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Time Buffer for Approximate Optimization of Production Systems: Concept, Applications and Structural Results

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ABSTRACT

Simulation Optimization is acquiring always more interest within the simulation community. In this field, Mathematical Programming Representation (MPR) has been applied for both simulation and sample path-based optimization of production systems performance. Although in the traditional literature these systems have been represented by means of Integer Programming (IP) models, recently, approximate Linear Programming (LP) models have been proposed to optimize and evaluate the performance of a category of production systems. This work deals with LP models developed based on the *Time Buffer* (TB) variable whose concept, applicability and structural properties will be presented. Moreover the models convergence, within the Sample Average Approximation (SAA) framework, will be characterized.

1 Contributions

MPR is deeply different from traditional simulation optimization techniques (Fu, Glover, and April 2005) since more information can be obtained from a single simulation (optimization) run (Chan and Schruben 2008).

TB-based LP models are here applied to approximately solve manufacturing optimization problems (Buzacott and Shantikumar 1993). These models, iteratively solved using the SAA approach, result in the optimal TB configuration and the related performance estimates (Sect. 3). These two outputs form the basis to derive the integer solution the TB is approximating. Indeed, the TB solution in the continuous domain is strongly related to a unique solution in the discrete domain.

Two applications of the TB have been developed so far: (1) approximation of the buffer capacity in an open flow line, (2) approximation of the number of pallets in a loop line. The optimization problems were the Buffer Allocation Problem (BAP) and the Pallet Sizing Problem (PSP) respectively. Although a model for the BAP was already proposed in (Alfieri and Matta 2012), for this case, the second order properties of the approximate model variables, the SAA solution approach and the convergence properties constitute a result of this work (Sect. 4).
Formulations are not reported for space limitations, hence refer to (Alfieri and Matta 2012).

2 Time Buffer for Simulation and Optimization

The TB is a continuous variable defined in the approximate model to replace the discrete variable defined in the original IP model. In general, to approximate a discrete variable with a TB, it must be possible to formally describe its effects on the events characterizing the system dynamics. This holds when the system dynamics can be formulated as a set of max-plus type equations, (Buzacott and Shantikumar 1993).
In the BAP case, for example, the space buffer capacity continuous counterpart has to be modelled. The capacity effect is to delay (anticipate) the time a customer enters the workstation upstream the buffer (start event). Hence, the start event time represents the continuous variable to trigger by means of the TB that

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will, directly, delay or anticipate this event as the space buffer, indirectly, does. Moreover, for the open line case, referring to the work of Shanthikumar et al., (Shanthikumar and Yao 1991), it was proved that: (1) the TB is increasing convex in the processing times if the processing times are convex in the parameters characterizing their distribution, (2) the completion time is decreasing convex in the TB and increasing convex in the processing times if these are increasing and convex in the parameters characterizing their distribution.

3 Simulation Optimization Algorithm

The TB simulation optimization models are iteratively solved to find the optimal TB configuration and its discrete counterpart, following the steps described below.

1. *Initialization*: Iteration $k = 0$. (1) set the parameters describing the system and the simulation optimization configuration parameters (e.g., the number of machines, the run length), (2) set the target average completion time, (3) generate the sample path of processing and arrival times.
2. *System Configuration Generation*: Solve the LP approximate optimization model, obtaining the $samplepath$-optimal TB. Go to Step 3.
3. *System Performance Evaluation*: Feed the approximate simulation model with the initialization data and the TB's obtained from the previous step. Solve the approximate simulation model. If $k > 0$ and the stopping condition is met, derive the *sp*-approximate integer solution and exit the procedure. Otherwise increase the sample path size and go to Step 2.

4 Convergence Properties

Let ε-SBAP define the finite Sample path approximate BAP problem to be solved for the ε-optimal solution in terms of TB, i.e. the solution characterized by a completion time higher than the target of maximum ε, with ε going to 0 as the size of the sample path $n \to \infty$, $\mathscr{P}$-almost surely.

For this problem, the asymptotic convergence was characterized for: (1) the feasible region, (2) the set of minimizers (in terms of TB solutions), (3) the value of the objective function. Based on the properties of the LP models, duality theory and epi-convergence theory, the asymptotic convergence for all the cases was proved.

Moreover, adopting the Large Deviation Theory ((Dembo and Zeitouni 2009), (Shapiro 1996)), it was proved that, in case the central limit theorem holds for the objective function and the constraints, the probability of obtaining a sample path solution which is not the optimal for the infinite sample path problem goes to 0 with exponential rate.

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