{k,t} \Delta t} \quad \forall i = 1...M \\ R_{j,t+\Delta t} &= R_{j,t} - \mathbb{1}_{\sum_{l=1}^{M} b_{lj} B_{l,t} \Delta t} \quad \forall j = 1...N \end{aligned} \tag{4}$$

3 Experiments

In this analysis, we conduct a crude Monte Carlo simulation for the model and compare the results to an approximation of the moments. In (1), B_t and R_t can be expressed as the sum of a bivariate Bernoulli trial with different probabilities; therefore, we can roughly approximate (B_t, R_t) as a bivariate Gaussian distribution with first and second moments. The accuracy levels and the computation times of the three models for homogeneously armed case and two models for inhomogeneously armed case are compared in Table 1. Taylor's model is used as a benchmark for Model 1,2,3, and Model 4 used as a benchmark for Model 5 because Taylor's model is valid only for homogeneously armed case. The accuracy is represented by the degree of Kullback-Leibler divergence from the benchmark.

Table 1: Kullback-Leibler divergence of five models from each benchmarks. Model 1 is (1) with a crude Monte Carlo(Rep:10^6), Model 2 is (1) with a moment matching scheme, and Model 3 is the model from Amacher and Mandallaz (1986) for homogeneously armed case. Model 4 is a crude Monte Carlo(Rep:10^6) and Model 5 is (4) with a moment matching scheme for inhomogeneously armed case.

Armed type	Homogeneously (1vs1)			Inhomogeneously (2vs2)	
Time	Model 1	Model 2	Model 3	Model 4	Model 5
1	0.000327	0.021887	0.046485	-	0.012664
2	0.000576	0.013663	0.02981	-	0.007065
3	0.000879	0.01022	0.017843	-	0.005565
Computation time(s)	373s	$\leq$ 0.1s		3338s	0.9s

The intent of this paper is to show a rapid approximation scheme using moments while maintaining a fairly high level of accuracy. Although the accuracy is reduced somewhat, the computation time is fairly short, and this difference in the computation time grows exponentially when the model is expanded to the inhomgeneously armed type. Further more, we can use the moment matching scheme suggested in (3) for any other Lanchester-type model with the proper matrix V. This can be used to evaluate and optimize complex problems in the military field like resource allocation problem with inhomogeneously armed case.

REFERENCES

Amacher, M., and Mandallaz D. 1986. "Stochastic version of Lanchester equations in wargaming". *European Journal of Operational Research* 24.1: 41-45

Lanchester, F. W. 1916. "Aircraft in warfare: The dawn of the fourth arm". *Constable and Company*, London

Taylor, James G. 1980. "Force-onforce attrition modelling". *Military Applications Section, Operations Research Society of America, c/o Ketron*