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NEW CONTROL VARIATES FOR LÉVY PROCESS MODELS

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ABSTRACT

We present a general control variate method for Monte Carlo estimation of the expectations of the functionals of Lévy processes. It is based on fast numerical inversion of the cumulative distribution functions and exploits the strong correlation between the increments of the original process and Brownian motion. In the suggested control variate framework, a similar functional of Brownian motion is used as a main control variate while some other characteristics of the paths are used as auxiliary control variates. The method is applicable for all types of Lévy processes for which the probability density function of the increments is available in closed form. We present the applications of our approach for simulation of path dependent options. Numerical experiments confirm that our method achieves considerable variance reduction.

1 INTRODUCTION

Let $\{L(t), t \geq 0\}$ be a Lévy process starting at zero $L(0) = 0$ and q be a function of the sample path of $L(t)$ on a discrete time grid $0 = t_0 < t_1 < t_2 < \ldots < t_d$ with equidistant time points $t_i = i\,\Delta t$. The unknown quantity that we are trying to estimate is the expectation

$$\mathrm{E}\left[q(L(t_1), \ldots, L(t_d))\right]. \tag{1}$$

For high dimensions and complicated functions, it is often not possible to find the exact value with a closed form solution. Such situations commonly arise e.g. when pricing options with path dependent payoffs.

Monte Carlo simulation is one of the widely used techniques for estimating the expectations of high dimensional problems. The availability of an error bound and the ease of implementation and parallelization make it quite attractive. However, an important disadvantage is that it is comparatively slow when precise results are required. Variance reduction techniques thus play an important role to increase the speed or precision of the simulation.

In the literature, there exist studies proposing variance reduction methods that work well for some special Lévy processes. However, there seem to exist only a limited number of studies proposing variance reduction methods, which are generally applicable to Lévy processes and different problem types. Dingeç and Hörmann (2012) developed a general control variate (CV) method for option pricing under Lévy processes. It is based on the idea of using the CV of the corresponding geometric Brownian motion (GBM) option and reaches strong correlation by using common random numbers for the path simulation. In this study, we extend the idea introduced by Dingeç and Hörmann (2012) to a larger class of problems and suggest to use some path characteristics as additional CVs together with a main CV. This results often in higher variance reduction due to the use of multiple CVs. We present the application of the new CV framework to path dependent options, which are contingent on the average and the maximum stock prices.

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2 GENERAL CV FRAMEWORK

Let $\{W(t), t \geq 0\}$ be a Brownian motion (BM) with parameters μ and $\sigma > 0$,

$$W(t) = \mu t + \sigma B(t),$$

where $B(t)$ is a standard Brownian motion. Assume that increments of $\{W(t), t \geq 0\}$ are correlated with that of the original Lévy process $\{L(t), t \geq 0\}$ with a comonotonic copula. Also, assume that there exist a functional ζ of W, which is equal or similar to q and there exist a solution for $\mathrm{E}[\zeta(W)]$. The idea is to use $\zeta(W)$ as main CV for $q(L)$. The comonotonicity between the increments and the similarity between $\zeta()$ and $q()$ should imply high correlation and thus lead to large variance reduction. One technical difficulty is the simulation of the comonotonic copula. It requires inversion of the cumulative distribution functions (CDFs) of the increments of both processes for a common uniform random number. However, for the Lévy processes considered in the literature, at most the probability density function (PDF) of the increments is available in closed form while the CDF and the inverse CDF are not tractable. So, to use inversion we need the fast numerical inversion algorithm, see Derflinger, Hörmann, and Leydold (2010), that requires as input only the PDF of the distribution.

When $\zeta(W)$ is used as single CV, the remaining variance is due to the difference between the two functions, $\zeta()$ and $q()$, and the two processes L and W. In practice, these functions are contingent on some characteristics of the paths such as average, maximum and terminal value. By using those characteristics as additional CVs we can further reduce the variance as they carry some information about the difference between $\zeta()$ and $q()$. The only requirement is the availability of their expectations. Moreover, in some cases, it is possible to obtain significant variance reduction by using only those characteristics as CVs without using $\zeta(W)$. Let $\gamma(W,L)$ be a function of the paths of W and L that evaluates the set of path characteristics. We call these additional CVs 'general CVs' since they are applicable to any $q()$, whereas $\zeta(W)$ is called 'special CV' as it is designed considering the special properites of $q()$.

In the suggested CV algorithm, the user has to provide the CV functions $\zeta()$ and $\gamma()$. Selection of $\zeta()$ of course depends on the problem type, as it is tailored to $q()$. However, the general CVs can be freely chosen from a large *pool* or *basket* of CV candidates. For the selection of the successful CVs from that basket, we can try to use our theoretical knowledge on the problem and guess which CV will be successful. In this study, we propose an alternative approach, which is more automatic and universally applicable to all problems. We make a stepwise backward regression to detect the useful CVs in a pilot simulation run. First we start with a full regression model where all possible CVs in the basket are used. The CV with the smallest absolute t statistic is removed from the model if its value is smaller than 5. After removal, the t statistics of the other CVs are recomputed for the new regression model. These two steps are repeated till all absolute t values are above 5. The CV candidates, which remains in the regression model, are used for the main simulation. Instead of using only the significant CVs, one can prefer to use all CVs in the basket. However, simulation or evaluation of the expectation of some CVs can be computationally expensive compared to that of the others. The backward regression automatically eliminates the CVs if they are not useful.

REFERENCES

Derflinger, G., W. Hörmann, and J. Leydold. 2010. "Random variate generation by numerical inversion when only the density is known". *ACM Transactions on Modeling and Computer Simulation* 20 (18): 1–25.

Dingeç, K. D., and W. Hörmann. 2012. "A general control variate method for option pricing under Lévy processes". *European Journal of Operational Research* 221 (2): 368–377.