Proceedings of the 2011 Winter Simulation Conference
S. Jain, R. R. Creasey, J. Himmelspach, K. P. White, and M. Fu, eds.

DISCRETE-VALUED, STOCHASTIC-CONSTRAINED SIMULATION OPTIMIZATION WITH COMPASS

Hélcio Vieira, Junior
Karl Heinz Kienitz
Mischel Carmen Neyra Belderrain

Technological Institute of Aeronautics
Praça Marechal Eduardo Gomes, 50
12228-900, São José dos Campos, BRAZIL

ABSTRACT

We propose an improvement in the random search algorithm called COMPASS to allow it to deal with a single stochastic constraint.

Our algorithm builds on two ideas: (a) a novel simulation allocation rule and (b) the proof that this new simulation allocation rule does not affect the asymptotic local convergence of the COMPASS algorithm.

It is shown that the stochastic-constrained COMPASS has a competitive performance in relation to other well known algorithms found in the literature for discrete-valued, stochastic-constrained simulation problems.

1 INTRODUCTION

"Simulation can be used to design a system to yield optimal expected performance" (Andradóttir 1998).

The tools that allow the above statement to be true are known as **simulation optimization**. What makes simulation optimization hard is the allocation of the computational budget between the search for a better solution versus a better estimation of the expected value of the candidate solutions. As we have estimates, it cannot be possible to determine if one system or alternative is better than another, hindering optimization algorithms based on hill-climbing movements (Banks et al. 2000).

Most of the work in simulation optimization has been in continuous-valued, single-performance-measure problems, i.e., problems in which the decision maker (DM) is concerned with only one performance measure and the variables are continuous. If the DM is interested in two performance measures, then the traditional approach, copied from mathematical programming, is to optimize one of the performance measures while constraining the other to be smaller/greater than some threshold. If we restrict our focus to discrete-valued variables, the problem is put mathematically as:

$$\begin{aligned} &Min\ \ h_x = E_\omega\left[H\left(x,\omega\right)\right] \\ &s.t. \\ &g_x = E_\omega\left[G\left(x,\omega\right)\right] \leq \gamma \\ &x \in \Theta \end{aligned} \tag{1}$$

Here, x is the decision vector; $H(\cdot)$ is the primary real-valued performance measure; $G(\cdot)$ is the secondary performance measure; ω represents the stochastic input to the simulation; and $\Theta = \left\{x \,\middle|\, x \in \mathbb{Z}^d, lb \leqslant x \leqslant ub\right\}$ is the finite feasible space. We assume that both $H\left(x,\omega\right)$ and $G\left(x,\omega\right)$ are measurable and integrable with respect to the distribution of ω. In addition, we assume that h_x and g_x are difficult (or impossible) to evaluate.

As the inequality constraint of (1) is also a stochastic outcome of the simulation, we are going to refer to it as a **stochastic constraint** as opposed to classical deterministic constraints (as is the second constraint of (1)). One example of stochastic outcomes (either objective functions or constraints) is found in a call center with several classes (types) of customers: there "are cost components associated with service level performance measures such as waiting times (most commonly the mean or the probability of waiting more than a certain amount of time, possibly weighted by class type) and operational costs associated with agent wages and network usage (trunk utilization). Abandonment rates of waiting customers, percentage of blocked calls (those customers that receive a busy signal), and agent utilization are other factors that are considered" (Fu 2002). One example of a deterministic constraint in the same environment (call center) is the number of telephone operator workstations being less than a threshold (due to a physical limitation).

The literature in stochastic-constrained, discrete-valued simulation optimization is not vast, but it has received some attention in the last years: Abspoel et al. (2001), Cezik and L'Ecuyer (2008), Atlason et al. (2008), Davis and Ierapetritou (2009), Andradóttir and Kim (2010) and Kleijnen et al. (2010).

The purpose of this paper is to propose an improvement in the random search algorithm called COMPASS to allow it to deal with a single **stochastic** constraint.

The organization of the rest of this paper is the following. We describe the original COMPASS algorithm in Section 2. In Section 3, we present our proposal. Numerical examples of our proposal utilization are given in Section 4, and we summarize our conclusions in Section 5.

2 COMPASS

2.1 Initial Considerations

COMPASS (Hong and Nelson 2006, Hong and Nelson 2007) stands for "Convergent Optimization via Most-Promising-Area Stochastic Search" and can be classified as a random search algorithm. Its main advantage is the novel neighborhood structure, which is large at the beginning of the search and gets smaller in the following iterations. The algorithm was designed to find local optimal solutions of discrete-valued simulation problems that are (a) fully deterministic-constrained or (b) partially deterministic-constrained or unconstrained.

We describe the basic algorithm (fully deterministic-constrained) in the next subsection. For more details of the other version, readers are referred to Hong and Nelson (2006).

2.2 COMPASS for Fully Deterministic-Constrained, Discrete-Valued Simulation Problems

2.2.1 Notation

- $H_l(x)$ is the l^{th} observation of $H(x,\omega)$;
- h_x is the sample mean of $N_k(x)$ observations of $H(x,\omega)$;
- x_0 is the starting solution;
- Θ is the search space (a d-dimensional set with integer elements);
- $S(k) = \bigcup_{i=0}^{k} S_i$ is the set of solutions sampled through iteration k;
- $\widehat{x}_k^*$ is the estimated optimal solution of iteration k;
- $a_k(x)$ is the additional number of simulation observations allocated to x on iteration k;
- SAR stands for simulation-allocation rule;
- $N_k(x)$ is the total number of simulation observations allocated to x on iteration k;
- $\mathrm{M}_k = \left\{x \,\middle|\, x \in \Theta \textit{ and } \left\|x - \widehat{x}_k^*\right\| \leqslant \|x-y\| \,\forall y \in \varepsilon_k \textit{ and } y \neq \widehat{x}_k^*\right\}$ is the most promising area on iteration k;
- ε_k is the set which includes all solutions that could be estimated (simulated) on iteration k;
- $\|i-j\|$ represents the Euclidean distance between i and j; and
- S_k is the set of unique solutions (i.e, with the duplicate solutions removed) sampled on iteration k.

2.2.2 Assumptions

1. $\lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} H_i(x) = h_x$ *with probability* 1.
   This assumption is always valid with IID outcomes of terminating simulations or when, under certain conditions, the outcomes are observations from a long-run, steady-state simulation (Law and Kelton 2000).
2. There exists a positive constant δ_0 such that the level set $\Gamma = \{x \in \Theta \,|h_x \leqslant h_{x_0} + \delta_0\}$ is finite.
   When Θ is finite (which is true for (1)), *Assumption 2* always holds.

2.2.3 Algorithm

1. Set iteration counter $k = 0$. Find $x_0 \in \Theta$, set $S(0) = \{x_0\}$ and $\widehat{x}_k^* = x_0$. Determine $a_0(x_0)$ according to the SAR. Take $a_0(x_0)$ observations from x_0, set $N_0(x_0) = a_0(x_0)$ and calculate h_{x_0}. Let $\mathrm{M}_0 = \Theta$.
2. Let $k = k+1$. Sample $x_{k1}, x_{k2}, \ldots, x_{km}$ uniformly and independently from M_{k-1}. Let $S_k = \{x_{k1}, x_{k2}, \ldots, x_{km}\}$ and $S(k) = S(k-1) \cup S_k$. Determine $a_k(x)$ according to the SAR for every x in $S(k)$. For all $x \in S(k)$, take $a_k(x)$ observations, and update $N_k(x)$ and h_x.
3. Let $\hat{x}_k^* = \arg\min_{x\in S(k)} h_x$. Construct M_k and go to step 2.

The simplest SAR proposed by COMPASS' authors is: $N_k(x) = N_k$ for all $x \in S(k)$ and $N_k \to \infty$ as $k \to \infty$.

3 OUR PROPOSAL

Our proposal is built on two ideas that are described here in subsections 3.2 and 3.3.

3.1 Notation

- $\varepsilon(k) = \bigcup_{i=0}^{k} \varepsilon_i$ is the set of solutions estimated through iteration k;
- $\eta\,(x) = \{y\,|y \in \Theta \textit{ and } \|x-y\| \leq 1\}$ is the local neighborhood of x;
- $|\cdot|$ denotes the cardinality of a set;
- σ_{h_x} is the standard deviation of $N_k(x)$ observations of $H(x,\omega)$;
- g_x is the sample mean of $N_k(x)$ observations of $G(x,\omega)$;
- $G_k(x)$ is the k^{th} observation of $G(x,\omega)$; and
- σ_{g_x} is the standard deviation of $N_k(x)$ observations of $G(x,\omega)$.

3.2 Locally Convergent Algorithms

Besides revising their original algorithm, Hong and Nelson (2007) offer two conditions that, under observation of 2.2.2, guarantee the local convergence of any random-search algorithm. They proved that the COMPASS algorithm obeys these two conditions. *Condition 2* is of especial interest to our proposal, so we reproduce it below.

Condition 2. *The estimation scheme satisfies the following requirements:*

1. ε_k *is a subset of* $S(k)$*;*
2. ε_k *contains* x_0*,* $\eta\left(\widehat{x}_{k-1}^*\right) \cap \varepsilon\,(k-1)$ *and* S_k*;*
3. $a_k(x)$ *is allocated such that* $\min_{x\in\varepsilon_k} N_k(x) \geq 1$ *for all* $k = 1,2,\ldots$ *and* $\min_{x\in\varepsilon_k} N_k(x) \to \infty$ *w.p. 1 as* $k \to \infty$*; and*
4. $|\varepsilon\,(\infty)| < \infty$ *with probability 1.*

The first two requirements are (a) that ε_k contains only solutions that have already been sampled and (b) that it contains, at least, x_0, the neighbors of $\widehat{x}^*_{k-1}$ that have been estimated through iteration $k-1$, and the newly sampled solutions. The third requirement assures that the solution can be estimated by allocating at least one observation to it and also that, asymptotically, the estimation will have no noise. The fourth requirement is that only a finite number of solutions are estimated in the limit.

3.3 Probability of False Selection

Hunter and Pasupathy (2010) propose a sampling allocation rule for stochastic-constrained simulation optimization that asymptotically minimizes the probability of false selection. In their framework, the DM chooses $0 < \alpha_1 < 1$ and the values of other α_k have to obey (2), where α_k is the percentage of the allowed budget that will be spent with system k; $\mathrm{I}(\cdot)$ is the indicator function; and the index 1 is associated with the best system so far ($\hat{x}^*_k$).

$$\frac{\alpha_i^*}{\alpha_j^*} \approx \frac{\left(\frac{h_1-h_j}{\sigma_{h_j}}\right)^2 \mathrm{I}(h_1 < h_j) + \left(\frac{\gamma-g_j}{\sigma_{g_j}}\right)^2 \mathrm{I}(g_j > \gamma)}{\left(\frac{h_1-h_i}{\sigma_{h_i}}\right)^2 \mathrm{I}(h_1 < h_i) + \left(\frac{\gamma-g_i}{\sigma_{g_i}}\right)^2 \mathrm{I}(g_i > \gamma)},\ i,j \neq 1 \tag{2}$$

Observe that the sampling allocation rule (2) takes into consideration both the objective function and the stochastic constraint. Formula (2) minimizes the asymptotic probability of false selection as long as the objective function and the constraint are mutually independent and normally distributed. As seen at 2.2.2, IID normality can be expected when the observations are either within-replication averages or a batch means of, respectively, a transient or steady-state simulation (Law and Kelton 2000).

3.4 Our Estimate of the Best

The estimate of the best made in step 3 of 2.2.3 should be modified to (3) in order to allow a feasibility check.

$$\widehat{x}^*_k = \underset{x \in S(k),\ g_x \leqslant \gamma}{\arg\min}\ h_x \tag{3}$$

3.5 Our SAR Proposal

As our goal is to find the system with the smallest expected objective-function value that is **also** feasible, the simulation allocation rule should somehow take these facts into consideration. We also want the new SAR to obey *condition 2* of Hong and Nelson (2007), so the stochastic-constrained COMPASS algorithm should maintain its desirable characteristic of asymptotic convergence to a local optimum.

The SAR we propose is described by (4) .

$$a_k(x) = \begin{cases} n_0,\ if\ x \in S_k = \{x_{k1}, x_{k2}, \ldots, x_{km}\} \\ \lambda \alpha_i,\ if\ x \in S(k-1) \end{cases} \tag{4}$$

Here, $\lambda + n_0 m$ is the computational budget allocated to each iteration of the stochastic-constrained COMPASS algorithm; $n_0 > 0$ is the initial sample size; $0 < \alpha_1 < 1$ is defined by the DM; and $\alpha_j, \forall j \neq 1$, is calculated through (2).

Theorem 1 The sampling allocation rule (4) obeys *condition 2* of Hong and Nelson (2007).

Proof. Requirements 1 and 2 are satisfied because $\varepsilon_k = S(k)$ in (4).

The first part of requirement 3 is true because $n_0 > 0$ by construction.

As h_1 is, by definition, the smallest value of the objective function among all systems that are feasible, $\mathrm{I}(h_1 < h_k)$ forms a subset that contains all feasible systems but the best. On the other side, $\mathrm{I}(g_k > \gamma)$

forms a subset that contains all infeasible systems. Because (a) the union of these two subsets contains all systems evaluated so far but the best system; (b) these subsets do not intersect each other; and (c) $\alpha_1 > 0$ by construction; then $\alpha_k > 0, \forall k$. These facts, together with the allowed budget $\lambda + n_0 m$ getting arbitrarily large, fulfill the second part of requirement 3.

Requirement 4 is accomplished because $|\Theta| < \infty$ by construction (vide (1)). □

4 EXPERIMENTAL RESULTS

4.1 First Experiment

When it is desired to select the *best* alternative among a (small) finite number of alternatives, it is usual to use a Ranking & Selection (R&S) procedure. The R&S was "developed to compensate for the limited inference provided by hypothesis test for the homogeneity of k population parameters (usually means). In many experiments, rejecting the hypothesis $H_0 : \mu_1 = \mu_2 = \cdots = \mu_k$, where μ_i is the parameter associated with the i^{th} population, leads naturally to questions about which one has the largest or smallest parameter. R&S tries to answer such questions" (Kim and Nelson 2006, p. 503).

We compare the efficiency of our SAR proposal with two constrained R&S algorithms proposed by Andradóttir and Kim (2010) in the Δ (difficult means) configuration. This comparison is only between the SAR rule and the other proposals, and not between the stochastic-constrained COMPASS and the proposals, because we did not use the search part of COMPASS (all the 25 candidate solutions were considered in all iterations). The results of this comparison are given by Table 1. The Δ configuration is listed by (5) and is one in which "it is difficult to distinguish between feasible and infeasible systems. In addition, all desirable and acceptable systems have h_i very close to that of the true best desirable system, which makes it difficult to eliminate inferior systems. On the other hand, all unacceptable systems have much smaller h_i than that of the true best desirable system" (Andradóttir and Kim 2010, p. 414).

$$H_i(x) = \begin{cases} N(-20^{-0.5}, 1), \ x = 1,\, 2, \ldots,\, b \\ N(0,1),\ x = b+1,\, b+2, \ldots,\, b+a \\ N(20^{-0.5}, 1),\ x = b+a+1,\, b+a+2, \ldots, 25 \end{cases} \tag{5}$$

$$G_i(x) = \begin{cases} N(0,1),\ x = 1,\, 2, \ldots,\, b-1 \\ N(-20^{-0.5}, 1),\ x = b \\ N(0,1),\ x = b+1,\, b+2, \ldots,\, b+a \\ N(-(x-1)20^{-0.5}, 1), x = b+a+1,\, b+a+2, \ldots, 25 \end{cases} \tag{6}$$

Where $N(\mu, \sigma^2)$ means normally distributed with mean μ and variance σ^2.

Table 1 shows that the SAR rule we adopt in the stochastic-constrained COMPASS algorithm needs fewer replications than the two rival proposals to achieve the same desirable probability of correct selection (PCS). The number of macroreplications used to compute the estimated PCS in table 1 was 20,000 and the PCS was calculated as the observed proportion of correct selections in the 20,000 replications.

4.2 Second Experiment

We also compare the performance of the stochastic-constrained COMPASS with the proposal of Kleijnen et al. (2010) for an optimization of an infinite-horizon, stochastic-constrained, periodic-review (s,S) inventory system with full back-logging. The assumptions used in the model are:

- Demand: exponentially distributed with an average $\lambda^{-1} = 100$ units;
- Check: the inventory is checked at the end of every time period. A replenishment order is placed if the inventory position is smaller than or equal to s. The size of the order is $S - s - \beta$, where

Table 1: Performance comparison of the SAR strategy in the Δ configuration. Desirable PCS=0.95.

	b	13	12	10	7	3	1
	a	0	1	3	6	10	12
AK*	REP	4063	4109	4184	4319	4502	4604
	PCS	0.973	0.973	0.973	0.974	0.975	0.975
AK+*	REP	3763	3749	3726	3686	3615	3581
	PCS	0.960	0.962	0.963	0.963	0.966	0.968
SAR	REP	3500	3500	3500	3500	3500	3500
	PCS	0.958	0.953	0.958	0.967	0.974	0.977

* Source: Andradóttir and Kim (2010).

β (outstanding orders) is the total size of the orders that have already been placed but have not arrived;

- Lead-time: Poisson distributed with an average of 6 units. Observe that this distribution of the replenishment lead-time allows orders to cross in time, i.e., the order in which they are placed is not necessarily the order in which they are received;
- Holding cost: $h = 1$ unit per period;
- Fixed ordering cost: $K = 36$ units per order;
- Variable ordering cost: $u = 2$ units per unit ordered;
- Replenishments: the orders are received at the beginning of a period;
- Objective: minimize the expected total cost $TC = hW_i^+ + \mathrm{I}\{X_i < s\}(K + u(S - X_i))$, where X_i is the inventory position at period i, $W_i = X_i + \beta$ is the inventory level at period i, β are the outstanding orders, $\mathrm{I}\{\cdot\}$ is the indicator function and $A^+ = max\{0, A\}$;
- Constraints:
  - Deterministic: $900 \leqslant s \leqslant 1250$ and $1 \leqslant Q \leqslant 500$, where $Q = S - s$;
  - Stochastic: stockout rate $\delta \geqslant 0.10$, where stockout rate is the fraction of demand not supplied from stock on hand;

The fact that orders are allowed to cross in time does not allow this model to be analytically tractable, so simulation is a need.

In order to have a fair comparison with the results of Kleijnen et al. (2010), we decided to simulate the optimal points found by them in our implementation of the (s,S) inventory model. Table 2 displays the simulation results. With the exception of the solution $(s,Q) = (1061,69)$, all other results were considered satisfactory. As a result of this discrepancy, we decided to check the accuracy of our implemented model. This check is showed in appendix A.

Due to the mentioned discrepancy and to our belief in our model accuracy, we decided to use our results of the optimal points found by Kleijnen et al. (2010) in all the comparisons hereafter.

The results of 10 macro-replications of the stochastic-constrained COMPASS are displayed in Table 3. Table 4 summarizes the results of both tables 2 and 3 (recall that we decided to use our results of of the optimal points found by Kleijnen et al. (2010)). Analysis of Tables 2, 3 and 4 shows that the stochastic-constrained COMPASS had the best mean result and also the best overall minimum that is feasible (the OptQuest (Arena 12) $m_i = 10$ had three better results, but they are all infeasible).

5 CONCLUSION

We proposed an improvement in the random search algorithm called COMPASS to allow it to deal with a single stochastic constraint.

We described the original COMPASS algorithm in Section 2. In Section 3, we presented our proposal, and numerical examples of our proposal utilization were given in Section 4.

Table 2: Our results of the solutions presented by Kleijnen et al (2010)

Proposal	Parameters		Kleijnen et al. (2010)		Our Results (300 replicates)			
					TC		SR	
	s	Q	TC^{*}	SR^{*}	μ_{TC}	σ_{TC}	μ_{SR}	σ_{SR}
OptQuest (Arena 12) $m_i = 10$	1009	287	716.16	0.0828	715.76	3.2900	0.0820	0.0037
OptQuest (Arena 11) $m_i = 10$	1047	108	660.91	**na**	661.61	2.8470	0.0889	0.0040
DOE-Kri-MP $m_i = 10$	1043	70	638.34	0.0992	638.69	2.8179	0.0982	0.0045
DOE-Kri-MP $\tau = 0.15$	1021	114	640.09	0.0991	640.99	2.8188	0.0989	0.0041
OptQuest (Arena 12) $m_i = 10$	1027	84	632.42	0.0993	631.63	2.7760	0.1018	0.0044
OptQuest (Arena 11) $m_i = 10$	1047	97	656.79	**na**	656.38	2.8966	0.0904	0.0041
DOE-Kri-MP $m_i = 10$	1043	70	638.34	0.0992	639.02	2.8606	0.0975	0.0045
DOE-Kri-MP $\tau = 0.15$	1061	31	634.74	0.0999	635.49	2.6992	0.1003	0.0042
OptQuest (Arena 12) $m_i = 10$	1050	44	632.77	0.0993	631.98	2.7342	0.1012	0.0048
OptQuest (Arena 11) $m_i = 10$	1047	85	650.60	**na**	650.36	3.0723	0.0928	0.0042
DOE-Kri-MP $m_i = 10$	1043	70	638.34	0.0992	639.07	2.9918	0.0978	0.0044
DOE-Kri-MP $\tau = 0.15$	1062	29	634.19	0.0999	635.52	2.8332	0.1000	0.0045
OptQuest (Arena 12) $m_i = 10$	1061	191	713.74	0.0746	716.21	3.3758	0.0731	0.0038
OptQuest (Arena 11) $m_i = 10$	1047	103	657.35	**na**	659.23	3.0888	0.0895	0.0042
DOE-Kri-MP $m_i = 10$	1057	41	636.36	0.0999	637.50	3.0586	0.0985	0.0047
DOE-Kri-MP $\tau = 0.15$	1076	12	637.39	0.0993	638.89	2.8771	0.1000	0.0043
OptQuest (Arena 12) $m_i = 10$	1129	35	700.19	0.0711	699.73	2.8992	0.0714	0.0035
OptQuest (Arena 11) $m_i = 10$	1047	99	657.90	**na**	657.04	2.9439	0.0904	0.0043
DOE-Kri-MP $m_i = 10$	1043	70	638.34	0.0992	638.86	2.7625	0.0979	0.0041
DOE-Kri-MP $\tau = 0.15$	1041	73	638.04	0.0983	639.07	3.0011	0.0975	0.0046
OptQuest (Arena 12) $m_i = 10$	1002	209	671.28	0.0936	671.03	2.9225	0.0929	0.0042
OptQuest (Arena 11) $m_i = 10$	1047	95	655.28	**na**	655.35	2.6980	0.0908	0.0039
DOE-Kri-MP $m_i = 10$	1043	70	638.34	0.0992	639.07	2.9905	0.0976	0.0048
DOE-Kri-MP $\tau = 0.15$	1047	58	635.20	0.0998	636.61	2.8626	0.0986	0.0041
OptQuest (Arena 12) $m_i = 10$	1027	84	632.02	0.0998	631.66	2.7763	0.1013	0.0043
OptQuest (Arena 11) $m_i = 10$	1047	99	657.86	**na**	656.91	3.1588	0.0904	0.0042
DOE-Kri-MP $m_i = 10$	1043	70	638.34	0.0992	639.10	2.8170	0.0976	0.0042
DOE-Kri-MP $\tau = 0.15$	1041	73	638.04	0.0983	638.57	2.8248	0.0979	0.0043
OptQuest (Arena 12) $m_i = 10$	1054	59	642.44	0.0956	643.01	2.8227	0.0956	0.0039
OptQuest (Arena 11) $m_i = 10$	1047	122	667.34	**na**	668.66	2.9712	0.0865	0.0042
DOE-Kri-MP $m_i = 10$	1046	59	635.63	0.0996	636.29	2.7352	0.0989	0.0043
DOE-Kri-MP $\tau = 0.15$	1057	40	634.62	0.0990	636.46	3.1857	0.0989	0.0046
OptQuest (Arena 12) $m_i = 10$	1061	69	714.79	0.0739	654.92	2.9306	0.0894	0.0041
OptQuest (Arena 11) $m_i = 10$	1047	92	652.85	**na**	653.28	2.8177	0.0915	0.0039
DOE-Kri-MP $m_i = 10$	1043	70	638.34	0.0992	638.97	3.1103	0.0979	0.0044
DOE-Kri-MP $\tau = 0.15$	1039	73	636.24	0.0992	637.00	2.8391	0.0985	0.0042
OptQuest (Arena 12) $m_i = 10$	999	185	655.21	0.0972	656.13	3.2160	0.0975	0.0044
OptQuest (Arena 11) $m_i = 10$	1047	109	661.05	**na**	662.07	3.1068	0.0881	0.0043
DOE-Kri-MP $m_i = 10$	1043	70	638.34	0.0992	639.01	2.8644	0.0976	0.0044
DOE-Kri-MP $\tau = 0.15$	1049	54	636.14	0.0999	636.46	2.8588	0.0991	0.0042

* source: Kleijnen et al. (2010); **na**: not available; TC: total cost; SR: stockout rate; μ_a: average of a; and σ_a: standard deviation of a.

Table 3: Stochastic-Constrained COMPASS results for the (s,S) inventory model.

Parameters		Total Cost		Stockout Rate	
s	Q	μ_{TC}	σ_{TC}	μ_{SR}	σ_{SR}
1010	135	641.91	2.7955	0.0996	0.0044
1052	46	634.27	3.1641	0.0979	0.0042
1033	79	635.18	2.9886	0.0997	0.0040
1037	72	634.48	3.0110	0.0997	0.0031
1040	66	634.45	2.5115	0.0987	0.0037
1028	90	635.91	3.0767	0.0999	0.0039
1087	1	643.28	2.9865	0.0996	0.0038
1060	31	634.99	2.7827	0.0993	0.0041
1054	40	633.91	2.6492	0.0998	0.0042
1003	156	645.84	2.6412	0.0988	0.0040

TC: total cost; SR: stockout rate; μ_a: average of a; and σ_a: standard deviation of a.

Table 4: Performance summary.

Proposal	Total Cost			Stockout Rate		
	Average	Min	Max	Average	Min	Max
Stochastic-Constrained COMPASS	637.42	633.91	645.84	0.0993	0.0979	0.0999
DOE-Kri-MP $\tau = 0.15$	637.51	635.49	640.99	0.0990	0.0975	0.1003
DOE-Kri-MP $m_i = 10$	638.56	636.29	639.10	0.0979	0.0975	0.0989
OptQuest (Arena 11) $m_i = 10$	658.09	650.36	668.66	0.0899	0.0865	0.0928
OptQuest (Arena 12) $m_i = 10$	665.21	631.63	716.21	0.0906	0.0714	0.1018

Our algorithm builds on two ideas: a novel simulation allocation rule based on the proposal of Hunter and Pasupathy (2010) and the proof that this new simulation allocation rule obeys the conditions established by Hong and Nelson (2007) for local convergence of any random-search algorithm.

It was shown that the stochastic-constrained COMPASS has a competitive performance in relation to other well known algorithms found in the literature: (a) two algorithms proposed by Andradóttir and Kim (2010) for constrained Ranking & Selection and (b) an algorithm proposed by Kleijnen et al. (2010) for general stochastic-constrained simulation optimization.

Future work shall focus on applying the stochastic-constrained COMPASS on a broader range of applications.

A ACCURACY TEST OF OUR SIMULATION MODEL IMPLEMENTATION

We checked the accuracy of our simulation model implementation with a reference (s,S) model that has a known analytical solution. Karlin (1958) showed that the analytical solution for a (s,S) system with exponentially distributed demand with average λ^{-1} , full back-logging with a penalty cost p applied when a demand is not satisfied, and zero lead time, is given by:

$$Q^* = \sqrt{{}^{2K}/_{\lambda h}} \tag{7}$$

$$s^* = \frac{-\ln\left(\frac{h+\sqrt{2Kh\lambda}}{h+p}\right)}{\lambda} \tag{8}$$

$$E[TC] = \frac{c}{\lambda} + \frac{K + h\left(s - {}^{1}/_{\lambda} + \lambda Q\,(s+0.5Q)\right)\left({}^{(h+p)\,e^{-\lambda s}}/_{\lambda}\right)}{1+\lambda Q} \tag{9}$$

Table 5 shows the comparison of the analytical results with our simulation outcomes for the following parameters: 30 replicates, $c = h = 1$, and number of periods per replicate of 30,000. Δ_{SE} is the representation of the difference between the analytical and simulated $E[TC]$ measured by the number of standard errors.

Table 5: Accuracy experiment for our implementation of the (s,S) inventory problem.

Parameters			Analytical			Our Implementation		Δ_{SE}
λ^{-1}	p	K	s	Q	E[TC]	E[TC]	Standard Error	
200.00	0.00	100.00	0.00	200.00	300.00	299.81	0.22	0.85
200.00	0.00	10000.00	0.00	2000.00	2018.18	2018.27	1.35	0.07
200.00	100.00	100.00	784.39	200.00	1184.39	1183.58	2.82	0.29
200.00	100.00	10000.00	443.45	2000.00	2643.45	2643.40	3.16	0.01
5000.00	0.00	100.00	0.00	1000.00	5166.67	5161.79	5.30	0.92
5000.00	0.00	10000.00	0.00	10000.00	11666.67	11663.78	6.81	0.42
5000.00	100.00	100.00	22163.99	1000.00	28163.99	28169.72	57.10	0.10
5000.00	100.00	10000.00	17582.54	10000.00	32582.54	32610.32	80.12	0.35

REFERENCES

Abspoel, S. J., L. F. P. Etman, J. Vervoort, R. A. van Rooij, A. J. G. Schoofs, and J. E. Rooda. 2001. Simulation based optimization of stochastic systems with integer design variables by sequential multipoint linear approximation. *Structural and multidisciplinary optimization* 22:125–138.

Andradóttir, S. 1998. Simulation optimization. In *Handbook of Simulation*, ed. J. Banks, 307–333. John Wiley & Sons.

Andradóttir, S., and S.-H. Kim. 2010. Fully sequential procedures for comparing constrained systems via simulation. *Naval Research Logistics* 57:403–421.

Atlason, J., M. A. Epelman, and S. G. Henderson. 2008. Optimizing call center staffing using simulation and analytic center cutting-plane methods. *Management Science* 54 (2): 295–309.

Banks, J., J. S. Carson, B. L. Nelson, and D. M. Nicol. 2000. *Discrete event systems simulation*. 3rd ed. Englewood Cliffs, NJ: Prentice Hall.

Cezik, M. T., and P. L'Ecuyer. 2008. Staffing multiskill call centers via linear programming and simulation. *Management Science* 54 (2): 310–323.

Davis, E., and M. Ierapetritou. 2009. A kriging based method for the solution of mixed-integer nonlinear programs containing black-box functions. *Journal of Global Optimization* 43:191–205.

Fu, M. C. 2002. Optimization for simulation: Theory vs. practice. *INFORMS Journal on Computing* 14 (3): 192–215.

Hong, L. J., and B. L. Nelson. 2006. Discrete optimization via simulation using compass. *Operations Research* 54 (1): 115–129.

Hong, L. J., and B. L. Nelson. 2007. A framework for locally convergent random-search algorithms for discrete optimization via simulation. *ACM Transactions on Modeling and Computer Simulation* 17 (4): 1–22. Article 19.

Hunter, S. R., and R. Pasupathy. 2010. Large-deviation sampling laws for constrained simulation optimization on finite sets. In *Proceedings of the 2010 Winter Simulation Conference*, ed. B. Johansson, S. Jain, J. Montoya-Torres, J. Hugan, and E. Yücesan: Piscataway, New Jersey: Institute of Electrical and Electronics Engineers, Inc.

Karlin, S. 1958. Steady state solutions. In *Studies in the Mathematical Theory of Inventory and Production*, ed. K. J. Arrow, S. Karlin, and H. Scarf. Stanford University Press.

Kim, S.-H., and B. L. Nelson. 2006. Selecting the best system. In *Handbooks in Operations Research and Management Science: Simulation*, ed. S. G. Henderson and B. L. Nelson, Volume 13, Chapter 17, 501–534. Elsevier.

Kleijnen, J. P. C., W. van Beers, and I. van Nieuwenhuyse. 2010. Constrained optimization in expensive simulation: Novel approach. *European Journal of Operational Research* 202:164–174.

Law, A. M., and W. D. Kelton. 2000. *Simulation modeling and analysis*. Third ed. McGraw-Hill.

AUTHOR BIOGRAPHIES

HÉLCIO VIEIRA, JUNIOR, a Lieutenant Colonel in the Brazilian Air Force, is a PhD candidate at the Technological Institute of Aeronautics in São José dos Campos, Brazil. He earned a Bachelors in Aeronautics Science at the Brazilian Air Force Academy and a Master's degree in Operations Research from the Federal University of Rio de Janeiro, RJ, Brazil. His research interest is in the area of Design of Experiments and Discrete-Event Simulation applied to military problems. His email address is junior_hv@yahoo.com.br.

KARL HEINZ KIENITZ was born in São Paulo, Brazil. He currently is an Associate Professor of Systems and Control at Technological Institute of Aeronautics (ITA), São José dos Campos, Brazil. He holds B.S. and M.E.E. degrees from ITA and a PhD degree in Electrical Engineering from the Swiss Federal Institute of Technology Zürich. His main research interests are in optimization, decision and robust control for aerospace applications. Karl is a Senior Member of the IEEE. His email address is kienitz@ita.br.

MISCHEL CARMEN NEYRA BELDERRAIN is a Professor at Technological Institute of Aeronautics, São José dos Campos, Brazil. She earned a bachelor in Operations Research, from Universidad Nacional Mayor de San Marcos; a Master in Computer Science – Optimization, from Federal University of Rio de Janeiro, RJ, Brazil; and Ph.D. in Operations Research from Technological Institute of Aeronautics. Her research interests include the following subjects: Multicriteria Decision Making Methods, Problem Structuring Methods (PSM) and OR Education. Her email address is carmen@ita.br.