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GREEN SIMULATION DESIGNS FOR REPEATED EXPERIMENTS

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ABSTRACT

In this article we present the concept of green simulation, which views simulation outputs as scarce resources that should be recycled and reused. Output recycling, if implemented properly, can turn the computational costs in an experiment into computation investments for future ones. Green simulation designs are particularly useful for experiments that are repeated periodically. In this article we focus on repeated experiments whose inputs are observations from some underlying stochastic processes. Importance sampling and multiple importance sampling are two particular output recycling implementations considered in this article. A periodic credit risk evaluation problem in the KMV model is considered. Results from our numerical experiments show significant accuracy improvements, measured by mean squared errors, as more and more outputs are recycled and reused.

1 Introduction

As a decision support tool, simulation experiments are usually designed for specific tasks and simulation outputs are often discarded when the designated tasks are completed. Consequently, in the field of design and analysis for simulation experiments, studies have usually been done for "simulation from scratch", which are designs that take little consideration of using preexisting simulation outputs or storing current simulation outputs for future usage. For instance, space-filling designs such as random sampling, 2^k factorial, and Latin Hypercube (LH), etc select design points without considerations of any other design point that exists in the design space (see Sanchez (2005), for example). There is little literature on experiment designs that consider storing simulation outputs to facilitate other tasks than the one for which the simulation is designated.

In many applications, simulation experiments, whether deterministic or stochastic, can be expensive to run because the increasing complexity of systems being modeled often outraced the advances in computational power. For example, Craig et al. (1997) consider an oil reservoir simulator using finite element grid in which simulating one output can take one to three days. In reinsurance industry, stochastic simulations of rare catastrophes could be time consuming to produce a result of demanded accuracy. In these cases, simulation outputs are scare resources and should be properly stored and reused for future experiments. Even for simulations that are not as computationally demanding, preexisting outputs from the same or similar experiments may provide useful information to improve the efficiency of the current experiment. Output recycling, if implemented properly, could turn computational costs in an experiment into investments for future experiments. As suggested by an anonymous reviewer, the idea of green simulation and output recycling could potentially be applicable to various optimization via simulation (OvS) algorithms (see Fu (1994), Hong and Nelson (2009) and references therein). In particular, in some OvS algorithms, such as stochastic ruler and its variants (see Yan and Mukai (1992) and Alrefaei and Andradóttir (2001), for

example), a common simulation model is used in each iteration at possibly different solutions. Hence one should recycle outputs from previous iterations to more accurately estimate the current objective and potentially be able to terminate the algorithm early. Maggiar et al. (2015) propose an application of output recycling in the context of stochastic optimization where the underlying function evaluations are computationally intensive.

We consider green simulation experiment designs in which outputs are recycled from one experiment to another to increase the efficiency of the latter. Although the idea of green simulation can be applied in more general settings, in this paper we limit our attentions to experiments that are repeated periodically under the same setups but different inputs, where the recycling scheme is seen to be the most effective. Repeated simulation experiments are not uncommon in practice. For example, risk management simulations such as those presented in Liu et al. (2010) and Liu and Staum (2010) may be repeated periodically to reevaluate risk exposures as market conditions changes over time. Simulations based on asset-liability management models (see Gerstner et al. (2008) and references therein, for example) may be run periodically due to changes in systemic and idiosyncratic risk factors. In repeated simulations, outputs in any previous experiment contribute to the understandings of the simulation model, i.e., the input-output relationship, and hence provides guidance to the current experiment. Metamodeling methods such as stochastic kriging (see Ankenman et al. (2010)) can be possible implementations of output recycling (see Feng (2016), for example). In this article we consider importance sampling, particularly multiple importance sampling (see Veach and Guibas (1995) and Owen and Zhou (2000)), to recycle simulation outputs from previous experiments. Our numerical experiments show that the recycling scheme increases the accuracy of the estimator over time even when a constant number of random samples are generated in each period.

The rest of this article is organized as follows. In Section 2 we present the mathematical framework in which green simulation is considered and the methods that are used to implement output recycling. In Section 3 we consider a periodic credit risk evaluation example and present numerical results of the example. In Section 4 we summarize the article and provides future outlooks for green simulation.

2 GREEN SIMULATION: OUTPUT RECYCLING IN REPEATED EXPERIMENTS

Let $\{X_k : k = 1,2,\cdots\}$ be a discrete-time Markov process with state space $\mathscr{X} \subseteq \mathbb{R}^n$ and known transition probability measure φ. We define a function $h(y;x)$ such that for any fixed $\bar{x} \in \mathscr{X}$ the function $h(y;\bar{x})$ is a well-defined probability density function with support $\mathscr{Y} \subseteq \mathbb{R}^d$. For any fixed time t_k, let $h(y;X_k)$ be the conditional density for random vector Y_k given the state X_k. Given the observed state x_k at time t_k, we are interested in approximating

$$f(x_k) = \mathbb{E}[F(Y_k)|X_k = x_k] = \int_{\mathscr{Y}} F(y)h(y;x_k)\,\mathrm{d}y \tag{1}$$

via simulation. The function $F(\cdot) : \mathscr{Y} \mapsto \mathbb{R}$ represents the simulation model of interest. We envision that output recycling is particularly useful in cases where the simulation model $F(\cdot)$ is computationally intensive.

The idea of green simulation can be broadly applicable with different frameworks and implementations. In this article we present a framework that is applicable to repeated simulations such that:

1. The same simulation model, $F(\cdot)$, is used when an experiment is launched at each time t_k, $k = 1,2,\cdots$.
2. The underlying stochastic process, $\{X_k : k = 1,2,\cdots\}$, affects the simulation model in that the distribution of the random vector Y_k depends on the state X_k.

We are interested in approximating (1) using not only outputs from the experiment launched at time k but also those from all experiments that were launched previously, if any. We assume that the simulation model depends on the state variable X_k only via the distribution of the random vector $Y_k \sim h(y;X_k)$. Although the methodologies presented in this article can be applied in more general settings, we restrict our attentions to the above framework for ease of exposure. For ease of discussions, hereinafter we will refer to time k as the

current time and the experiment launched then as the *current experiment*. In addition, let $Y_k^{(j)}$ be random samples of the random vectors in the current experiment, we will refer to $\left\{F\left(Y_k^{(j)}\right) : j = 1, \cdots, N_k\right\}$ as the outputs from the current experiment, or simply the *current outputs*.

To put the above abstract settings into perspective, consider the following repeated risk evaluation problem that may occur in financial applications. Suppose one needs to evaluate the risk exposure of a given portfolio via simulation to assess the adequacy of its risk capital. The random vectors Y_k, used in the simulation model $F(\cdot)$ depend on the state variables X_k, for example, certain stock prices. Given the current stock prices x_k at time k, a simulation experiment is launched and random vectors $Y_k^{(j)}$, $j = 1, \cdots, N_k$ are simulated. The simulation output $F\left(Y_k^{(j)}\right)$ is the indicator of incurring a high loss due to default given the j-th random vector. The sample average of all these outputs is regarded as the standard Monte Carlo estimate of the risk measure in question. Suppose that such risk evaluation is conducted periodically to ensure consistent compliance to certain regulatory requirements. Simulation outputs from one experiment contains useful information for another because the risk value evaluations in the experiments are the same. One should seek ways to recycle and reuse simulation outputs from all previous experiments to increase the efficiency of the current experiment.

2.1 Standard Monte Carlo

Given an observation of the current state $X_k = x_k$ at time t_k, $k = 1, 2 \cdots$. Suppose $Y_k^{(1)}, \cdots, Y_k^{(N_k)}$ are sampled from $h(y; x_k)$, it is common to use the standard Monte Carlo (SMC) estimator of $f(x_k)$ of the form

$$\hat{f}^{SMC}(x_k) = \frac{1}{N_k} \sum_{j=1}^{N_k} F\left(Y_k^{(j)}\right). \tag{2}$$

For fixed x_k, $\hat{f}^{SMC}(x_k)$ is a unbiased estimator and the convergence of $\hat{f}^{SMC}(x_k)$ to $f(x_k)$ as the sample size $N_k \to \infty$ is elementary by the strong law of large number.

As (2) reveals, the current SMC estimator uses only the N_k outputs generated in the current experiment while outputs from previous experiments are ignored, even when available. Moreover, in traditional experiment designs the current outputs are usually discarded when the designated tasks at the current time are completed.

In repeated simulations, since the simulation model is the same at all times so outputs from one experiment contain valuable information for another. Therefore recycling simulation outputs from previous experiments, if available, could significantly increase the efficiency of the current one. If the simulation model is complex, the computations involved in $F(y)$ could be much more time-consuming than evaluating the densities $h(y; x)$. Therefore recycling outputs from previous experiments could be beneficial even at the expense of some additional density evaluations. This observation hints towards using importance sampling as a way to recycle simulation outputs when available, which will be presented in the next section.

2.2 Ordinary Importance Sampling

Importance sampling is a collection of variance reduction methods to estimate properties of a *target distribution* via random samples from other *sampling distributions*. Readers are encouraged to refer to Tokdar and Kass (2010), Owen (2013) and references therein for a detailed review on importance sampling. In the following discussions we present a output-recycling technique in the context of repeated simulations based on the idea of importance sampling.

Given an observation of the current state $X_k = x_k$, we are interested in approximating $f(x_k)$ using all available outputs. When the current experiment is launched at time t_k, we assume that all outputs up to the current time, i.e, $\left\{F\left(Y_i^{(j)}\right) : j = 1, \cdots, N_i, i = 1, \cdots, k\right\}$, are available. We further assume that an oracle is available to evaluate all density functions $h(y; x_i)$, where i, $i = 1, \cdots, k$ are all observed states up to the

current time. Using the idea of importance sampling, we can combine outputs by averaging the products of all outputs and their corresponding likelihood ratios. The resulting ordinary importance sampling estimator is given by

$$\hat{f}^{OIS}(x_k) = \frac{1}{\tilde{N}_k}\sum_{i=1}^{k}\sum_{j=1}^{N_i}\frac{h\left(Y_i^{(j)};x_k\right)}{h\left(Y_i^{(j)};x_i\right)}F\left(Y_i^{(j)}\right) \tag{3a}$$

$$= \sum_{i=1}^{k} p_i^k\left[\frac{1}{N_i}\sum_{j=1}^{N_i}\frac{h\left(Y_i^{(j)};x_k\right)}{h\left(Y_i^{(j)};x_i\right)}F\left(Y_i^{(j)}\right)\right] \tag{3b}$$

where $\tilde{N}_k = \sum_{i=1}^{k} N_i$ is the cumulative sample size up to the current time and $p_i^k = N_i/\tilde{N}_k$, $i = 1,\cdots,k$ is the proportion of samples in experiments up to the current time. The likelihood ratios h_{x_k}/h_{x_i} respects the sampling distribution of simulated scenario $Y_i^{(j)}$ and the target distribution in the current experiment. One can show that $\hat{f}^{OIS}(x_k)$ is a unbiased estimator of $f(x_k)$.

The two expressions in (3) suggests two viewpoints for $\hat{f}^{OIS}(x_k)$:

1. A weighted average of all available outputs, weighted by the respective likelihood ratios, as shown in (3a).
2. A weighted average of k single-batch importance sampling estimators, weighted by the proportions of random samples generated in all experiments up to the current time, as shown in (3b). In this view, one can see that $\hat{f}^{SMC}(x_k)$ is part of $\hat{f}^{OIS}(x_k)$: the squared bracketed terms in (3b) for $i = k$ is precisely $\hat{f}^{SMC}(x_k)$.

Intuitively speaking, the OIS estimator is superior than the SMC estimator because the former includes all available outputs while the latter includes only the current one. However, since importance sampling is used as a way to exploit the "free" outputs, the OIS estimator may suffer variance inflation when the sample likelihood ratios are too imbalanced. It is well-known that, with poorly chosen sampling distributions, the OIS estimator could have variance larger than that of a SMC estimator or even an infinite variance. Repeated simulations that we consider in this article may magnify this drawback because the target and sampling distributions are determined by the underlying stochastic process, $\{X_k\}$, so the users have no control over them. The OIS estimator may fail even when there is just one sampling distribution that creates imbalanced likelihood ratios. This problem could be aggravated over time in an OIS estimator because more and more sampling distributions are added to the estimator. It is not clear that whether the benefit of including more outputs is greater than the drawback of imbalanced likelihood ratios. We will consider a more robust estimator in repeated simulations in the next section.

2.3 Multiple Importance Sampling

Multiple importance sampling (MIS) is first introduced by Veach and Guibas (1995) for rendering problems in computer graphics and is studied by Owen and Zhou (2000) as one effective importance sampling method. It is initially proposed in cases where it is not clear what sampling distribution is suitable for a particular problem so an alternative is to combine different sampling distributions.

In repeated simulations, the simulated scenarios $Y_i^{(j)}$ in each experiment are sampled from a different sampling distribution. As a group, however, the samples up to time t_k have the following *combined mixture distribution*

$$h_k^*(y) = \sum_{i=1}^{k} p_i^k h(y;x_i) \tag{4}$$

where p_i^k are proportion of samples generated at time i, as defined in (3). More precisely, $h_k^*(y)$ is the distribution of a random variable Y which is equal to each $Y_i^{(j)}$ with probability $1/\tilde{N}_k$. From this point of view, consider the following MIS estimator that combines outputs from all experiments up to time k:

$$\hat{f}^{MIS}(x_k) = \frac{1}{\tilde{N}_k} \sum_{i=1}^{k} \sum_{j=1}^{N_i} \frac{h\left(Y_i^{(j)}; x_k\right)}{h_k^*\left(Y_i^{(j)}\right)} F\left(Y_i^{(j)}\right). \tag{5}$$

Note that the above estimator is referred to as the "balance heuristic" in Veach and Guibas (1995) or "deterministic mixture sampling" in Owen and Zhou (2000). It is a member of a broad class of estimators bearing the term "multiple importance sampling". In this article $\hat{f}^{MIS}(x_k)$ is the only member in the MIS class we consider so we refer it as the MIS estimator for simplicity.

We observe that (5) differs from (3a) in only the denominator in the likelihood ratios due to a different viewpoint in the sampling distributions. This subtle difference makes (5) a better estimator than (3). Since the time k target distribution is mixed into the combined mixture distribution, the likelihood ratio is uniformly bounded above. In particular, for any $y \in \mathscr{Y}$ we have

$$\frac{h(y;x_k)}{h_k^*(y)} = \frac{h(y;x_k)}{\sum_{i=1}^{k} p_i^k h(y;x_i)} \leq \frac{h(y;x_k)}{p_k^k h(y;x_k)} = \frac{\tilde{N}_k}{N_k}.$$

Since all likelihood ratios are bounded in (5), they cannot be too imbalanced.

In a way the combined mixture distribution provides a safeguard to the likelihood ratios and prevents the estimator from failing dramatically, as it could be the case in (3). Moreover, Veach and Guibas (1995) shows that $\hat{f}^{MIS}(x_k)$ is a "nearly optimal" way to combine available outputs from different sampling distributions.

From a qualitative perspective, the combined mixture density $h_k^*\left(Y_i^{(j)}\right)$ is evaluated without regard to which distribution was $Y_i^{(j)}$ sampled from. This ignorance seems questionable because some information is lost. However, (Hesterberg 1988) argues from the Rao-Blackwell theorem that taking consideration of which mixture component generated the sample may be disadvantageous.

In the next section we will compare and contrast the aforementioned three estimators in a more realistic financial example where a risk assessment simulation experiment is carried out periodically.

3 EXAMPLE: PERIODIC CREDIT RISK EVALUATION IN KMV MODEL

In this section we consider a credit risk evaluation example that resembles characteristics of a Kealhofer-McQuown-Vasicek (KMV) model. The KMV model was developed by the KMV corporation by applying the framework of Merton (1974), in which the equity of a firm is a call option on the underlying asset value of the firm with a strike price equal to the face value of the firm's debt. A credit event, or a default, happens when a firm depletes all its equity and the asset falls below its debt. Given a portfolio of corporate bonds, we are interested in evaluating the probability of it incurring high losses by the end of a projection period (e.g., 6 months) due to issuers' defaults. The issuers' assets are marginally log-normally distributed and the correlation structure is specified by a student-t copula. This credit risk evaluation is conducted periodically (e.g., weekly) for risk monitoring purpose.

3.1 Problem Statement

Consider a portfolio of n corporate bonds indexed by $i = 1, \cdots, n$. At any time t_k the financing structure of a solvent issuer i is given by the accounting equation

$$S_{t_k,i} = E_{t_k,i} + D_i \tag{6}$$

where $S_{t_k,i}$, $E_{t_k,i}$, and $D_{k,i}$ denote the issuer's asset, equity, and debt and time t_k. We assume that asset values $\{S_{t,i}, t \geq 0\}$ follows a Geometric Brownian Motion (GBM) with annualized return μ_i and volatility

σ_i, for all $i = 1, \cdots, n$. We assume further that the debts for all issuers are constants for all time $t_k \geq 0$. We say that the i-th issuer defaults at time $t_k \geq 0$ if $E_{t_k,i} < 0$ or equivalently $S_{t_k,i} < D_i$. When the i-th issuer defaults, the portfolio incurs a loss of G_i, $i = 1, \cdots, n$. This is often referred to as the *loss given default*, or LGD, in the literature. We assume that the LGDs are constants for all time $t_k \geq 0$. The observed asset values, $x_k = (S_{t_k,1}, \cdots, S_{t_k,n})$ is the current state based on which the credit risk evaluation is conducted.

Experiments are launched periodically with a repeating period Δt (e.g., one week) so the k-th experiment is launched at time $t_k = k\Delta t$, for $k = 1, 2, \cdots$. Note that although the asset values $\{S_t, t \geq 0\}$ are modeled as GBMs, which are continuous stochastic processes, the states X_k are observed periodically at discrete times t_k, $k = 1, 2, \cdots$. Therefore the state process can nevertheless be defined as a discrete-time Markov process as in Section 2. Let t be the projection period (e.g., 6 month) and let T be an evaluation period (e.g., 5 years) of interest. Given the current state, we are interested in estimating the probability that the portfolio incurs a discounted loss higher than a threshold κ at time $t_k + t$, for loss that incurs at time $t_k + t + T$.

To elaborate, given an observation of the current state $x_k = (S_{t_k,1}, \cdots, S_{t_k,n})$, the projected stock prices $Y_k = (S_{t_k+t,1}, \cdots, S_{t_k+t,n})$ is the random vector involved in the current experiment. Let the $S_{t_k+t,i}$ be a particular random sample of i-th issuer's asset value at time $t_k + t$, the i-th issuer's default loss random variable at time $t_k + t + T$ is given by

$$L_i = \begin{cases} e^{rT} G_i, & \text{if } S_{t_k+t,i} < D_i \\ G_i, & \text{if } S_{t_k+t,i} \geq D_i \text{ and } S_{t_k+t+T,i} < D_i \\ 0, & \text{if } S_{t_k+t,i} \geq D_i \text{ and } S_{t_k+t+T,i} \geq D_i \end{cases} \tag{7}$$

where r is the risk free interest rate. The above random variable states that: If the i-issuer defaults at the end of the projection period $t_k + t$, it incurs the LGD then and accumulates such loss at the rate of interest. If the i-th issuer is solvent at the end of the projection period but defaults at the end of the evaluation period $t_k + t + T$, then it incurs the LGD in the latter. If the i-th issuer is solvent at the end of both the projection periods the evaluation periods, then it does not incur any cost.

Under the GBM assumption for assets, one can calculate the time-$(t_k + t)$ value of (7) using risk neutral pricing. In particular, the last two lines in (7) resembles the payoff of a digital put option with strike price D_i. The discounted loss of the i-th issuer at time $t_k + t$, given particular scenario $y = (S_{t_k+t,1}, \cdots, S_{t_k+t,n})$, is given by

$$\begin{aligned} l_i(y) &= e^{-rT} \mathbb{E}^Q [L_i | S_{t_k+t,i}] \\ &= [\mathbf{I}_{i,k+t} + (1 - \mathbf{I}_{i,k+t}) e^{-rT} \Phi(d_2^i)] G_i \\ &= \max\{\mathbf{I}_{i,k+t}, e^{-rT} \Phi(d_2^i)\} G_i \end{aligned}$$

where $\mathbf{I}_{i,t_k+t} = \mathbb{1}\{S_{t_k+t,i} < D_i\}$ is the default indicator, $\Phi(\cdot)$ is the standard normal cumulative density function, and

$$d_2^i = \frac{\ln(S_{t_k+t,i}/D_i) + (r - \sigma_i^2/2)T}{\sigma_i \sqrt{T}}$$

is the i-th issuer's *distance to default* in the KMV model. Therefore given a particular future scenario $Y_k = y$ the portfolio's discounted loss at time $t_k + t$ is given by

$$l(y) = \sum_{i=1}^{n} l_i(y). \tag{8}$$

Note that $l(y)$ is a constant for a given future scenario $Y_k = y$ but is a random variable given the current state because the future scenario is a random variable given the current state. In particular, the future scenario given current state follows a joint lognormal distribution whose correlation structure is specified by a student-t copula. Please refer to Cherubini et al. (2004) and Frey et al. (2001) backgrounds on copulas. The risk evaluation problem at time t_k is to estimate the following probability

$$f(x_k) = \Pr(l(Y) > \kappa | x_k) = \Pr(l(Y) > \kappa | S_{t_k}) \tag{9}$$

using simulation models that evaluates indicator functions

$$F(y) = \mathbb{1}\{l(y) > \kappa\} \tag{10}$$

for given scenario y.

Figure 1 provides a graphical illustrations for the first two experiments launched for a portfolio consisting of only one corporate bond. The upper panel of Figure 1 illustrates the experiment at time t_1: The left half of the x-y plane shows three sample paths of asset price, conditioning on the observed state $x_1 = 100$. Note that simulating the whole sample path is only for illustration purpose and is unnecessary in the actual experiment. The end of the sample paths denotes the samples of projected stock prices $Y_1^{(j)}$, $j = 1,2,3$. The density function of Y_1 is shown in the right half of the x-y plane. For each projected stock price a corresponding output $F\left(Y_1^{(j)}\right)$, $j = 1,2,3$, is calculated, which are marked by circles in Figure 1. As defined in (9), $F(y)$ is an indicator function and therefore only take values in $\{0,1\}$, therefore the outputs in Figure 1 have values equal to zero (lying on the x-y plane) or one (with a stem of height 1). According to the outputs shown in Figure 1, one would have $\hat{f}^{SMC}(x_1) = \hat{f}^{OIS}(x_1) = \hat{f}^{MIS}(x_1) = 1/3$.

Assuming that the observed state at time t_2 is $x_2 = 95$ (with transition probability $\varphi(X_2 = 95|X_1 = 100)$), the lower panel of Figure 1 depicts the second experiment. The second experiment differs from the first one in the observed state, which affects the density of the projected stock prices Y_2, as shown by the difference between $h(y;x_1)$ and $h(y;x_2)$ in the right half of the x-y plane. In addition to the outputs generated in the second experiment, marked by circles, the outputs from the first experiment, marked by stars are also available in the second experiment. Therefore the three estimators presented in Section 2 may give different estimates.

3.2 Numerical Example

We implement a numerical example for the aforementioned risk evaluation problem using the following parameters:

1. The portfolio consists of two corporate bonds whose initial prices, debts, and LGDs are given by

$$S_{t_1} = \begin{bmatrix} 100 \\ 90 \end{bmatrix}, \quad D = \begin{bmatrix} 85 \\ 85 \end{bmatrix}, \quad \text{and} \quad G = \begin{bmatrix} 5 \\ 4 \end{bmatrix}.$$

2. The risk management decisions such as the repeating period, projection period, and evaluation period as well as the high-loss threshold are given by

$$\Delta t = 1/52, \quad t = 0.5, \quad T = 5, \quad \text{and} \quad \kappa = 6.$$

   This mean the risk evaluation is conducted weekly. The goal of each experiment is to estimate the probability of incurring a high loss in 6 months, where loss is defined as the 5-years discounted loss given default.
3. The parameters for the underlying economy model are given by

$$\mu = \begin{bmatrix} 15\% \\ 10\% \end{bmatrix}, \quad \sigma = \begin{bmatrix} 30\% \\ 20\% \end{bmatrix}, \quad \text{and} \quad r = 5\%.$$

4. The correlation structure between the two assets is model by the student-t copula with 3 degrees of freedom and a correlation of 0.5.

We consider 10^3 sample paths, each with 26 weekly observations of asset values and thus induces 26 weekly risk evaluation experiments. We consider a fixed-budget experiment design and compare the

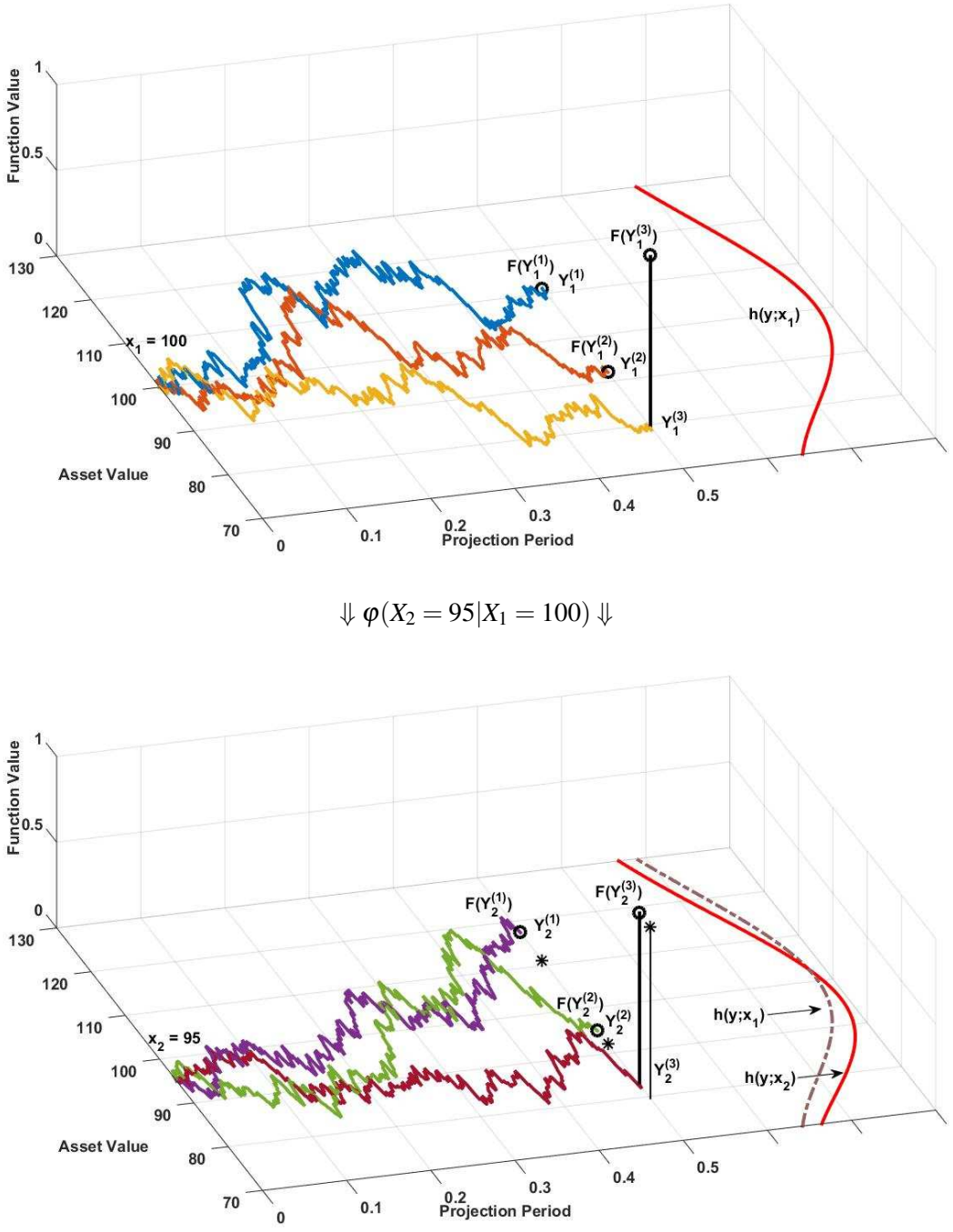

Figure 1: Credit risk evaluation simulation experiments launched at times t_1 and t_2. The end point of a sample path denotes a future scenario Y. Outputs of the current experiment is marked by a circle while the preexisting outputs are marked by stars on top of a scenario. The density functions are drawn to scale.

accuracies of the three estimators over time. For all experiments, $N = 10^3$ new outputs are generated and the three estimators are compared to an *accurate estimate*, which is obtained from the SMC estimator using 10^6 outputs. In each experiment, the squared error to the accurate estimate is recorded. For each experiment time t_k, $k = 1, \cdots, 26$, the mean squared error (MSE) over 1000 sample paths is the figure of merit of each of the three estimators. Figure 2 compares and contrast the accuracies of the three estimators.

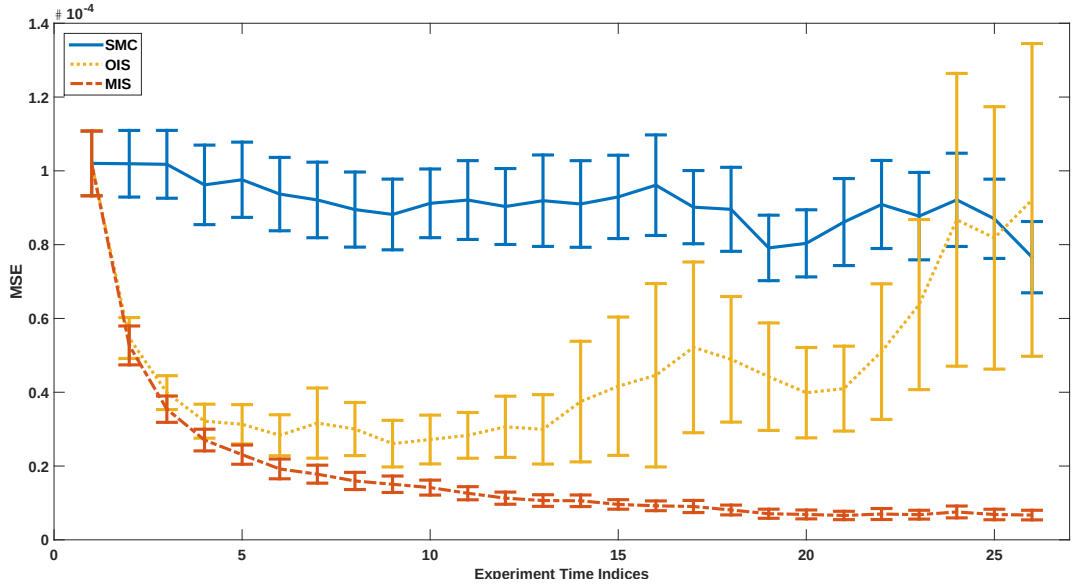

Figure 2: MSEs and error bars for the SMC, OIS, and MIS estimators over a period of 26 weeks, as shown by the solid, dotted, and dashed lines, respectively.

We see from the solid blue line in Figure 2 that the accuracy of the SMC estimator remains relatively constant over time. This is expected because a fixed number of independent new outputs are generated and used by the SMC estimator.

The yellow dotted line in Figure 2 shows that the accuracy of the OIS estimator increases initially, i.e., the first five weeks. This shows that recycling preexisting outputs is beneficial because the proximity of observed states ensures the similarities of the density functions and thus the balance of sample likelihood ratios. However, as the state variable evolves further from the initial state, the accuracy of the OIS estimator worsens over time. This shows that the OIS estimator suffers the drawback imbalanced likelihood ratios due to dissimilarities of sampling densities to the target density. Moreover, we can see from the last few weeks in Figure 2 that OIS estimator can in fact be worse than the SMC estimator when more outputs are included.

The dashed red line in Figure 2 shows an encouraging result: the accuracy of the MIS estimator, measured by its MSE, improves over time. In particular, its accuracy is consistently better than that of an SMC estimator by more than 10 times after the 15-th week. This shows that the MIS estimator is a robust method to recycle and reuse preexisting simulation outputs. It can extract useful information from preexisting outputs without suffering from the imbalanced likelihood ratios, as the case in OIS estimator. Its improvement slows down because some outputs from the distant past may not be very useful of the current experiment. Since the accurate value to which the MIS estimator compares is an SMC estimate and thus itself has uncertainties. This partially explains why the accuracy of the MIS estimator has little improvement after 20 weeks.

4 SUMMARY AND FUTURE DIRECTIONS

In this article we propose the idea of green simulation designs: recycling preexisting simulation outputs to improve the efficiency of the current experiment. We envision that green designs are particularly useful for experiments that are repeated periodically. Importance sampling and multiple importance sampling are considered as particular implementations of output recycling. Numerical example of periodic credit risk evaluation problem is studied. Our numerical result shows that multiple importance sampling is an effective and robust way to recycle and reuse preexisting outputs.

In the future we will theoretically study the multiple importance sampling estimator in the context of green simulation and identify the conditions under which it will have certain convergence property. We will also study other methods for output recycling, e.g., metamodeling, statistical learning, machine learning, etc. Other green simulation designs such as fixed accuracy designs are also of interest to us. Last but not least, we will explore various efficient implementations, such as parallel programming, of green simulations.

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