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ESTIMATING GREEKS FOR VARIANCE-GAMMA

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ABSTRACT

Assuming the underlying assets follow a Variance-Gamma (VG) process, we consider the problem of estimating sensitivities such as the Greeks on a basket of stocks when Monte Carlo simulation is employed. We focus on a class of derivatives called mountain range options, comparing indirect methods (finite difference techniques such as forward differences) and two direct methods: infinitesimal perturbation analysis (IPA) and the likelihood ratio (LR) method, where the latter is also implemented via a recently proposed numerical technique developed by Glasserman and Liu (2007) using the characteristic function. We carry out numerical simulation experiments to evaluate the efficiency of the different estimators and discuss the strengths and weakness of each method.

1 INTRODUCTION

Simulation-based derivative estimates are useful in financial engineering in estimating the Greeks, which are critical for hedging financial derivatives such as options. Gradient estimation techniques were first applied to option pricing using infinitesimal perturbation analysis (IPA) for European and American options by Fu and Hu (1995) and using both IPA and the likelihood ratio (LR) method for European and Asian options by Broadie and Glasserman (1996); see also Glasserman (2004) and Fu (2006, 2008) for more details on Monte Carlo simulation for financial engineering and various methods for estimating the Greeks using simulation.

The Variance-Gamma (VG) process was introduced to the finance community as a model for log-price returns and option pricing by Madan and Seneta (1990), and developed in Madan and Milne (1991) and Madan, Carr, and Chang (1998). Fu (2007) gives a general introduction to the VG Process in the context of stochastic (Monte Carlo) simulation and shows how to price and simulate the stock price. Hall (2009) considered gradient estimation for a class of financial derivatives on a basket of stocks called mountain range options under an asset price model of geometric Brownian motion. This paper also considers gradient estimation for mountain range options, but assuming that the underlying assets follow VG processes. We derive IPA and LR estimators for the various sensitivities where applicable and compare them in numerical experiments with each other and with finite difference estimates. We also compare these estimators with an LR estimator using the recently developed numerical approximation technique of Glasserman and Liu (2007), which relies only on the characteristic function and does not require the explicit probability density function for the transition. This method is especially relevant for simulation of Lévy processes, where the characteristic function is readily available. We discuss the strengths and weaknesses of each method.

2 BACKGROUND

2.1 Problem Setting

The objective is to estimate

$$\frac{\partial V}{\partial \xi},$$

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where V is the value (or price) of the financial derivative and ξ is the parameter of interest. For example, if V is the price of an option written on a single underlying stock and ξ is a current stock price, then this would correspond to estimating the most famous financial Greek, the "Delta" of the option in Hull(2002).

In this paper, the derivative price will take the following form:

$$V = e^{-rT} E[J_T],$$

where T is the maturity or expiration date, r is the risk-free interest rate (assumed deterministic and constant), and J_T is an option payoff function. The setting assumes that the expected payoff $E[J_T]$ cannot be easily computed, so that Monte Carlo simulation is required to estimate it. This paper focuses on options written on a basket of underlying assets following VG processes.

2.2 Variance-Gamma Process

The VG process is a Lévy process, having independent and stationary increments, with three parameters σ, ν, θ. The characteristic function of the process at a fixed time T, VG(σ, ν, θ, T), is given by

$$\phi_{VG}(u, \sigma, \nu, \theta, T) = (1 - iu\theta\nu + 0.5\sigma^2\nu u^2)^{-T/\nu}. \tag{1}$$

Two ways to define the VG process $\{X_t\}$ are as follows:

- **Gamma-time-changed Brownian motion**, with the subordinator being a gamma process. Let $\{W_t\}$ denote standard Brownian motion, $B_t^{\mu,\sigma} = \mu t + \sigma W_t$ denote Brownian motion with constant drift rate μ and volatility σ, $\gamma_t^{(\nu)}$ be the gamma process with drift $\mu = 1$ and variance parameter ν.

$$X_t = B^{(\theta,\sigma)}_{\gamma_t^{\nu}} = \theta\gamma_t^{(\nu)} + \sigma W_{\gamma_t^{(\nu)}}. \tag{2}$$

- **Difference of two gamma processes**. Let $\gamma_t^{(\mu,\nu)}$ be the gamma process with drift parameter μ and variance parameter ν.

$$X_t = \gamma_t^{\mu_+,\nu_+} - \gamma_t^{\mu_-,\nu_-}, \tag{3}$$

  where $\mu_{\pm} = (\sqrt{\theta^2 + 2\sigma^2/\nu} \pm \theta)/2$, and $\nu_{\pm} = \mu_{\pm}^2 \cdot \nu$.

Under the risk-neutral measure, with no dividends, the stock price is given by

$$S_t = S_0 \exp((r + \omega)t + X_t),$$

where $\omega = \ln(1 - \theta\nu - \sigma^2\nu/2)/\nu$ is the parameter that makes the discounted asset price is a martingale, which makes $E[e^{-rt}S_t] = S_0$.

In Madan, Carr and Chang(1998), the density function of the log-price $z = \ln(S_t/S_0)$ is

$$h(z) = \frac{2\exp(\theta x/\sigma^2)}{\nu^{t/\nu}\sqrt{2\pi}\sigma\Gamma(\frac{t}{\nu})} \left(\frac{x^2}{2\sigma^2/\nu + \theta^2}\right)^{\frac{t}{2\nu} - \frac{1}{4}} \kappa_{\frac{t}{\nu} - \frac{1}{2}}\left(\frac{1}{\sigma^2}\sqrt{x^2(2\sigma^2/\nu + \theta^2)}\right), \tag{4}$$

where κ is the modified Bessel function of 2nd kind, and $x = z - rt - \frac{t}{\nu}\ln(1 - \theta\nu - \sigma^2\nu/2)$.

2.3 LR Method using Characteristic Function

Glasserman and Liu (2007) propose an LR method to estimate the Greeks using only the characteristic function by numerically approximating the density function $g_\xi(x)$ and derivative of the density function $\frac{dg_\xi(x)}{d\xi}$. The algorithm is as follows: Pick a finite grid of x values, pre-compute values of $G_\xi, g_\xi, \dot{g}_\xi$, through numerical transform inversions.

- Using the Abate-Whitt (1992) algorithm, each transform inversion is approximated using a finite weighted sum of transform values:

$$I^{N,h}_{\xi,x}(L_f) = \frac{he^{\sigma x}}{2\pi} L_f(\sigma) + \frac{he^{\sigma x}}{\pi} \sum_{k=1}^{N} \Big(\mathrm{Re}[L_f(\sigma + ikh)]\cos(khx) - \mathrm{Im}[L_f(\sigma + ikh)]\sin(khx) \Big),$$

where N is the truncation point, and L_f is the characteristic function. Then calculate G_j on the grid, pick $\sigma_+ \in (0, \sigma_u)$ and $\sigma_- \in (\sigma_l, 0)$ and let

$$G_j = \begin{cases} I^{N,h}_{\sigma_+, x_j}(L_{G_\xi}), & if \;\; x_j \leq 0; \\ 1 - I^{N,h}_{\sigma_-, x_j}(L_{\bar{G}_\xi}), & if \;\; x_j > 0. \end{cases}$$

- Generate $\hat{X}$ from the approximation $\hat{G}_\xi$ by setting $X = \hat{G}_\xi^{-1}(U), U \sim U(0,1)$:

$$\hat{X} = \frac{U\delta + x_{j-1}G_j - x_j G_{j-1}}{G_j - G_{j-1}}.$$

- At each simulation step, approximate g_ξ and $\dot{g}_\xi$ through G_j:

$$\hat{g}_\xi(x) = \begin{cases} (G_j - G_{j-1})/\delta, & if \;\; x \in [x_{j-1}, x_j), j \in J \\ 0, & if \;\; x < x_{\min} \;\; or \;\; x > x_{\max} \end{cases}$$

$$\dot{\hat{g}}_\xi(x) = \begin{cases} (\dot{G}_j - \dot{G}_{j-1})/\delta, & if \;\; x \in [x_{j-1}, x_j), j \in J \\ 0, & if \;\; x < x_{\min} \;\; or \;\; x > x_{\max} \end{cases}$$

  where $\dot{G}_j \approx \dot{G}_\xi(x_j)$ is calculated through $\dot{G}_\xi = \frac{dG_\xi}{d\xi}$. Then estimate the approximation score function $\hat{S}_\xi = \frac{\dot{\hat{g}}_\xi}{\hat{g}_\xi}$ at X.
- At the end of each path, return the LR estimate $e^{-rT} J_T(\hat{X}) \hat{S}_\xi(\hat{X})$ with the option price estimate $e^{-rT} J_T(\hat{X})$.

To distinguish this from the usual LR estimator, we will henceforth refer to it as the GL estimator.

3 MOUNTAIN RANGE OPTIONS

Mountain range options are exotic options originally marketed by Société Générale in 1998; see also Overhaus (2002), Quessette (2002) and Meaney(2007). The options combine characteristics of basket options and range options by basing the value of the option on several underlying assets, and by setting a time frame for the option. Here, we will consider only the case where the underlying assets are independent, and treat four types of mountain range options: Everest, Atlas, Altiplano/Annapurna, and Himalayan. To price these options, we take $(X_1(t), X_2(t))^T$ as a two-dimensional independent VG process. Since they are independent, we can deal with them separately. Two different ways of representing each X_i are as follows:

- Gamma-time-changed Brownian motion

$$X_j(t) = B^{(\theta_j, \sigma_j)}_{\gamma_j^{(\nu_j)}(t)} = \theta_j \gamma_j^{(\nu_j)}(t) + \sigma_j W_{\gamma_j^{(\nu_j)}(t)}, \text{for } \; j = 1, 2.$$

- Difference of two gamma processes

$$X_j(t) = \gamma_j^{(\mu_j^+, \nu_j^+)}(t) - \gamma_j^{(\mu_j^-, \nu_j^-)}(t),$$

  where

$$\mu_j^\pm = (\sqrt{\theta_j^2 + 2\sigma_j^2/\nu_j} \pm \theta_j)/2 \text{and} \nu_j^\pm = (\mu_j^\pm)^2 \cdot \nu_j \;\; , \text{for } \; j = 1, 2.$$

The characteristic function of VG process $X_j(t)$ is given by

$$\phi_{VG}(u, \sigma_j, \nu_j, \theta_j, t) = (1 - iu\theta_j\nu_j + 0.5\sigma_j^2 \nu_j u^2)^{-t/\nu_j}. \tag{5}$$

Under the risk-neutral measure, the stock price would be

$$S_j(t) = S_0(t) \exp((r + \omega_j)t + X_j(t)),$$

where $\omega_j = \frac{1}{\nu_j} \log(1 - \theta_j \nu_j - \sigma_j^2 \nu_j / 2)$, for $j = 1, 2$.

The density function of log-price $z_j = ln(S_j(t)/S_0(t))$ is:

$$h_j(z_j) = \frac{2\exp(\theta_j x_j/\sigma_j^2)}{\nu_j^{t/\nu_j}\sqrt{2\pi}\sigma_j\Gamma(\frac{t}{\nu_j})}\left(\frac{x_j^2}{2\sigma_j^2/\nu_j+\theta_j^2}\right)^{\frac{t}{2\nu_j}-\frac{1}{4}}\kappa_{\frac{t}{\nu_j}-\frac{1}{2}}\left(\frac{1}{\sigma_j^2}\sqrt{x_j^2(2\sigma_j^2/\nu_j+\theta_j^2)}\right), \tag{6}$$

where κ is the modified Bessel function of 2nd kind, and $x_j = z_j - rt - \frac{t}{\nu_j}\ln(1-\theta_j\nu_j - \sigma_j^2\nu_j/2)$, for $j = 1,2$.

3.1 Everest Option

Given n stocks $S_1, S_2, \cdots, S_n$, the payoff for an Everest option is given by

$$J_T = \min_{i=1,\cdots,n}\left(\frac{S_i^T}{S_i^0}\right). \tag{7}$$

Notice that the payoff function is a continuous and monotonically non-decreasing piecewise linear function of S_i^T.

3.1.1 IPA for Everest Option

The IPA estimator is given by

$$\begin{aligned}\frac{dJ_T}{d\xi} &= \sum_{i=1}^{n}\frac{dJ_T}{dS_i^T}\frac{dS_i^T}{d\xi}\\ &= \sum_{i=1}^{n}\left(\frac{1}{S_i^0}\frac{dS_i^T}{d\xi} - \frac{S_i^T}{(S_i^0)^2}\frac{dS_i^0}{d\xi}\right)1_{\{\frac{s_i^T}{s_i^0}\le\frac{s_j^T}{s_j^0},\forall j\neq i\}},\end{aligned}$$

and we find that (as pointed out in Hall 2009)

$$\Delta = \frac{dJ_T}{dS_i^0} = \sum_{i=1}^{n}\left(\frac{1}{S_i 0}\frac{S_i^T}{S_i^0} - \frac{S_i^T}{(S_i^0)^2}\right) = 0. \tag{8}$$

For other Greeks, $\frac{dS_i^0}{d\xi} = 0$, where ξ could be $\sigma_i, \nu_i, \theta_i, T$. Therefore

$$\frac{dJ_T}{d\xi} = \frac{1}{S_i^0}\frac{dS_i^T}{d\xi}1_{\{\frac{s_i^T}{s_i^0}\le\frac{s_j^T}{s_j^0},\forall j\neq i\}}. \tag{9}$$

The derivative of S_i^T w.r.t to different parameters ξ are given as in Fu (2007).

3.1.2 LR for Everest Option

The LR estimator is

$$\min_{i=1,\cdots,n}\left(\frac{S_i^T}{S_i^0}\right)\frac{d\ln f(X_1^T, X_2^T, \cdots, X_n^T;\xi)}{d\xi}, \tag{10}$$

where $n = 2$, $f(X_1^T, X_2^T;\xi) = h_1(z_1)\cdot h_2(z_2)$, where $h_1(z_1)$ and $h_2(z_2)$ are the density functions of X_1^T, X_2^T in (6) for $j = 1,2$ respectively. Due to space limitations, the details of the calculation of $\frac{d\ln f(X_1^T, X_2^T;\xi)}{d\xi}$ are not included here, but can be found in Cao and Fu (2010).

3.1.3 GL for Everest Option

We use the LR estimates as in (10), but approximate $h_i(z_i)$ and $\frac{dh_i(z_i)}{d\xi}$ separately through the characteristic function as in (5).

Table 1: Everest option simulation results

VG1	Rho	Theta	$\frac{dV}{d\sigma_1}$	$\frac{dV}{d\sigma_2}$	$\frac{dV}{d\nu_1}$	$\frac{dV}{d\nu_2}$	$\frac{dV}{d\theta_1}$	$\frac{dV}{d\theta_2}$
FD	0.1953469	-0.05653285	-0.0990439	-0.1190807	-0.05605222	-0.0098866	-0.031017	-0.0248973
StdErr	8.55E-05	0.03169108	0.0027073	0.0026424	0.03232712	0.03254652	0.0012334	0.001405
IPA	0.195152	0.338435	-0.097177	-0.117733	-0.053776	-0.036777	-0.030057	-0.023645
StdErr	8.54E-05	0.0317364	0.0027252	0.00265496	0.0078798	0.01990643	0.001247	0.001421
LR		-0.794334	3.94683	2.106137	1.937903	1.404842	-1.318293	-0.895224
StdErr		0.027162	0.078509	0.049966	0.02226	0.017368	0.023043	0.016883
VG2								
FD	0.1952389	-0.06273382	-0.1072506	-0.1182797	0.00896037	0.00838822	-0.0314099	-0.0216959
StdErr	8.54E-05	0.06059977	0.00256686	0.0025537	0.05930252	0.05833827	0.0009187	0.00095
IPA	0.195043	0.4117492	-0.012775	-0.014911	-0.1373324	-0.1010475	0.021193	0.030463
StdErr	8.53E-05	0.02875367	0.000124	0.0001542	0.01560542	0.012627	0.001825	0.001848
LR		-1.151322	4.270414	2.120657	2.177208	1.469639	-1.400568	-0.880311
StdErr		0.0277723	0.080310	0.049307	0.023406	0.018371	0.023728	0.016677
GL FD		-0.04114368	-0.0915674	-0.100493	-0.0030287	0.0021092	-0.0330786	-0.02243387
StdErr		0.0013093	0.00244272	0.0024285	0.0002222	0.0002243	0.000707	0.0006621
GL		0.257925	0.836115	0.67904	0.779812	0.642608	-0.073425	-0.051864
StdErr		0.025401	0.0593164	0.0404484	0.029419	0.028962	0.029509	0.027148

3.1.4 Numerical Results

To compare the performance of the IPA, LR, GL and FD estimators for the Everest option, 10000 independent replications were simulated, with parameter settings of $T = 0.2$ years, $\nu_0 = 0.2686, \nu_1 = 0.2976, \theta_0 = 0.1436, \theta_1 = 0.1033, \sigma_0 = 0.1213, \sigma_1 = 0.1532$, $r = 0.0570$. The numerical results are shown in Table 1, where VG1 and VG2 correspond to the time-changed Brownian motion and the difference-of-gammas representations of the VG processes, respectively. The results indicate that the IPA estimator matches the FD estimator closer than the LR and GL estimates for Rho and some others, with considerably lower standard error. However, there are problems with Theta, something that was also reported in Hall (2009) for the geometric Brownian motion setting.

3.2 Atlas Option

Given two positive integers n_1, n_2 where $n_1 + n_2 < n$, and n stocks $S_1, S_2, \cdots, S_n$, with strike K, the payoff for an Atlas option is given by

$$J_T = \left(\sum_{j=1+n_1}^{n-n_2} \frac{R_{(j)}^T}{n-(n_1+n_2)} - K \right)^+, \tag{11}$$

where $R_{(i)}^t$ is the ith smallest return from $\left\{ \frac{S_1^t}{S_1^0}, \frac{S_2^t}{S_2^0}, \cdots, \frac{S_n^t}{S_n^0} \right\}$.

3.2.1 IPA for Atlas Option

The IPA estimator is given by

$$\begin{aligned} \frac{dJ_T}{d\xi} &= \frac{dJ_T}{dS_i^T} \frac{dS_i^T}{d\xi} 1_{\{1+n_1 \le i \le n-n_2\}} \\ &= \frac{1}{(n-(n_1+n_2))S_{(i)}^0} 1_{\{\sum_{j=1+n_1}^{n-n_2} \frac{R_{(j)}^T}{n-n_1-n_2} > K\}} \frac{dS_i^T}{d\xi} 1_{\{1+n_1 \le i \le n-n_2\}}, \end{aligned}$$

where $\frac{dS_i^T}{d\sigma_j}$, $\frac{dS_i^T}{dr}, \frac{dS_i^T}{dT}$ are the same as in the Everest option.

Table 2: Atlas option simulation results

VG1	Rho	Theta	$\frac{dV}{d\sigma_1}$	$\frac{dV}{d\sigma_2}$	$\frac{dV}{d\nu_1}$	$\frac{dV}{d\nu_2}$	$\frac{dV}{d\theta_1}$	$\frac{dV}{d\theta_2}$
FD	0.159036	-0.0158852	-0.0362382	-0.0336875	-0.0401362	-0.012172	-0.033765	-0.0313695
StdErr	0.0007886	0.0246182	0.0015954	0.0012604	0.0255424	0.02511867	0.0010829	0.001176
IPA	0.157945	0.3483325	-0.037779	-0.034934	-0.055719	-0.055719	-0.033171	-0.030723
StdErr	7.99E-04	0.03122565	0.0016719	0.001311	0.0198458	0.4747499	0.0010973	0.001188
LR		-0.032638	0.059033	0.05023	0.138973	0.097045	0.034971	0.036539
StdErr		0.0025664	0.0027265	0.0024749	0.0016691	0.0010944	0.0019362	0.0015127
VG2								
FD	0.157255	-0.016965	-0.038218	-0.033474	0.0152349	0.02292447	-0.0333294	-0.0290269
StdErr	8.00E-04	0.0447099	0.00141597	0.0011544	0.04361048	0.0432855	0.0008513	0.0007886
IPA	0.15585	0.370754	-0.01066	-0.011347	-0.070123	-0.0311865	-0.016232	-0.016798
StdErr	8.14E-04	0.009441	0.0001239	0.000151	0.0064307	0.0012461	0.0011146	0.000945
LR		-0.076233	0.054035	0.049993	0.14383	0.109235	0.033789	0.039159
StdErr		0.0022163	0.0030254	0.0023068	0.0013697	0.0010415	0.0021623	0.001686
GL		0.109806	-0.005084	0.005352	0.025381	0.026887	0.084429	0.076695
StdErr		0.0032667	0.0027291	0.0025486	0.0013492	0.0012727	0.0019843	0.0016859

3.2.2 LR for Atlas Option

The LR estimator is

$$\left(\sum_{j=1+n_1}^{n-n_2} \frac{R_{(j)}^T}{n-(n_1+n_2)} - K\right)^+ \frac{d\ln f(X_1^T, X_2^T, \cdots, X_n^T; \xi)}{d\xi}, \tag{12}$$

where $n=2$, the density $f(X_1^T, X_2^T; \xi) = h_1(z_1)\cdot h_2(z_2)$. The calculation of $\frac{df(X_1^T, X_2^T; \xi)}{d\xi}$ is the same as in the Everest option.

3.2.3 GL for Atlas Option

We use the LR estimator in (12) to estimate, but approximate $h_i(z_i)$ and $\frac{dh_i(z_i)}{d\xi}$ through the characteristic function as in (5).

3.2.4 Numerical Results

Again, the performance of the IPA, LR, GL and FD estimators are compared using 10000 independent replications were simulated, with strike price $K = 0.95$ and $n_1 = 0, n_2 = 1$ and using the same values as in the Atlas option for the other parameter settings: $T = 0.2$ years, $\nu_0 = 0.2686, \nu_1 = 0.2976$, $\theta_0 = 0.1436, \theta_1 = 0.1033$, $\sigma_0 = 0.1213, \sigma_1 = 0.1532$, $r = 0.0570$. From the results in Table 2, again IPA is generally closer to the FD results, with smaller standard error than the LR and GL method, and there are significant discrepancies between all of the Theta estimates.

3.3 Altiplano/Annapurna Option

Given n stocks $S_1, S_2, \cdots, S_n$, a coupon amount C, a limit L and strike K, the barrier period from t_1 to t_2, the payoff for Altiplano option is

$$J_T = \begin{cases} C & \text{if } \max\left(\frac{S_i^t}{S_i^0}\right) \le L, \forall i, \forall t \in (t_1, t_2) \\ \left(\sum_{j=1}^{n} \frac{S_j^T}{S_j^0} - K\right)^+ & \text{otherwise} \end{cases}. \tag{13}$$

If the limit is a floor rather than a ceiling, the option is Annapurna.

Due to the discontinuities in the payoff functions, IPA will not be applicable for Altiplano and Annapurna options.

3.3.1 Numerical Results for Annapurna Option

Again, 10000 independent replications were simulated to compare the performance of the LR, GL and FD estimators, with strike price $K = 1.8$, boundary levels $L = 0.75, C = 0.75$, barrier period $t_1 = 0, t_2 = 1/3$, and all other parameter values remaining the same. The results in Table 3 indicate similar conclusions as before, with the LR estimates all having

Table 3: Altiplano and Annapurna options simulation results

VG1	Rho	Theta	$\frac{dV}{d\sigma_1}$	$\frac{dV}{d\sigma_2}$	$\frac{dV}{d\nu_1}$	$\frac{dV}{d\nu_2}$	$\frac{dV}{d\theta_1}$	$\frac{dV}{d\theta_2}$
FD	0.0125041	0.01151162	-0.00044710	-0.00010735	-0.00185042	0.00670405	1.77E-05	-2.17E-06
StdErr	7.23E-05	0.0239784	0.0008394	0.0008443	0.0237760	0.0224050	0.000448	0.0004515
LR		-0.632261	-0.018207	-0.016837	0.179233	0.237734	0.003234	0.010078
StdErr		0.007175	0.0082798	0.0244308	0.0038626	0.0078192	0.0024114	0.006493
VG2								
FD	0.012013	0.0994565	0.00817122	0.0134781	-0.02935161	0.0237006	0.0799004	0.079851
StdErr	7.24E-05	0.0917235	0.00276237	0.00294089	0.090495	0.0892023	0.00167481	0.00169140
LR		-3.49639	1.168193	0.812599	2.4657	1.669322	0.03548	0.128527
StdErr		0.012140	0.032039	0.024548	0.010850	0.007281	0.014214	0.011926
GL		0.705108	0.207643	0.25539	0.199728	0.199733	0.422369	0.422562
StdErr		0.0146927	0.016973	0.017243	0.008698	0.008734	0.011636	0.0112303

much larger standard error than the FD estimates. Furthermore, not surprisingly the GL method is computationally far more intensive than the usual LR method, so knowing the density saves a lot of computational burden.

3.4 Himalayan Option

Define

$$\begin{aligned}
\mathscr{R}_i &= \left\{\frac{S_1^t}{S_1^0}, \frac{S_2^t}{S_2^0}, \cdots, \frac{S_n^t}{S_n^0}\right\} \\
i_1^* &= arg\max \mathscr{R}_1 \\
i_2^* &= arg\max \mathscr{R}_2 \backslash \left\{\frac{S_{i_1^*}^t}{S_{i_1^*}^0}\right\} \\
i_3^* &= arg\max \mathscr{R}_3 \backslash \left\{\frac{S_{i_1^*}^{t_1}}{S_{i_1^*}^0}, \frac{S_{i_2^*}^{t_2}}{S_{i_2^*}^0}\right\}
\end{aligned}$$

Given a basket of n stocks and a number of time points $\{t_0, t_1, \cdots, T\}$, first construct

$$\left\{R_{i_1^*}^{t_1}, R_{i_2^*}^{t_2}, \cdots, R_{i_n^*}^{T}\right\},$$

the payoff of a Himalayan option is given by

$$J_T = \begin{cases} \left(\sum_{j=1}^{n} (R_{i_j^*}^{t_j} - 1)\right)^+ & \text{if globally floored,} \\ \sum_{j=1}^{n} (R_{i_j^*}^{t_j} - 1)^+ & \text{if locally floored.} \end{cases} \tag{14}$$

Again, since the Himalayan option has a discontinuous payoff, IPA is not applicable.

3.4.1 Numerical Results for the Himalayan Option

Again, 10000 independent replications were simulated to compare the performance of the LR, GL and FD estimators, using a local floor over 0.2 year with strike price $K = 1.8$, and all other parameter values remaining the same. Preliminary results reported in Table 4 indicate potential implementation problems, indicated by the large discrepancies.

4 CONCLUSION

The IPA method performs well where applicable, but it is not applicable in many cases. And the discrepancy in estimating Theta, reported also in Hall (2009), is still unresolved. Discrepancies for some of the other examples indicate that implementation of the estimators can present practical implementation challenges. When the density is available, the direct LR method is preferred to the numerical approximation, as they have essentially the same statistical properties, but the numerical approximation is computationally intensive.

Table 4: Himalayan option simulation results

VG1	Rho	Theta	$\frac{dV}{d\sigma_1}$	$\frac{dV}{d\sigma_2}$	$\frac{dV}{d\nu_1}$	$\frac{dV}{d\nu_2}$	$\frac{dV}{d\theta_1}$	$\frac{dV}{d\theta_2}$
FD	0.07465529	0.02429409	0.01116208	0.0096664	-0.0224674	-0.0245313	0.00385225	0.0031703
StdErr	5.68E-04	0.05225252	0.00262158	0.0027304	0.0502685	0.0522712	0.00332053	0.0022075
LR		-0.527853	0.203279	0.120401	0.356572	0.243691	-0.033431	-0.013259
StdErr		0.0090514	0.0075663	0.0047919	0.0058844	0.0040752	0.0027434	0.0020601
VG2								
FD	0.0746191	0.01365887	0.0105495	0.00999457	-0.00211135	0.00113118	0.00527929	0.00472026
StdErr	5.68E-04	0.06573117	0.00263391	0.00311503	0.0653485	0.0623135	0.0025863	0.0024531
LR		-0.524836	0.197253	0.127338	0.355567	0.242128	-0.034231	-0.013318
StdErr		0.0090048	0.0069403	0.0050826	0.0058075	0.004076	0.0026761	0.002136
GL		0.201545	0.082342	0.09934	0.0893	0.086913	0.102032	0.096517
StdErr		0.0238168	0.026162	0.020448	0.013089	0.012830	0.014724	0.013302

In ongoing work (Cao and Fu 2010), in addition to trying to resolve and/or explain the apparent discrepancies in the reported numerical experiments, we are developing IPA and LR estimators for the case where the underlying assets are correlated. Also, the detailed derivations for the estimators are included there.

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