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DERIVATION AND ASSESSMENT OF INTEREST IN CASH FLOW CALCULATIONS FOR TIME-COST OPTIMIZATIONS IN CONSTRUCTION PROJECT MANAGEMENT

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ABSTRACT

This paper fills a gap in the financial and project management literature of examining how financing fees, particularly interest, are determined accurately for planning and management of cash flows in construction projects. For planning purposes, most models assign costs at the activity level, as individual transactions at their actual date of occurrence as yet unknown. The interplay of cash outflows from numerous purchases, salaries, and payments for materials, labor, and equipment and regular cash inflows from progress payments by the owner to the contractor create a characteristic 'sawtooth' pattern. However, interest calculations for such continuously changing balances traditionally used averaging approximations that deviate from the exact solution. The derivation for such financing fee is presented and its logarithmic expression is compared with the approximations. It is concluded that more detailed research is merited as to how assuming a linearization used in manifold examples of cash flow analysis matches with practice.

1 INTRODUCTION

Business relies on cash flow, the regular outflow and inflow of money, for its vitality and financial health. Construction contractors must carefully plan and manage their cash flow to cover direct costs and payments to subcontractors and suppliers. Shortfalls can be disruptive even if all physical operations progress as planned. Borrowing can temporarily cover such shortfalls, but its financing fees impact profitability. Lack of capital from poor cash flow planning can lead to bankruptcy (Harris and McCaffer 2001) and is a "leading cause of contractor's failure" (Touran et al. 2004), necessitating a very detailed analysis.

1.1 Cash Outflows

Construction contractors face continued individual expenses as work ensues. These cash outflows include intermittent operational expenses, internal payroll, and overhead. In the construction industry, they are assumed *a priori* as linearized within each activity (Kenley 2003) over time or even simplified further. The timing of activities as well as the payment terms to subcontractors and 'trade credit' extended by suppliers may add flexibility, but overall cash outflows incurs continuous growth. Note that this study did not examine how well the linearization assumption holds for typical activities of different types of construction.

1.2 Cash Inflows

Cash inflows are defined as receipts of cash. In the construction industry they can have a lower frequency than in other industries, as bills are paid regularly, e.g. monthly, only after a specific portion of the work is completed. Phasing and payment terms like retainage may further exacerbate these regular accumulated

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payments. Depicting the pay cycle with its billing-to-payment delay gives a stepped pattern that is unbalanced from previous expenses. As contractors may undertake several projects of different sizes and activities in parallel subject to different payment terms, inflows may almost behave as a quasi-random function.

1.3 Importance of Combined Cash Flows

The importance of modeling and optimizing cash outflows versus inflows can hardly be overstated. Combining these disparate and shifted phenomena gives an uneven 'sawtooth' pattern (Kenley 2003) that typically will result in shortfalls. Contractors must therefore augment and 'front-load' their inflows with borrowing. This can take two forms, commercial borrowing from financial institutions with associated fees or self-financing from retained earnings, profits, or direct equity investment (i.e. self-lending) and the associated opportunity cost of capital (e.g. the expected rate of return or interest income that is foregone by utilizing the given cash for other activities). For the purpose of this paper these methodologies are considered equivalent and the discussion focuses on commercial borrowing and its predictable fee structure.

1.4 Financing Fees as Part of Cash Flow

Financing fees are assessed by financial institutions that act as lenders as charges for temporarily turning over funds to borrowers for their beneficial use. Such commercial loans are an important for enabling companies, e.g. construction contractors, to perform ventures that are ultimately profitable, but whose financial burden currently exceeds the liquidity of the company. The *principal* is a term used under two different but closely related scenarios; it denotes the currently remaining amount of money that has been borrowed and is subject to financing fees (from the perspective of an entity borrowing from a bank) and conversely it denotes the currently accumulated amount of money that has been saved and that continues to earn interest (from the perspective of an entity investing in a bank). While companies certainly seek to growth their cash holdings through investing them for interest, their daily transactions from operating capital are often performed in business accounts that are non-interest bearing or have a minimum balance requirement to waive fees. Like the reviewed previous studies, this paper takes the perspective of contractors in order to examine the case of borrowing money from a lender for funding a construction project.

Once financing has been procured, the borrower must perform revenue generating operations whose income exceeds the required payback of principal plus the regularly accumulating financing fees to earn a profit. One type of financing fee, the most well known one, is *interest* on debt. Additional fees are charged by banks for performing various service actions, e.g. printing and mailing certain statements, etc. Their amounts are posted in a catalog and not treated further, as they are known and easily implemented.

An additional type that merits closer examination, however, are fees for opening and maintaining a commercial credit line (up to a credit limit) and that may be incurred for not using credit that was available. These two varieties were termed *commitment fee* (Garner *et al.* 1994). The case of a fixed one-time or periodic fee also is not treated further. However, if the fee depends on the varying amount of unused credit, it is subject to the same time value of money considerations as interest, but has a counteracting effect, i.e. borrowing more increases interest but reduces the unused credit fee and vice versa, up to the maximum of the credit limit. For clarity, it is called *unused credit fee* hereinafter. It was reported that "[o]ften .25 to .5 percent is charged on the unused portion of the credit line" (Garner et al. 1994), whereas studies in more recent literature assumed even higher values, e.g. 0.8% of the unused credit (Elazouni and Metwally 2005). Typically, the percentage of the unused credit fee is lower than the interest rate on actual negative balances, because the bank can likely lend the unused funds to another borrower. In effect, this allows the bank to earn money from the company for any scenario between no borrowing whatsoever to borrowing at the credit limit, while the earnings from interest will typically be the highest.

Financing fees become part of the cash outflows (from the perspective of the borrower) directly after the finish of every period and in previous cash flow models were determined in two different ways; either they were assessed based on one specific balance, e.g. the value at the start of finish of a period, or based on an averaging these values. The following sections examine these different approaches in more detail.

2 LITERATURE REVIEW

Numerous studies presented example calculations of cash flows for construction projects to demonstrate their functioning and to present improvements in analyzing and optimizing the relationship between the timing of activities in the schedule, their direct costs plus any indirect costs, and the rules and limitations imposed by the available credit line. Cui *et al.* (2010) developed a systems model of cash flows that considered interest on borrowing and interest earnings on savings, but calculated it based only on the balance at the finish of each previous period and omitted the unused credit fee. Senouci and El-Rayes (2009) analyzed the tradeoff between timing and costs of different crew configurations versus possible profit after financing fees. They calculated interest based on the finish balance and also omitted the unused credit fee. Elazouni and Metwally (2005) performed optimization with a genetic algorithm and were the only study that explicitly included unused credit. Directly succeeding studies, e.g. Elazouni and Metwally (2007) and Elazouni (2009) did not include it, nor did Liu and Wang (2008) who optimized the same example project with constraint programming. An example by Singh (2001) gave a flowchart of a computer implementation of cash flow calculations but even omitted interest. Halpin and Woodhead (1998) gave a small example whose approach was later used by Senouci and El-Rayes (2009) and – shifted – by Cui et al. (2010).

The literature review reveals that previous studies used several approximations of financing fee *within* one period as per the chronological list of Table 1, where i is the interest rate per period in percent, u is the unused credit rate per period in percent, l is the credit limit, b is the balance, and $t - 1$ and t are indices that mark the ends of the previous and current periods. All of the equations in Table 1 are written using these variables, although some only described the interest calculation in words or it had to be derived from calculated values. It is noted that with one exception all studies omitted the unused credit fee. Related topics were retainage, which most studies included explicitly or at least mentioned, and a potential correction for inflation applied to longer projects, which was omitted by most. All of the studies included detailed discussions of cash flows and most used specific examples to demonstrate their calculations.

In several cases it was unclear how interest was calculated for each period, because it was provided as a percentage without specifying what *basis* it had (e.g. "percent" instead of "percent of the balance at time *x*"). While most percentages appeared to apply per each period, some potential for confusion existed if interest was expressed over a different duration than the periods themselves, e.g. as an *annual percentage rate* (APR) for monthly periods. The *APR* or nominal rate i_{nom} is the interest rate i per period multiplied by the number of periods per year m, $APR = i_{nom} = i \cdot m$. It omits any exponential compounding for periods that each are shorter than one year. While by law the *APR* must be disclosed, it cannot be used directly in calculating the actual financing costs. For example, an $APR = 12\%$ indicates that $i = 1\%$ p.p. (per period), which gives an effective annual rate $EAR = 1.01^{12} = 1.1268$ or 12.68%. The *APR* thus always understates the actual financing costs whenever the interest is actually assessed with the true periodic rate i.

2.1 Approximations Used by Previous Studies

The following two major approximations were identified from the literature review as commonly used:

<u>Approximation 1:</u> Assessing interest based on the finish-of-the-period balances b_t only (or b_{t-1}). This assumes that all costs during the period are incurred immediately after its start and thus approximates their balance as a rectangle. Halpin and Woodhead (1998) described this as interest charged on "the amount of the overdraft at the end of the month". Harris and McCaffer (2001, p. 231) explained further that "[a] measure of the interest payable is obtained by calculating the area between the cash-out and the cash-in curves". While this approach is intuitive, it neglects the *time value of money* within periods and – as will be demonstrated in the following – *overstates* interest by *more than doubling* it for all periods of the cash flow that have the characteristic 'sawtooth' shape. Only ¼ of the 'triangular debt' of the linearization as-

sumption occurs in the first half of the time period but ¾ of it occurs in the second half, for which the borrowing time is always less than half of a period long. This approximation thus is expected to perform worst.

Table 1: Approximations of Interest and Unused Credit Fee in the Literature.

Study	Interest	Unused Credit Fee	Remarks
Cui et al. 2010	Unclear, $i \cdot b_{t-1}$? (given 12% per year or 0.3% per week)	No	Purchase discounts; interest on savings account
Elazouni 2009	$i \cdot b_{t-1} + i \cdot 0.5 \cdot (b_t - b_{t-1})$	No	Retainage; no inflation
Senouci and El-Rayes 2009	$i \cdot b_t$	No	Retainage; no inflation
Liu and Wang 2008	$i \cdot b_{t-1} + i \cdot 0.5 \cdot (b_t - b_{t-1})$	Mentioned	Retainage; no inflation
Elazouni 2007	$i \cdot b_{t-1} + i \cdot 0.5 \cdot (b_t - b_{t-1})$	No	No retainage; no inflation
Elazouni and Metwally 2007	Unclear, $i \cdot b_{t-1} + i \cdot 0.5 \cdot (b_t - b_{t-1})$? (given 0.3% per week)	No	Retainage; no inflation
Chen et al. 2005	No	No	No retainage; mentions inflation
Elazouni and Metwally 2005	$i \cdot b_{t-1} + i \cdot 0.5 \cdot (b_t - b_{t-1})$	$u \cdot (l - b_{t-1})$	Retainage; no inflation
Motawa et al. 2005	Mentioned, company-level model	No	No retainage; but inflation
Elazouni and Gab-Allah 2004	$i \cdot b_{t-1} + i \cdot 0.5 \cdot (b_t - b_{t-1})$	No	Retainage mentioned
Touran *et al.* 2004	Mentioned as APR (no details)	No	Retainage; no inflation
Akgun 2001	Mentioned as APR, like Halpin and Woodhead (1998)	No	Retainage; no inflation
Barbosa and Pimentel 2001	Unclear, $i \cdot b_{t-1}$?	No	None mentioned
Harris and McCaffer 2001	Mentioned as APR (15 per year / 12 months per year)	No	Mentions retainage; no inflation
Khosrowshahi 2001a	Mentioned, different for borrowing and investing	No	Retainage; no inflation
Khosrowshahi 2001b	Mentioned, different for borrowing and investing	No	Mentions retainage; no inflation
Singh 2001	No	No	None mentioned
Halpin and Woodhead 1998	$i \cdot b_t$	No	Retainage; no inflation
Navon 1995	No	No	Retainage; no inflation
Kirkpatrick 1994	Yes (not explained)	No	None mentioned
Au and Hendrickson 1986	$i \cdot b_{t-1} + i \cdot 0.5 \cdot (b_t - b_{t-1})$	No	Mentions retainage; inflation, but payments already start after 1 period
McCaffer 1979	Mentioned as APR (15 per year / 12 months per year)	No	Mentions retainage; no inflation
Ashley and Teicholz 1977	$i \cdot b_t$	No	Mentions retainage; no inflation
Reinschmidt and Frank 1976	Mentioned (no details)	No	None mentioned
Fondahl 1973	Mentioned as compound interest	No	None mentioned
Peterman 1973	Mentioned (no details)	No	Retainage; no inflation

Approximation 2: Assessing it based on any start-of-the-period balance b_{t-1} plus the interest rate multiplied by one half of the difference between the finish-of-the-period and the start-of-the-period balances b_t and b_{t-1}. This approach assumes that 50% of the difference between the start and finish balances is incurred immediately after the start of the period and no additional costs thereafter. It approximates their balance as a rectangle of half the height over the full period. This constitutes an *average balance* approach that again ignores the *time value of money* within periods for any linearly growing balance, as all previous studies clearly assumed for their cash flows. Its approximation distributes the debt as ½ in the first half of the period and ½ in the second half. The amount of interest alone is calculated by halving the interest rate used in this approximation.

Yet another approximation is theoretically possible, although it was not used in any previous studies:

Approximation 3: Assessing it based on the assumption that none of the costs are incurred during the first half of the period but 100% of the difference between the start and finish balances during the second half. It approximates them as a delayed rectangle of half the width, with the same area as 2. This approach would still be an approximation but better reflects the *time value of money* within periods by unbalancing the borrowed amount to the second half of one time period.

Equations (1) through (3) express these three possible approximating approaches mathematically.

$FV = A \cdot (1+i)$ for approximation 1. (1)

$FV = A \cdot (1+i)/2$ for approximation 2, where the interest portion itself is $A \cdot i/2$. (2)

$FV = A \cdot (1+i)^{\frac{1}{2}}$ for approximation 3. (3)

2.2 Need for Derivation of Exact Equation

Identifying two different approximations for calculating interest as used in the literature, both of which do not reflect the compounding nature of interest for linearly growing balances and may significantly deviate from the true value, plus one analogously possible but hitherto unused approximation that is expected to give results of somewhat better accuracy show a clear need to derive the exact equation. The following section gives the background from financial mathematics for repeated payments and use its approach of taking the difference between two series that are shifted by one period as inspiration for the new equation.

3 DERIVATION OF EXACT EQUATION

3.1 Time Value of Money Derivation for Annuity

Among the most common equations used in financial management are those that describe the time value of money, i.e. the behavior of funds that are subject to financing fees over time, which themselves often are compounded, i.e. that interest is draws interest itself. An individual payment of the *present value PV* that draws the periodic interest rate *i* thus generates the *future value* $FV = PV \cdot (1+i)^n$ over the duration of *n* periods. Inverting this equation isolated the required *PV* for a desired *FV* of an investment decision.

Payments are often repeated each period, e.g. monthly payments to grow a savings account or pay off a loan. An *annuity* is a finite series of such payments, whose amount is often constant. The derivation of its equation (e.g. Park 2008, Newnan et al. 2004) inspires deriving the new equation for exact interest and unused credit fee on changing balances, which was lacking in previous cash flow models and has not been identified in any of the literature. Writing an annuity as a series with exponential interest gives equation (4), whose terms are sorted from left to right by the decreasing number of periods over which the compounding has occurred. The payments *A* are assumed to begin at the finish of the first period (a so-called ordinary or annuity-immediate), not at the start, and continue to be paid over n periods. Note that their durations in the exponent of each term in (4) are the number of periods n that remain until the final payment.

$$A \cdot (1+i)^{n-1} + A \cdot (1+i)^{n-2} + \ldots + A \cdot (1+i)^{1} + A \cdot (1+i)^{0} = FV_{annuity} \tag{4}$$

The series of additive terms in equation (5) mathematically is a geometric series written in the inverse order. Assuming a second independent stream of payments A that already begins at the start of the first period (a so-called annuity-due, which represents the common case of e.g. investing in a savings account, etc.) gives equation (5), whose terms are shifted by the duration of one period from their counterparts.

$$A \cdot (1+i)^{n} + A \cdot (1+i)^{n-1} + \ldots + A \cdot (1+i)^{2} + A \cdot (1+i)^{1} = FV_{annuity} \cdot (1+i) \tag{5}$$

Subtracting equation (4) from (5) cancels out all terms except for the very first and last ones in (6).

$$FV_{annuity} \cdot (1+i) - FV_{annuity} = A \cdot (1+i)^{n} - A \cdot (1+i)^{0} \tag{6}$$

Extracting FV on the left side of equation (6) and dividing by the interest rate i gives the FV in (7).

$$FV_{annuity} = A \cdot \frac{(1+i)^{n} - 1}{i} \tag{7}$$

The elegance of equation (6) lies in its *collapsing* nature, i.e. it comprises a series of n payments, each with their own streams of compounding interest, which are staggered over time and 'stacked' additively to the final FV. Using an analogous approach, the PV of a limited series of annuity payments can be derived to be $PV_{annuity} = A \cdot \left(1 - (1/1+i)^{n}\right)/i$. Moreover, financial texts, e.g. Newnan et al. (2004) also provide the PV for series whose payment amounts grows either *arithmetically* or *geometrically* from one period to the next one. These extensions of equation (7) or its inverted $PV_{annuity}$ can also be derived by again assuming series whose difference is collapsed to its short solution. For an arithmetic, i.e. linear, growth of the integer multiples of a payment A for an annuity-immediate, i.e. paying $0 \cdot A$, then $1 \cdot A$, then $2 \cdot A$, etc. at the finish of each period, the present value is $PV = A \cdot \left(\left((1+i)^{n} - i \cdot n - 1\right)/\left(i^{2} \cdot (1+i)^{n}\right)\right)$. For a geometric, i.e. exponential, growth of a payment A that is multiplied by $(1+g)$ where g is the growth rate per period, for an annuity immediate, the present value is $PV = A \cdot \left(\left(1 - \left((1+g)^{n} \cdot (1+i)^{-n}\right)\right)/(i-g)\right)$ (Newnan et al. 2004). Shortcut equations also exist e.g. to convert annuities of variable payments into annuities with constant payments (note that using the terms increasing or decreasing annuities can be misleading, as they may denote e.g. adding to an investment or e.g. drawing from retirement savings). The issue of the varying number of calendar days in each period for higher frequencies of compounding is excluded here for brevity.

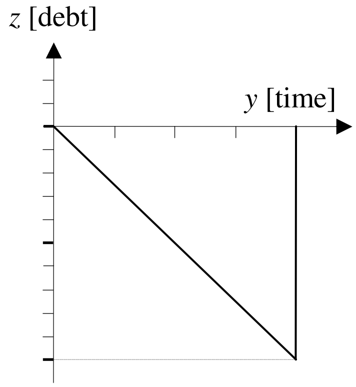
However, the problem that is analyzed in this paper asks a more detailed question over a shorter duration – that applied to all cash flow models of previous studies, but does not appear to have been asked yet – what is the exact interest and any other financing fees if the balance changes already *within one period*?

3.2 Derivation for Interest

Assume a linearly growing balance over one period as shown in Figure 1a. To calculate the exact interest on such triangular debt it is decomposed into a series of stepped borrowings as shown in Figure 1b. Each of these incremental borrowings generates a separate stream of interest, all of which must be added. Note that the previous annuity payment A is now divided into k portions within one period as per equation (8).

$$FV_{annuity} = \sum_{j=0}^{k-1}\left[\frac{A}{k}\cdot(1+i)^{\frac{j}{k}}\right] = \frac{A}{k}\cdot\left((1+i)^{\frac{0}{k}} + (1+i)^{\frac{1}{k}} + \ldots + (1+i)^{\frac{k-2}{k}} + (1+i)^{\frac{k-1}{k}}\right) \tag{8}$$

where *j* is a variable counting index within the total number of incremental borrowings *k* and *i* is the interest rate. Each of the *k* terms draws $(1+i)^{\frac{j}{k}}$ in interest over its partial duration *j* within one complete period. They are sorted by *j* from zero to $k-1$, thus inverting the decreasing order used in equation (4).



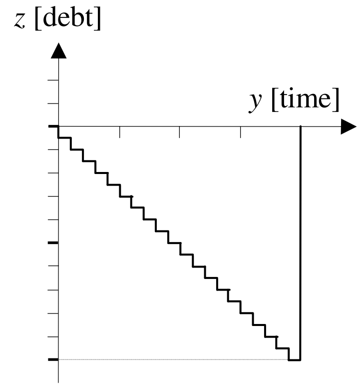


Figure 1a: Linearly Growing Debt. Figure 1b: Incremental Decomposition.

Equation (9) again is a *geometric series*, because it is finite and each term is the $(1+i)^{\frac{1}{k}}$ -fold of the previous one. Converting this series into an expression that is independent of *j* first uses an approach that is inspired by equation (6). It creates the second stream of equation (9) that is shifted by one incremental period against equation (8). Equation (10) subtracts equation (8) from equation (9). This technique of canceling out all interior terms is very similar to *telescoping series* that collapses to a single value, but here is deliberately introduced for taking the difference of two overlapping versions of the same series.

$$FV_{annuity}\cdot(1+i)^{\frac{1}{k}} = \frac{A}{k}\cdot\left((1+i)^{\frac{1}{k}} + (1+i)^{\frac{2}{k}} + \ldots + (1+i)^{\frac{k-1}{k}} + (1+i)^{\frac{k}{k}}\right) \tag{9}$$

$$FV_{annuity}\cdot(1+i)^{\frac{1}{k}} - FV_{ann} = \frac{A}{k}\cdot(1+i)^{\frac{k+1}{k}} - \frac{A}{k}\cdot(1+i)^{\frac{0}{k}} \tag{10}$$

Equation (11) extracts common factors from (10). Dividing it by the bracket on its left half gives (12).

$$FV_{annuity}\cdot\left((1+i)^{\frac{1}{k}} - 1\right) = \frac{A}{k}\cdot\left((1+i)^{\frac{k+1}{k}} - 1\right) \tag{11}$$

$$FV_{annuity} = \sum_{j=0}^{k-1}\left[\frac{A}{k}\cdot(1+i)^{\frac{j}{k}}\right] = A\cdot\frac{(1+i)^{\frac{k+1}{k}} - 1}{k\cdot\left((1+i)^{\frac{1}{k}} - 1\right)} \tag{12}$$

The geometric series, a summation of the *k* portions of the annuity payment *A*, has thus first been converted into a ratio whose value depends only on *k* and is now independent of the previous variable *j*. Second, finding its exact value so that it does not even depend on *k* anymore requires making the duration of each incremental step infinitesimally small, i.e. equivalently making their number *k* infinitely large, i.e. $k \to \infty$. Following the mathematical rule of equation (13) that the limit of a fraction of two any functions

$f_1(k)$ and $f_2(k)$ is the fraction of their limits, the limit operator can be applied separately to the numerator and denominator of equation (12) to give equation (14) that is examined further for its limit values.

$$\lim_{k\to\infty}\left[\frac{f_1(k)}{f_2(k)}\right]=\frac{\lim_{k\to\infty}[f_1(k)]}{\lim_{k\to\infty}[f_2(k)]} \tag{13}$$

$$\lim_{k\to\infty}\left[\sum_{j=0}^{k-1}\left[\frac{1}{k}\cdot(1+i)^{\frac{j}{k}}\right]\right]=\frac{\lim_{k\to\infty}\left[(1+i)^{\frac{k+1}{k}}-1\right]}{\lim_{k\to\infty}\left[k\cdot\left((1+i)^{\frac{1}{k}}-1\right)\right]} \tag{14}$$

The numerator of equation (14) is determined by inserting the variable *k* directly into its exponent, which cancels out for $k\to\infty$ in (15). The entire term is thus simplified to being the interest rate *i* itself.

$$\lim_{k\to\infty}\left[(1+i)^{\frac{k+1}{k}}-1\right]=(1+i)^{\frac{\infty+1}{\infty}}-1=i \tag{15}$$

The denominator of (14) is determined by replacing $h=1/k$ and $1+i=v>1$ temporarily. This leads to the somewhat surprising finding that the term approaches the natural logarithm of $1+i$ for $k\to\infty$ in (16).

$$\lim_{k\to\infty}\left[k\cdot\left((1+i)^{\frac{1}{k}}-1\right)\right]=\lim_{h\to 0}\left[\frac{1}{h}\cdot\left((1+i)^h-1\right)\right]=\lim_{h\to 0}\left[\frac{1}{h}\cdot\left(v^h-1\right)\right]=\ln(v)=\ln(1+i) \tag{16}$$

Finally, equation (17) provides the final new expression for interest on a linearly growing balance.

$$FV_{annuity}=A\cdot\frac{i}{\ln(1+i)} \tag{17}$$

A comprehensive literature search, including scholarly papers as well as various textbooks on accounting, finance, and economics, did not identify any previous publication of this exact calculation of the interest, as all of them focus on discrete, period-by-period calculations, not this continuous phenomenon.

3.3 Derivation for Unused Credit Fee

As mentioned before, the unused credit fee has a counteracting effect to the interest. From the bank's perspective it ensures earnings even if the borrower does not use the available credit line. It is also subject to the *time value of money* within periods and has a linearly shrinking balance. It applies *u* to the difference between credit limit *l* and b_{t-1} (i.e. a triangle or even a trapezoid if $l < b_t$) by deducting (17) from a 'block' to give the fee as $(l-b_{t-1})\cdot(u-((u/\ln(1+u))-1))$. Just like interest, is it assessed and charged directly after the finish of every period and contributes to the compounding of negative balances and financing fees.

4 ANALYTICAL COMPARISON

The behavior of these approximations is now examined for a unit model with a duration of only one period over which the interest accrues. For brevity, the unused credit fee is excluded. Figure 2 shows equations (1) through (3) graphically as gray shaded areas overlaid upon the linearly growing cash outflow.

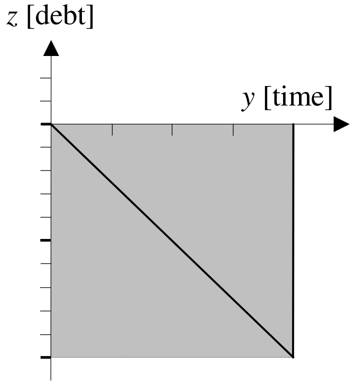


Figure 2a: Approximation 1.

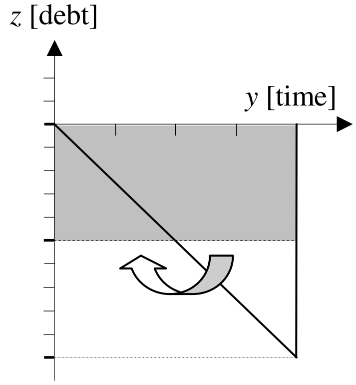


Figure 2b: Approximation 2.

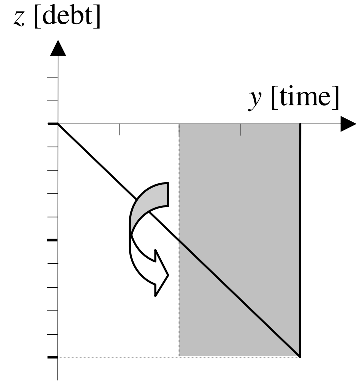


Figure 2c: Approximation 3.

Figure 3 shows the actual interest that accrues on a linearly growing balance over one period. The balance itself remains constant and can be assumed as an even value, e.g. $100. These approximations depend only on the periodic interest rate *i* as the independent variable with the actual interest costs as the dependent variable. It is normalized to a percentage of the finish-of-period balance *A* to allow neutral comparisons. The possible input *i* ranges from 0% to 50% to cover any interest rates that could be reasonably expected to occur. The diagonal of $A \cdot (1+i)$ describes a linear relationship of input and output.

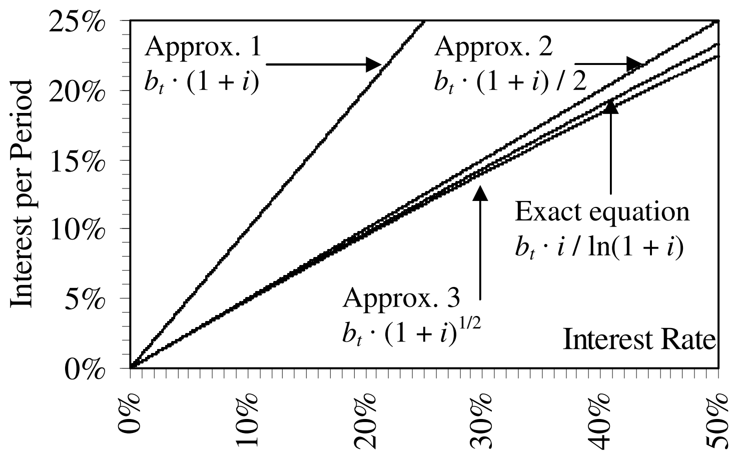


Figure 3: Exact Interest and Approximations within Period.

Figure 4 is based on Figure 3 but provides comparisons of the *relative differences* between the three approximations and their baseline, the new exact equation, thus showing only three curves. Examining the steepness and growth rates of the interest curves in Figure 3 and the deviation curves in Figure 4 shows that approximation 1, used recently by Senouci and El-Rayes (2009) and also by Halpin and Woodhead (1998), strongly overestimates interest to more than doubling the exact value. The rectangular approximation 1 of the interest based on only the finish-of-the-period balance b_t should therefore not be used in any cash flow calculations, where balances are continuously changing and offset only by periodically stepped cash inflows from progress payments. An important observation is made for the average balance approximation 2 (Liu and Wang 2008, Elazouni and Metwally 2005, Au and Hendrickson 1986). The curve is consistently slightly above the curve with the exact values. Moreover, it is also concave, thus deviating stronger from the exact value for larger interest rates. In other words, this approximation of financing fees that construction contractors incur from using their overdraft business account is always slightly in favor of the bank. The larger i and/or the balance itself, the larger becomes this deviation. This effect is not due to any rounding error, but results from the purposeful averaging, which creates the balance of 50% of A that is subject to compound interest over the entire period. Figure 2b shows that adding such triangular area during the first half of the period and subtracting it from the second half (indicated by an arrow and a dashed line) significantly increases the duration over which the bulk of interest accumulates. Such 'redistribution' of areas within the cash flow diagram to simplify interest calculations ignores the time value of money for its compounding. Interest thus is overstated, which is contrary to the cost minimization strategy of the contractor. While the percentage of the deviation itself is small, as it is applied to a percentage, the dollar amount of such deviation can give an order of hundreds of dollars monthly or thousands of dollars total for large multi-month projects whose cash flows and respective balances that must be financed can reach the order of millions of dollars monthly. Note that this consideration applies regardless of whether a contractor finances via a bank or uses their own capital; in the latter case the financing costs would be incurred as opportunity costs from not being able to invest said capital elsewhere for growth.

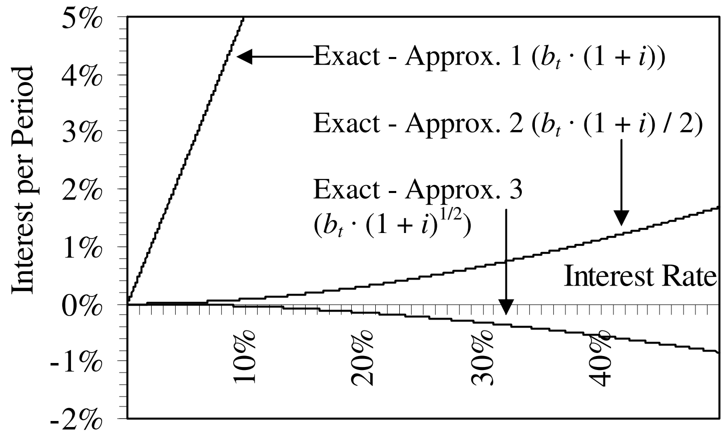


Figure 4: Relative Differences of Exact Interest and Approximations.

The new approximation 3 as per equation (3) redistributes the area between the first and second halves of the period and provides a counterpoint to (2). Once again, the balance that is financed remains constant, but shifting all of it into the second half of the period significantly decreases the duration over which interest accumulates. Figures 3 and 4 confirm that it always slightly underestimates the exact value. The larger i and/or the balance itself, the larger becomes this deviation due to its convex shape. The exponential growth of compound interest causes the underestimation to be smaller in absolute value than the overestimation of approximation 2. In light of the simplicity of the exact equation (17), neither of the approximation with their exacerbating positive or negative deviations is recommended for use in practice.

5 CONCLUSIONS

In an ever more competitive marketplace, calculating financing fees precisely and being able to perform a detailed analysis toward minimizing them is a competitive advantage. While the dollar amount of the inconsistency in applying the time value of money is small in comparison to the balances that must be financed, it should nonetheless be taken seriously. In practice, interest is typically charged on average daily balances in a way that is favorable to the lender and worsens with increased balances and today's preponderance of prolonged schedules and mega-projects. The current outward focus – using period approximations of interest – rather than inward, i.e. within the period of triangular debt, falls short of the proverbial 'valuing the cent to earn the dollar'. The new exact model can produce greater accuracy and positive financial results. It is recommended that future research perform a comprehensive study of actual cost data to validate – or augment – the widely used assumption of linearized cash outflows of individual activities.

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