## **RESOURCE LEVELING OF LINEAR SCHEDULES WITH SINGULARITY FUNCTIONS**

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## ABSTRACT

This paper builds on a new methodology of modeling linear schedules with singularity functions. These unique functions have been used successfully for criticality and float analyses. The approach is extended to deriving one flexible equation for the complete resource profile of a schedule, including any changes in the resource rates of activities. A subsequent equation describes the first moment of area of the resource profile. Minimizing the moment is the objective function for leveling the resource profile. A genetic algorithm with inverse ranking is computerized to perform successive iterations. Chromosomes contain different permutations resource rates at which the activities can be performed. Probabilistic reproduction, crossover, and mutation steps mimic a biological selection process. Step-by-step descriptions of the calculations and a detailed example of a construction project illustrate how singularity functions can provide a powerful model that integrates the linear schedule with its resource profile and facilitates the overall optimization process.

## **1** INTRODUCTION

Linear scheduling is an intuitive method of scheduling that can be applied to a wide variety of construction projects. Its twodimensional diagrams display work amount and time in a coordinate system. If the work amount is tied to a longitudinal geometric feature of the project, e.g. the length of a road or the height of a building, the linear schedule can indicate progress in terms of locations that have been reached. Other possible measures of work amount that are applicable to any type of construction project are the percentage or dollar value of the work that has been completed. Activities are plotted as lines whose slope depends on their productivity. Changes in productivity and their impact on the project duration, the direction of work, parallel 'balanced' productivities, planned breaks or sudden interruptions, and any pairs of activities that are in proximity can be identified visually. Introducing buffers between sequentially dependent activities can prevent such conflicts of congestion.

Important goals of scheduling are to control ongoing construction operations through regular schedule updates and comparing planned versus actual progress and to plan future operations in terms of sequencing, direction, and resource usage. Various performance measures are sought to be optimized, i.e. minimized, including the project duration, i.e. makespan, the project cost, and generally any deviations from the planned execution of the work that could cause the projects to overrun their deadline or budget. Schedulers attempt to identify those activities whose delays would directly impact other activities or, worse, the project duration itself. Such activities and their entire sequence within the project are called critical, which gave the well-known critical path method its name. Researchers continue to examine how the linear scheduling method can be formalized beyond its graphical origins and grown into a powerful mathematical planning and analysis method for project managers. They have examined the criticality of activities and segments thereof (Ammar and Elbeltagi 2001, Kallantzis *et al.* 2007), the flexibility called float that activities have to absorb delays (Awwad and Ioannou 2007, Lucko and Peña Orozco 2008), the desirability of resource continuity (El-Rayes and Moselhi 2001, Liu and Wang 2007), and how changes in the resource profile can be minimized (Nassar 2005, Georgy 2008). The latter focus – resource usage – is the topic of this paper.

# 2 **RESOURCE LEVELING**

The assumptions and heuristic algorithm of resource leveling were described by Harris (1990). Its premise is that a smooth or 'flat' resource profile is desirable because an uneven one causes costs of hiring, firing, and training and negatively impacts

the workers' morale. A suitable measure of 'flatness' was found in the structural engineering concept of the *first moment of area*. It is calculated as the area times the perpendicular lever from its center of gravity, i.e. centroid, to the axis around which a rotation is assumed. Harris (1990) divided the resource profile into vertical strips, multiplied their height and width (one time unit) by their lever (half the height), and added the incremental moments. Resource leveling requires that all noncritical activities are placed into their earliest start position. The heuristic algorithm started with the noncritical activities that were last in sequence. It calculated an improvement factor for each possible shift that included its subtractions and additions to the resource profile, i.e. whether an activity would shift from a peak into a valley and how much this would reduce the profile.

# **3** SINGULARITY FUNCTIONS

Singularity functions have been used in structural analyses of beams and other elements under various loads. Their basic concept has been made suitable for construction scheduling by Lucko (2008). Equation (1) is the elementary term of these *singularity functions*. The pointed bracket operator is evaluated as a case distinction of inactive versus active range of the function.

$$b \cdot \langle x - a \rangle^n = \begin{cases} 0 \text{ for } x < a \\ b \cdot (x - a)^n \text{ for } x \ge a \end{cases}$$
(1)

where *b* is a scaling factor, *x* is a variable on the horizontal *x*-axis, *a* is the cutoff value on that axis at which the function becomes valid, and *n* is the order of the function. For example, n = 0 models a step of the height *b* at *a*, n = 1 is a ramp of the slope *b* growing from *a*, and n = 2 is a quadratic curve. Equation (1) acts like a 'switch' for all values of *x* from *a* onward. It is right continuous and defined for all values. It can be differentiated and integrated normally using equations (2) and (3).

$$\frac{d}{dx}b\cdot\left\langle x-a\right\rangle ^{n}=b\cdot n\cdot\left\langle x-a\right\rangle ^{n-1}$$
(2)

$$\int b \cdot \langle x - a \rangle^n dx = (b/(n+1)) \cdot \langle x - a \rangle^{n+1} + C$$
(3)

where *C* is an integration constant. Complicated shapes of y(x) are modeled by *superposition* of several (up to infinitely many) elementary terms that each describe one change. Singularity functions are *cumulative*, meaning that with growing *x* more terms become active. To inactivate a term it is subtracted. The  $b_i$  of terms can be added if they have the same *a* and *n*.

#### 3.1 General Model

Linear schedules can be plotted with a horizontal or a vertical time axis y depending on their application. The general model of a singularity function (for illustration with exponents n = 0 and n = 1 only) is given by equation (4) and shown in Figure 1. For clarity, terms are sorted by x-value and within that by exponent. The first term is the intercept, subsequent ones are slope changes. The last two terms subtract the cumulative slope and height to 'bring the curve back down to zero' at  $x_{max}$  and can often be omitted. Otherwise it would continue at the final slope until infinity, which would cause errors if it were integrated.

$$y(x) = \Delta t_0 \cdot \langle x - 0 \rangle^0 + \sum_{j=1}^m \left( \frac{\Delta t_j - \Delta t_{j-1}}{\Delta a_j - \Delta a_{j-1}} \cdot \langle x - \sum_{q=1}^{j-1} \Delta a_q \rangle^1 \right) - \frac{\Delta t_m}{\Delta a_m} \cdot \langle x - \sum_{j=1}^m \Delta a_j \rangle^1 - \langle x - \sum_{j=1}^m \Delta a_j \rangle^0 \cdot \sum_{j=0}^m \Delta t_j$$
(4)

where y is the time, x is the work,  $\Delta t_j$  are durations on the y-axis,  $\Delta a_j$  are ranges on the x-axis, j numbers segments up to a total number of m, and within that q numbers active segments. For construction projects like roads or buildings the work amount is often expressed as a location (a point on the x-axis) or as a distance (a range on the x-axis) to measure its progress.

#### 3.2 Transposition

It may be necessary to rotate the general model by 90°, i.e. switch its *x*-axis and *y*-axis, e.g. if the user wishes to have a horizontal time axis *y* to plot the elevation of a building on the vertical work axis *x* or – as in the case of this paper – to display resources r(y) as the typical profile, a histogram. Different from classic linear equations of the type  $y = m \cdot x + b$ , this does not create any problems for horizontal or vertical lines, as singularity functions can model either. Equation (4) provides the trans-

position. The first term is simplified to  $x(y)=0\cdot(y-\Delta t_0)^0+...$  to avoid creating non-existing negative work amounts beneath the *y*-axis near the origin. Any slopes described by (5) are proportional to the productivity, which is defined as work per time.

$$x(y) = -\frac{\Delta t_0 \cdot \Delta a_1}{\Delta t_1} \cdot \langle y - 0 \rangle^0 + \sum_{j=1}^m \left( \frac{\Delta a_j - \Delta a_{j-1}}{\Delta t_j - \Delta t_{j-1}} \cdot \langle y - \sum_{q=0}^{j-1} \Delta t_q \rangle^1 \right) - \frac{\Delta a_m}{\Delta t_m} \cdot \langle y - \sum_{j=0}^m \Delta t_j \rangle^1 - \langle y - \sum_{j=0}^m \Delta t_j \rangle^0 \cdot \sum_{j=1}^m \Delta a_j$$
(5)



Figure 1: General Model for Singularity Function

#### 4 SMALL EXAMPLE

Modeling the resource profile is introduced with a small example as shown in Figure 2a. Activity A has a unit productivity of 3/2 units of work per day and worker  $[u / (d \cdot w)]$  and activity B has one of 3/4 u / d. Both require a resource rate of 1 w / d.

The singularity functions for activities A and B are created using equation (1) and yield (6) and (7). Their slope terms are expanded by their resource rate of 1 w. The previously discussed last two terms of the downward end correction are shown.

$$y(x)_{A} = 0 d \cdot \langle x - 0u \rangle^{0} + (2 d \cdot w/3u \cdot w) \cdot \langle x - 0u \rangle^{1} - (2 d \cdot w/3u \cdot w) \cdot \langle x - 3u \rangle^{1} - 2 d \cdot \langle x - 3u \rangle^{0}$$
(6)

$$y(x)_{B} = 1 d \cdot \langle x - 0 u \rangle^{0} + (4 d \cdot w/3 u \cdot w) \cdot \langle x - 0 u \rangle^{1} - (4 d \cdot w/3 u \cdot w) \cdot \langle x - 3 u \rangle^{1} - 5 d \cdot \langle x - 3 u \rangle^{0}$$

$$\tag{7}$$

The axis orientation in Figure 2a and its equations is that the resource profile cannot be added 'sideways' to e.g. derive that its maximum occurs on the second day. Equations (6) and (7) are therefore transposed using (5) to yield (8) and (9). Evaluating (8) at  $y_{\text{max A}} = 2 d$  and (9) at  $y_{\text{max B}} = 5 d$  yields their necessary end correction of  $x_{\text{max}} = 3 u$ , i.e. the scope of work.

$$x(y)_{A} = 0u \cdot \langle y - 0d \rangle^{0} + (3u/2d) \cdot \langle y - 0d \rangle^{1} - (3u/2d) \cdot \langle y - 2d \rangle^{1} - 3u \cdot \langle y - 2d \rangle^{0}$$
(8)

$$x(y)_{B} = -(3/4)u \cdot \langle y - 0d \rangle^{0} + (3u/4d) \cdot \langle y - 0d \rangle^{1} - (3u/4d) \cdot \langle y - 5d \rangle^{1} - 3u \cdot \langle y - 5d \rangle^{0}$$
(9)

The first term of (9) is simplified to  $x(y)_B = 0 u \cdot \langle y - 1d \rangle^0 + ...$  to avoid negative values. Figure 2b shows the transposed linear schedule and its resource profile, which requires applying the differentiation operator of equation (2) to (8) and (9).

#### 4.1 Modified Differentiation

Equations (8) and (9) are differentiated using (2) and yield (10) and (11). The resource rates [w / d] must be retained because the resource profile contains only profile steps, but the rest of these factors with the unit [u / w] in (10) and (11) are removed.

$$x(y)_{A}'=0+(3u\cdot w/2d\cdot w)\cdot\langle y-0d\rangle^{0}-(3u\cdot w/2d\cdot w)\cdot\langle y-2d\rangle^{0}-0$$
(10)

$$x(y)_{B}'=0+(3u\cdot w/4d\cdot w)\cdot \langle y-0d\rangle^{0}-(3u\cdot w/4d\cdot w)\cdot \langle y-5d\rangle^{0}-0$$
(11)

Adding them yields the equation (12) for the resource profile. This approach is mathematically independent of Figure 2.

$$r(y) = \frac{1}{w} d \cdot \langle y - 0d \rangle^{0} + \frac{1}{w} d \cdot \langle y - 1d \rangle^{0} - \frac{1}{w} d \cdot \langle y - 2d \rangle^{0} - \frac{1}{w} d \cdot \langle y - 5d \rangle^{0}$$
(12)

Overall, the general equation (13) for the resource profile is the sum of many strips of the positive or negative height  $b_i$ .

$$r(y) = \sum_{i=1}^{k} \left( b_i \cdot \left\langle y - y_i \right\rangle^0 - b_i \cdot \left\langle y - \left( y_i + \Delta t_i \right) \right\rangle^0 \right)$$
(13)

where r is the unit of resources, k is the total number of activities,  $b_i$  are the resource rates, and  $y_i$  are the cutoff values.



Figure 2b: Transposed Linear Schedule and Resource Profile

## **5 DERIVATION OF MOMENT**

Since the resource profile of a linear schedule is expressed with singularity functions, it is necessary to derive formulas for its area and centroid. The area underneath a function is its integral as per equation (3), which is applied to equations such as (10) and (11) and yields (14). It describes the area of a resource profile such as in Figure 2b, i.e. the total resource consumption.

$$A_r = \int_{y_1}^{y_k} r(y) = \sum_{i=1}^k \left( b_i \cdot \left\langle y - y_i \right\rangle^1 - b_i \cdot \left\langle y - \left(y_i + \Delta t_i\right) \right\rangle^1 \right)$$
(14)

where *A* is the area and  $\Delta t_i$  are the activity durations. The heuristic moment minimization algorithms by Harris (1990) and other researchers (Martínez and Ioannou 1993, Mattila and Abraham 1998, Hiyassat 2000, Georgy 2008) all divided the resource profile into vertical strips. The innovation of the approach presented in this paper is to divide the resource histogram into horizontal strips over the time axis *y* that begin where activities (or their segments) begin and end where these end. Each resource block creates a positive term  $b_i$  at its start and a negative  $-b_i$  at its finish, because each block is modeled as a superposition of one positive and one negative strip. Terms with the same *a* and *n* are simplified by adding their  $b_i$ . Strips in (14) are measured to  $y_{max}$ . Equation (15) is the area-weighted lever from the time axis *y* to the centroid *C* of a composite shape.

$$l(y)_{C} = \sum_{i=1}^{2 \cdot k} (l_{i} \cdot A_{i}) / \sum_{i=1}^{2 \cdot k} A_{i}$$
(15)

where  $l_i$  are levers of horizontal strips and  $A_i$  are areas of individual strips. Note again that each activity as per equation (14) contributes a pair of positive and negative strips, which is why equation (15) counts up to  $2 \cdot k$ . The numerator of (15) is expanded to yield (16) for the moment of the resource profile. The lever for a newly added strip is equal to the cumulative sum of all steps  $b_i$  minus one half the current step height. Simplification of adding  $b_i$  may reduce the number of terms in (16).

$$M_r = \sum_{i=1}^{2 \cdot k} \left( b_i \cdot \left\langle y_k - y_i \right\rangle^1 \cdot \left( \left( \sum_{\nu=1}^i b_\nu \right) - 0.5 \cdot b_i \right) \right)$$
(16)

where *M* is the first moment of area, *i* numbers activities up to a total number of *k*, and within that *v* numbers activities. Applying equation (16) to the small example yields  $M_r = 1 \cdot \langle 5 - 0 \rangle^1 \cdot (1 - 0.5) + 1 \cdot \langle 5 - 1 \rangle^1 \cdot (2 - 0.5) - 1 \cdot \langle 5 - 2 \rangle^1 \cdot (2 - 0.5) = 4$ .

## 6 EXAMPLE FROM HARRIS AND IOANNOU (1998)

Harris and Ioannou (1998) introduced an example of a linear schedule and analyzed its duration and criticality. It was chosen for this paper because several subsequent studies (Ioannou and Srisuwanrat 2006, Awwad and Ioannou 2007, Ioannou and Srisuwanrat 2007, Lucko 2008, and Srisuwanrat *et al.* 2008) examined aspects of this example, including resource continuity and probabilistic durations. It consists of six activities to construct a project of six units [u], e.g. miles of road. Its dependency structure is shown in Figure 3. The unit durations of activities *A* through *F* to produce one unit are listed in Table 1. Note that activity *A* incurs two production changes, *B* incurs a mandatory interruption, and *C* does not perform work from 5 *u* to 6 *u*.



Figure 3: Network Diagram for One Unit

Table 1: Activity	y Dependency	and Productivity
	/	

Activity	<b>Unit Duration</b>	Successors	Production
А	2 <i>d</i>	B, C	Half productivity for units 3 and 4
			After unit 1, finish-to-start lead time of 2 days to C
В	1 <i>d</i>	D	5 days interruption between units 3 and 4
С	4 <i>d</i>	D, E	No work on unit 5, but still continuous
D	3 <i>d</i>	F	Regular activity
Е	1 <i>d</i>	F	Regular activity
F	1 <i>d</i>	-	Regular activity

## 6.1 **Previous Criticality and Float Analysis**

The left half of Figure 4 shows the criticality and float that were calculated by Lucko and Peña Orozco (2008). They used singularity functions to precisely determine the different types of float and their numerical values. The reader is referred to that manuscript for a more detailed description of the float calculations. Solid thick lines in the left half of Figure 4 represent critical activity segments. Dotted thick lines represent the critical path continuing across buffers. Dark gray areas represent mandatory minimum buffers that must be maintained between activities, which causes them these to not touch directly. Light gray areas represent how noncritical activity segments can consume free float at their early or late ends by rotating, which is

equivalent to modifying their resource rate. Their lowest possible resource rate is shown by dashed lines. Note that due to the vertically plotted time axis *y*, the slopes of all activity segments are inversely proportional to their respective productivity.



Figure 4: Linear Schedule with Criticality and Resource Profile with Variable Rates

Analysis of this linear schedule yields the singularity functions of equations (17) through (22) that fully describe the six activities. The analytical steps for obtaining these equations as per the method described by Lucko (2008) included (a) describing activities and their buffers with tentative intercept values, (b) stacking them alternatingly under consideration of their dependency structure, while applying the critical path method concept that the maximum finish date of all predecessors becomes the start date of one direct successor, (c) taking pairwise differences between adjacent activities and buffers, (d) differentiating these differences, thus exploiting any possible overlap between them. Following these steps guarantees reaching the shortest possible project duration, i.e. makespan, for the given productivities and dependency structures (Lucko 2007). Considering all constraints from productivity, dependency, buffers, and interruptions yields a project duration of 30 days [d].

$$y(x)_{A} = 0 \cdot \langle x - 0 \rangle^{0} + \frac{2}{1} \cdot \langle x - 0 \rangle^{1} + \frac{2}{1} \cdot \langle x - 2 \rangle^{1} - \frac{2}{1} \cdot \langle x - 4 \rangle^{1}$$
(17)

$$y(x)_{B} = 9 \cdot \langle x - 0 \rangle^{0} + \frac{1}{1} \cdot \langle x - 0 \rangle^{1} + 5 \cdot \langle x - 3 \rangle^{0}$$
(18)

$$y(x)_{c} = 2 \cdot \langle x - 0 \rangle^{0} + \frac{4}{1} \cdot \langle x - 0 \rangle^{1} - \frac{4}{1} \cdot \langle x - 4 \rangle^{1} + \frac{4}{1} \cdot \langle x - 5 \rangle^{1}$$
(19)

$$y(x)_{D} = 11 \cdot \langle x - 0 \rangle^{0} + \frac{3}{1} \cdot \langle x - 0 \rangle^{1}$$
<sup>(20)</sup>

$$y(x)_{E} = 17 \cdot \langle x - 0 \rangle^{0} + \frac{1}{1} \cdot \langle x - 0 \rangle^{1}$$
(21)

$$y(x)_{F} = 24 \cdot \langle x - 0 \rangle^{0} + \frac{1}{1} \cdot \langle x - 0 \rangle^{1}$$
(22)

The ranges of activity segments that were identified to have free float, i.e. the flexibility to be delayed without impacting any adjacent activity in the dependency structure, are listed in Table 2 along with other pertinent results from prior analysis, i.e. start and finish coordinates of each activity segment, slope, duration, the type of criticality, the initial number of resources, the float rate, and the resource consumption in the unit worker-days  $[w \cdot d]$ . Activities A, D, and F are partially critical with either early or late float at their start or finish end, respectively, activity B is critical only at 3 u through its interruption, activity C is fully critical due to lacking free float by touching its immediate successor E at the interface  $\{5, 22\}$  (but noncritical if considering only the possible impact of any delay of it on the project finish), and activity E is fully noncritical. The calculated float rate, i.e. possible productivity change, is expressed in units of work per time [u / d], inverse to the slope.

		Start	Finish	Slope	Initial		Initial	Float	Worker-
Activity	Segment	x, y	x, y	y/x	Duration	Criticality	Resources	Rate x/y	Days
А	1	0, 0	2, 4	2/1	4	Yes 6 -		-	24
	2	2,4	4, 12	4/1	8	Yes	3	-	24
	3	4, 12	6, 16	2/1	4	No, late float	6	1/6	24
В	1	0, 9	3, 12	1/1	3	No, early float	12	7/10	36
	2	3, 19	6, 22	1/1	3	No, late float	12	2/3	36
С	1	0, 2	4, 18	4/1	16	Yes 3 -		-	48
	2	5, 18	6, 22	4/1	4	Yes	3	-	12
D	1	0,11	2, 17	3/1	6	No, early float 4 1/21		24	
	2	2,17	6, 29	3/1	12	Yes	4	-	48
Е	1	0,17	5,22	1/1	5	No, early float	12	1/2	60
	2	5,22	6,23	1/1	1	No, late float	12	6/7	12
F	1	0,24	5, 29	1/1	5	No, early float 12 6/11		6/11	60
	2	5, 29	6,30	1/1	1	Yes	12	-	12

Table 2: Linear Schedule Analysis with Criticality and Float

## 6.2 Resource Analysis

The total number of worker-days in the project is  $3 \cdot 24 + 2 \cdot 26 + 48 + 12 + 24 + 48 + 60 + 12 + 60 + 12 = 420 \ w \cdot d$  during the 30 d project duration, considering that activity C does not perform work from 5 u to 6 u. Table 3 lists the inverse of the slope, i.e. the initial productivity and the lowest productivity that can be achieved if the float of an activity segment is fully exploited. This lowest productivity is achieved if the lowest possible resources are deployed to work on the project. To facilitate a meaningful analysis, the assumption is made for this example that the standard productivity is 12 workers can produce one unit per day, i.e. the unit productivity is  $p = 1/12 \ u / (d \cdot w)$ . Subtracting the float rate in Table 2 from the initial productivity in Table 3 yields the lowest possible productivity. This is converted into an equivalent number of lowest resources via the unit productivity. At this stage, any uneven resources are rounded to integer values, which are marked with brackets and boldface, as partial workers cannot exists. To be equivalent to the these integer values, the mathematically lowest productivity-ities are then corrected, which are also marked with brackets and boldface. The resources thus can range from the rounded lowest integer value to the initial higher value from Table 2. This creates the large number of  $1 \cdot 1 \cdot 3 \cdot 9 \cdot 9 \cdot 1 \cdot 1 \cdot 1 \cdot 7 \cdot 11 \cdot 7 \cdot 1 = 130,977$  permutations in which this project could be carried out. Lowering the number of resources to integer values are listed in Table 3.

The right half of Figure 4 shows the resource profile for this example. Dark gray shaded blocks represent the resources of critical activity segments that cannot be decreased without directly impacting the project finish. For clarity, only the highest and lowest possible integer resources are shown, the latter ones as dashed blocks. The 'flattening' of the resource blocks is shown overlapping in some cases due to space constraints, whereas in reality the resources are strictly additive on their resource axis r. Activity D is shown as being fully critical because the rounding to integer values yielded only the possible resource rate of 4 w, rendering it unchangeable. To perform the calculations that are necessary for resource leveling, equations (17) through (22) are first rewritten in the more detailed form of equations (23) through (28) that break out the terms of the inverse of the unit productivity p multiplied by the inverse of the number of resources (due to slope being the inverse of productivity). Changes between segments are indicated in brackets where the new slope is added and previous one is sub-tracted.

		Initial	Lowest	Lowest	Rounded	Highest
Activity	Segment	Productivity x/y	Productivity x/y*	Resources	<b>Resource Range*</b>	Duration
А	1	1/2	-	6.00	6	4
	2	1/4	-	3.00	3	8
	3	1/2	1/3	4.00	4, 5, <u>6</u>	6
В	1	1/1	3/10 (1/3)	3.60	<b>(4)</b> , 5, 6, 7, 8, 9, 10, 11, <u>12</u>	10
	2	1/1	1/3	4.00	4, 5, 6, 7, 8, 9, 10, 11, <u>12</u>	9
С	1	1/4	1/4	3.00	3	16
	2	1/4	1/4	3.00	3	4
D	1	1/3	2/7 (1/3)	3.42	<u>(4)</u>	7
	2	1/3	1/3	4.00	4	12
Е	1	1/1	1/2	6.00	6, 7, 8, 9, 10, 11, <u>12</u>	10
	2	1/1	1/7 <b>(1/6)</b>	1.71	<b>(2)</b> , 3, 4, 5, 6, 7, 8, 9, 10, 11, <u>12</u>	7
F	1	1/1	5/11 (1/2)	5.45	<b>(6)</b> , 7, 8, 9, 10, 11, <u>12</u>	11
	2	1/1	1/1	12.00	12	1

Table 3: Possible Resource Rates

\* Initial values are underlined. Rounded values are in brackets and boldface.

$$y(x)_{A} = 0 d \cdot \langle x - 0u \rangle^{0} + \frac{1}{p} \cdot \frac{1}{6w} \cdot \langle x - 0u \rangle^{1} + \frac{1}{p} \cdot \left(\frac{1}{3w} - \frac{1}{6w}\right) \cdot \langle x - 2u \rangle^{1} + \frac{1}{p} \cdot \left(\frac{1}{r_{A3}} - \frac{1}{3w}\right) \cdot \langle x - 4u \rangle^{1}$$
(23)

$$y(x)_{B} = \left[12d - \frac{1}{p} \cdot \frac{1}{r_{B1}} \cdot 3u\right] \cdot \left\langle x - 0u \right\rangle^{0} + \frac{1}{p} \cdot \frac{1}{r_{B1}} \cdot \left\langle x - 0u \right\rangle^{1} + 5d \cdot \left\langle x - 3u \right\rangle^{0} + \frac{1}{p} \cdot \left(\frac{1}{r_{B2}} - \frac{1}{r_{B1}}\right) \cdot \left\langle x - 3u \right\rangle^{1}$$
(24)

$$y(x)_{c} = 2d \cdot \langle x - 0u \rangle^{0} + \frac{1}{p} \cdot \frac{1}{3w} \cdot \langle x - 0u \rangle^{1} + \frac{1}{p} \cdot \left( -\frac{1}{3w} \right) \cdot \langle x - 4u \rangle^{1} + \frac{1}{p} \cdot \frac{1}{3w} \cdot \langle x - 5u \rangle^{1}$$
(25)

$$y(x)_{D} = 11d \cdot \langle x - 0u \rangle^{0} + \frac{1}{p} \cdot \frac{1}{4w} \cdot \langle x - 0u \rangle^{1}$$
(26)

$$y(x)_{E} = \left[22d - \frac{1}{p} \cdot \frac{1}{r_{E1}} \cdot 5u\right] \cdot \langle x - 0u \rangle^{0} + \frac{1}{p} \cdot \frac{1}{r_{E1}} \cdot \langle x - 0u \rangle^{1} + \frac{1}{p} \cdot \left(\frac{1}{r_{E2}} - \frac{1}{r_{E1}}\right) \cdot \langle x - 5u \rangle^{1}$$
(27)

$$y(x)_{F} = \left[29d - \frac{1}{p} \cdot \frac{1}{r_{F1}} \cdot 5u\right] \cdot \langle x - 0u \rangle^{0} + \frac{1}{p} \cdot \frac{1}{r_{F1}} \cdot \langle x - 0u \rangle^{1} + \frac{1}{p} \cdot \left(\frac{1}{12w} - \frac{1}{r_{F1}}\right) \cdot \langle x - 5u \rangle^{1}$$
(28)

Late float in activities A, B, and E is expressed through the inverse values of their variable final resource rates  $r_{A3}$ ,  $r_{B2}$ , and  $r_{E2}$ . The initial terms of (24), (27), and (28) for activities B, E, and F have a variable intercept whose coordinate on the time axis y from the pivot coordinate downward depends on the inverse of their variable initial resource rates  $r_{B1}$ ,  $r_{E1}$ , and  $r_{F1}$ . These pivots are critical points between segments that had previously been identified (Lucko 2008) and are listed in Table 2.

Following equation (5), (23) through (28) must now be transposed from describing a diagram with time axis y over work amount axis x in the left half of Figure 4 to a traditional schedule where time is displayed on the horizontal axis and the work amount on the vertical axis. Equations (29) through (34) show the detailed singularity functions after this axis switch. They contain the unit productivity and integer number of resources in their non-inverted form familiar, where a higher number of resources result in a proportionally higher productivity and steeper slope as the transposed of the diagram of Figure 4. Note that the variable intercept terms are also subject to the transposition and must now appear inside the pointed bracket terms.

$$x(y)_{A} = 0u \cdot \langle y - 0d \rangle^{0} + p \cdot 6w \cdot \langle y - 0d \rangle^{1} + p \cdot (3w - 6w) \cdot \langle y - 4d \rangle^{1} + p \cdot (r_{A3} - 3w) \cdot \langle y - 12d \rangle^{1}$$

$$(29)$$

$$x(y)_{B} = 0u \cdot \left\langle y - \left[ 12d - \frac{1}{p} \cdot \frac{1}{r_{B1}} \cdot 3u \right] \right\rangle^{0} + p \cdot r_{B1} \cdot \left\langle y - \left[ 12d - \frac{1}{p} \cdot \frac{1}{r_{B1}} \cdot 3u \right] \right\rangle^{1} + p \cdot (-r_{B1}) \cdot \left\langle y - 12d \right\rangle^{0} + p \cdot (r_{B2}) \cdot \left\langle y - 17d \right\rangle^{0}$$
(30)

$$x(y)_{c} = 0u \cdot \langle y - 2d \rangle^{0} + p \cdot 3w \cdot \langle y - 2d \rangle^{1} + 1u \cdot \langle y - 18d \rangle^{0}$$
(31)

$$x(y)_{D} = 0u \cdot \langle y - 11d \rangle^{0} + p \cdot 4w \cdot \langle y - 11d \rangle^{1}$$
(32)

$$x(y)_{E} = 0u \cdot \left\langle y - \left[ 22d - \frac{1}{p} \cdot \frac{1}{r_{E1}} \cdot 5u \right] \right\rangle^{0} + p \cdot r_{E1} \cdot \left\langle y - \left[ 22d - \frac{1}{p} \cdot \frac{1}{r_{E1}} \cdot 5u \right] \right\rangle^{1} + p \cdot (r_{E2} - r_{E1}) \cdot \left\langle y - 22d \right\rangle^{1}$$
(33)

$$x(y)_{F} = 0u \cdot \left\langle y - \left[ 29d - \frac{1}{p} \cdot \frac{1}{r_{F1}} \cdot 5u \right] \right\rangle^{0} + p \cdot r_{F1} \cdot \left\langle y - \left[ 29d - \frac{1}{p} \cdot \frac{1}{r_{F1}} \cdot 5u \right] \right\rangle^{1} + p \cdot (12 - r_{F1}) \cdot \left\langle y - 29d \right\rangle^{1}$$
(34)

In the following step, the resources are described in their typical profile form, a histogram. Each activity segment contributes a rectangular block to the profile. To model them correctly, the modified differentiation demonstrated in equations (10) and (11) is applied, where not only the exponent is reduced by one and the original exponent becomes a multiplicative factor as per equation (2), but also the unit productivity as units per worker [u / w] is dropped from the singularity functions. To ensure that resources are subtracted from the profile when activities end, instead of continuing as a too long block until the project finish at  $y_{max} = 30 d$ , an end term is appended to equations (35) through (40) that subtracts the final resource rate at the (possibly variable if late float exists) end coordinate. A prime ' indicates that the modified differentiation has been used.

$$x(y)'_{A} = 0 + 6 w \cdot \langle y - 0d \rangle^{0} - 3 w \cdot \langle y - 4d \rangle^{0} + (r_{A3} - 3w) \cdot \langle y - 12d \rangle^{0} - r_{A3} \cdot \langle y - \left[12d + \frac{1}{p} \cdot \frac{1}{r_{A3}} \cdot 2u\right] \rangle^{0}$$
(35)

$$x(y)'_{B} = 0 + r_{B1} \cdot \left\langle y - \left[ 12d - \frac{1}{p} \cdot \frac{1}{r_{B1}} \cdot 3u \right] \right\rangle^{0} - r_{B1} \cdot \left\langle y - 12d \right\rangle^{0} + r_{B2} \cdot \left\langle y - 17d \right\rangle^{0} - r_{B2} \cdot \left\langle y - \left[ 17d + \frac{1}{p} \cdot \frac{1}{r_{B2}} \cdot 3u \right] \right\rangle^{0}$$
(36)

$$x(y)'_{c} = 0 + 3w \cdot \langle y - 2d \rangle^{0} - 3w \cdot \langle y - 22d \rangle^{0}$$
(37)

$$(y)'_{D} = 0 + 4 w \cdot \langle y - 11d \rangle^{0} - 4 w \cdot \langle y - 29d \rangle^{0}$$
(38)

$$x(y)'_{E} = 0 + r_{E1} \cdot \left\langle y - \left[ 22d - \frac{1}{p} \cdot \frac{1}{r_{E1}} \cdot 5u \right] \right\rangle^{0} + \left( r_{E2} - r_{E1} \right) \cdot \left\langle y - 22d \right\rangle^{0} - r_{E2} \cdot \left\langle y - \left[ 22d + \frac{1}{p} \cdot \frac{1}{r_{E2}} \cdot 1u \right] \right\rangle^{0}$$
(39)

$$x(y)'_{F} = 0 + r_{F1} \cdot \left\langle y - \left[ 29d - \frac{1}{p} \cdot \frac{1}{r_{F1}} \cdot 5u \right] \right\rangle^{0} + \left( 12w - r_{F1} \right) \cdot \left\langle y - 29d \right\rangle^{0} - 12w \cdot \left\langle y - 30d \right\rangle^{0}$$
(40)

Finally, (35) through (40) are added in the final singularity function r(y) of (41) that models the entire resource profile with all of its 130,977 permutations. Terms in the pointed brackets are 'switches' that activate each one at its correct time. Terms without variables are constant resource blocks of the dark gray critical activity segments in the right half of Figure 4.

x

$$r(y) = 6w \cdot \langle y - 0d \rangle^{0} + 3w \cdot \langle y - 2d \rangle^{0} - 3w \cdot \langle y - 4d \rangle^{0} + 4w \cdot \langle y - 11d \rangle^{0} + (r_{A3} - 3w) \cdot \langle y - 12d \rangle^{0} - r_{B1} \cdot \langle y - 12d \rangle^{0} + r_{B2} \cdot \langle y - 17d \rangle^{0} + (r_{E2} - r_{E1} - 3w) \cdot \langle y - 22d \rangle^{0} + (8w - r_{F1}) \cdot \langle y - 29d \rangle^{0} - 12w \cdot \langle y - 30d \rangle^{0} + r_{B1} \cdot \langle y - \left[12d - \frac{1}{p} \cdot \frac{1}{r_{B1}} \cdot 3u\right] \rangle^{0} - r_{A3} \cdot \langle y - \left[12d + \frac{1}{p} \cdot \frac{1}{r_{A3}} \cdot 2u\right] \rangle^{0} - r_{B2} \cdot \langle y - \left[17d + \frac{1}{p} \cdot \frac{1}{r_{B2}} \cdot 3u\right] \rangle^{0} + r_{E1} \cdot \langle y - \left[22d - \frac{1}{p} \cdot \frac{1}{r_{E1}} \cdot 5u\right] \rangle^{0} - r_{E2} \cdot \langle y - \left[22d + \frac{1}{p} \cdot \frac{1}{r_{E2}} \cdot 1u\right] \rangle^{0} + r_{F1} \cdot \langle y - \left[29d - \frac{1}{p} \cdot \frac{1}{r_{F1}} \cdot 5u\right] \rangle^{0}$$

$$(41)$$

## 6.3 Moment Minimization for Resource Leveling

The objective function for resource leveling that should be minimized is the first moment of area from structural engineering applied to the resource profile. Assuming a rotation around the horizontal axis, it is calculated as an area times the lever from the axis to the center of gravity of said area. For rectangles, this is found at one half of their height. Different from previous approaches, the length of each horizontal strip is directly tied to the duration of its activity or segment thereof and the height is directly tied to its resource rate. A change in any activity segment thus will only change one horizontal strip, whereas many vertical strips would be affected. Equation (42) is generated from (41) by increasing the exponents to one (because a zero exponent would only yield "1" for all terms), extending each term by the total height of the resource profile minus one half of the newly added or subtracted strip, and by inserting  $y_{max} = 30 d$  as the end of all strips. Note that each resource block of an activity segment contributes two terms to the resource and moment equations, i.e. one term each for adding and subtracting it.

$$M(y) = 6w \cdot \langle 30d - 0d \rangle^{1} \cdot [r(0d) - 0.5 \cdot 6w] + 3w \cdot \langle 30d - 2d \rangle^{1} \cdot [r(2d) - 0.5 \cdot 3w] - 3w \cdot \langle 30d - 4d \rangle^{1} \cdot [r(4d - \varepsilon) - 0.5 \cdot |- 3w|] + 4w \cdot \langle 30d - 11d \rangle^{1} \cdot [r(11d) - 0.5 \cdot 4w] + r_{A3} \cdot \langle 30d - 12d \rangle^{1} \cdot [r(12d) - 0.5 \cdot r_{A3}] + (-r_{B1} - 3w) \cdot \langle 30d - 12d \rangle^{1} \cdot [r(12d - \varepsilon) - 0.5 \cdot |(-r_{B1} - 3w)]] + r_{B2} \cdot \langle 30d - 17d \rangle^{1} \cdot [r(17d) - 0.5 \cdot r_{B2}] + r_{E2} \cdot \langle 30d - 22d \rangle^{1} \cdot [r(22d) - 0.5 \cdot r_{E3}] + (-r_{E1} - 3w) \cdot \langle 30d - 22d \rangle^{1} \cdot [r(22d - \varepsilon) - 0.5 \cdot |(-r_{E1} - 3w)]] + 8w \cdot \langle 30d - 29d \rangle^{1} \cdot [r(29d) - 0.5 \cdot 8w]$$

$$-r_{F_{1}} \cdot \langle 30d - 29d \rangle^{1} \cdot [r(29d - \varepsilon) - 0.5 \cdot |(-r_{F_{1}})] - 12w \cdot \langle 30d - 30d \rangle^{1} \cdot [r(30d - \varepsilon) - 0.5 \cdot 12w] + r_{B_{1}} \cdot \langle 30d - [12d - \frac{1}{p} \cdot \frac{1}{r_{B_{1}}} \cdot 3u] \rangle^{1} \cdot [r(12d - \frac{1}{p} \cdot \frac{1}{r_{B_{1}}} \cdot 3u] - 0.5 \cdot r_{B_{1}}] - r_{A_{3}} \cdot \langle 30d - [12d + \frac{1}{p} \cdot \frac{1}{r_{A_{3}}} \cdot 2u] \rangle^{1} \cdot [r(12d + \frac{1}{p} \cdot \frac{1}{r_{A_{3}}} \cdot 2u - \varepsilon) - 0.5 \cdot |-r_{A_{3}}|] - r_{B_{2}} \cdot \langle 30d - [17d + \frac{1}{p} \cdot \frac{1}{r_{B_{2}}} \cdot 3u] \rangle^{1} \cdot [r(17d + \frac{1}{p} \cdot \frac{1}{r_{B_{2}}} \cdot 3u - \varepsilon) - 0.5 \cdot |-r_{B_{2}}|] + r_{E_{1}} \cdot \langle 30d - [22d - \frac{1}{p} \cdot \frac{1}{r_{E_{1}}} \cdot 5u] \rangle^{1} \cdot [r(22d - \frac{1}{p} \cdot \frac{1}{r_{E_{1}}} \cdot 5u] - 0.5 \cdot r_{E_{1}}] - r_{E_{2}} \cdot \langle 30d - [22d + \frac{1}{p} \cdot \frac{1}{r_{E_{2}}} \cdot 1u] \rangle^{1} \cdot [r(22d + \frac{1}{p} \cdot \frac{1}{r_{E_{2}}} \cdot 1u - \varepsilon) - 0.5 \cdot r_{E_{2}}] + r_{F_{1}} \cdot \langle 30d - [29d - \frac{1}{p} \cdot \frac{1}{r_{F_{1}}} \cdot 5u] \rangle^{1} \cdot [r(29d - \frac{1}{p} \cdot \frac{1}{r_{F_{1}}} \cdot 5u) - 0.5 \cdot r_{F_{1}}]$$

$$(42)$$

Singularity functions by their definition in equation (1) are right continuous, i.e. their term becomes active at the cutoff value *a* itself, no immediately thereafter. If steps have a different sign at the same cutoff value *a* they cannot be added for simplification as they will cancel each other out and the lever will be too short. The time of each subtraction is therefore reduced by an infinitesimal amount  $\varepsilon$  to ensure that the height of the centroid is correct if both an addition and a subtraction occur at that cutoff value *a*. Achieving the minimization of the moment equation (42) with its 130,977 permutations depends only on the six variables  $r_{A3}$ ,  $r_{B1}$ ,  $r_{B2}$ ,  $r_{E1}$ ,  $r_{E2}$ , and  $r_{F1}$  that can take on the values as listed in the second-to-last column of Table 3. Each pair of addition and subtraction terms in (42) quantifies the exact contribution of one activity segment to the moment of the resource profile. For example, the moment of the unchanged permutation (sorted by time *y*, with the values of variable resources underlined, and summarized additions and subtractions at identical cutoff values *a*) is  $M_r = 6 \cdot 30 \cdot 3 + 3 \cdot 28 \cdot 7.5 - 3 \cdot 26 \cdot 7.5 + 12 \cdot 21 \cdot 12 + 4 \cdot 19 \cdot 20 + (6 - 12 - 3) \cdot 18 \cdot 17.5 - 6 \cdot 14 \cdot 10 + (12 + 12) \cdot 13 \cdot 19 - 12 \cdot 10 \cdot 25 + (-12 - 3 + 12) \cdot 8 \cdot 17.5 - 12 \cdot 7 \cdot 10 + 12 \cdot 6 \cdot 10 + (8 - 12) \cdot 1 \cdot 14 - 12 \cdot 0 \cdot 6 = 3,786$  worker-days times workers  $[w^2 \cdot d]$ , including an unchangeable part of 666  $w^2 \cdot d$ . Calculating vertical strips yields  $M_r = 6 \cdot 3 \cdot 2 + 9 \cdot 4.5 \cdot 2 + 6 \cdot 3 \cdot 5 + 18 \cdot 9 \cdot 2 + 22 \cdot 11 \cdot 1 + 13 \cdot 6.5 \cdot 4 + 7 \cdot 3.5 \cdot 1 + 31 \cdot 15.5 \cdot 3 + 19 \cdot 9.5 \cdot 2 + 16 \cdot 8 \cdot 1 + 4 \cdot 2 \cdot 1 + 16 \cdot 8 \cdot 5 + 12 \cdot 6 \cdot 1 = 3,786 w^2 \cdot d$ .

## 7 GENETIC ALGORITHM

Finding the permutation(s) with the minimum moment among the 130,977 possible ones requires an optimization algorithm, as would be tedious to list all of them and difficult to find a direct solution due to the complex interactions between overlapping activities. Real-world schedules with more activities, resources, and productivities would see extremely large numbers of permutations. Genetic algorithms are inspired by evolutionary processes (Bäck *et al.* 1997) and allow an efficient randomized search of a large and complex solution space. Its efficiency depends on user-selected probabilities but cannot guarantee that the absolute optimum will be found. The approach follows Srisuwanrat and Ioannou (2007), who optimized work breaks, with modifications to evaluate equation (42), the singularity function that models the entire moment of the resource profile.

- *Randomization*: First, a *chromosome* of six *genes* is created, i.e. randomly chosen inputs for the variable resource rates of Table 3. The 'bandwidth' of the algorithm is four chromosomes. Further *generations* will use *offspring* of these values.
- *Reproduction*: The moment of each chromosome determines its *fitness* as an inverse ranking where low moments receive a higher probability of survival into the next generation and high moments may soon die out due to lacking reproduction.
- *Crossover*: Two *parent* chromosomes may randomly exchange genes from a random position onward, whose probability was set to 0.6. This recombination (only one per iteration) could yield offspring with a higher fitness than either parent.
- *Mutation*: In rarer cases, with a probability set to 0.2, individual genes could spontaneously mutate to another resource rate from Table 3 (several per iteration possible). This fluctuation prevents that the algorithm settles into local optima.

#### 7.1 Computer Implementation

A computer implementation was modularly created in Microsoft<sup>®</sup> Excel to perform the evaluation of the singularity functions and the iterative steps of the genetic algorithm until 50 iterations had been reached. The case distinction of equation (1) was expressed as an IF condition. Numerical values of each of the four resource profiles for the four chromosomes were calculated in four detailed table with a resolution on the time axis y of 0.05 d to incur only negligible rounding errors, if any. Column diagrams provided the resource profiles visually in addition to the numerical results. Additions and subtractions at the same cutoff value a (reduced by  $\varepsilon$  for subtractions) were recognized by combining the COUNTIF and SUMIF commands.

Figure 5 shows the results of minimizing the moment, i.e. leveling the resource profile, in four 'threads' from the four chromosomes. The algorithm was seeded with two randomly chosen chromosomes and two permutations that yielded very high moments,  $4,092 w^2 \cdot d$  and  $4,004 w^2 \cdot d$ , from manual experimentation. All subsequent values were entirely computer-

generated. The genetic algorithm successfully minimized the moment as the chromosomes with higher moments died out and lower ones were randomly found and reproduced to subsequent generations. The somewhat asymptotic nature of the moments indicates that the genetic algorithm performed efficiently. Upward spikes were crossovers and mutations with a low fitness that became extinct within a few iterations. The minimum moment was  $3,193 w^2 \cdot d$  for the permutation of  $r_{A3} = 5 w$ ,  $r_{B1} = 4 w$ ,  $r_{B2} = 4 w$ ,  $r_{E1} = 7 w$ ,  $r_{E2} = 8 w$ , and  $r_{F1}$  The correct functioning of the computer implementation and the results that it produced was verified with several manual calculations. Current research at the Catholic University of America includes a collaboration with several major construction contracting and consulting firms to accomplish validation of the new method.



Figure 5: Results of Resource Leveling with Genetic Algorithm

## 8 CONCLUSIONS

This paper has described how the linear schedule analysis with singularity functions is extended to include resource leveling. Singularity functions can model the schedule as well as its resource profile. The equation of the resource profile is calculated via transposition and differentiation from the activity equations. It includes all possible changes from changing resource rates and allows deriving the objective function for an optimization. A computerized genetic algorithm minimized the first moment of area of the resource profile to achieve leveling. The results of an example were validated with manual calculations. Future research will investigate how singularity functions can solve various other scheduling problems for linear schedules, e.g. resource allocation, resource interruptability versus continuity, probabilistic activity durations, time-cost tradeoffs that shorten the project duration at increasing costs, cash flow optimization, and generalization of the approaches for more dimensions.

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