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A MARKOV PROCESS BASED DILEMMA ZONE PROTECTION ALGORITHM

Pengfei Li

Via Department of Civil and Environmental Engineering
301-D Patton Hall
Virginia Tech
Blacksburg, VA 2406, USA

Montasir M. Abbas,

Via Department of Civil and Environmental Engineering,
301-A Patton Hall
Virginia Tech
Blacksburg, VA 24061, USA

ABSTRACT

Dilemma zone (DZ) is an area at high-speed signalized intersections, where drivers can neither cross safely nor stop comfortably at the yellow onset. The dilemma zone problem is a leading cause for crashes at high-speed signalized intersections and is therefore a pressing issue. This paper presents a novel Markov-chain-based dilemma zone protection algorithm that considers the number of vehicles caught in DZ as a Markov process. The new algorithm can predict the numbers of vehicles in DZ and determine the best time to end the green so as to reduce the number of vehicles caught in DZ per hour. The algorithm was compared to the traditional green extension system and the results showed that the new algorithm was superior.

1 INTRODUCTION

Dilemma zone (DZ) is an area at high-speed signalized intersections, where drivers are indecisive of stopping or crossing when presented with yellow indicator. The dilemma zone problem is a leading cause for rear-end collisions and red-light running. According to latest safety surveys, there are more than one million crashes at intersections a year and most of them occurred at signalized intersections(National Safety Council 2007). As a result, there is a strong need for real time algorithms that can predict the number of vehicles in DZ so as to make the correct decision of when to end the green phase.

Traffic controllers typically provide a window of time with a variable length to end the green. If the green continued until its maximum time (known in traffic jargon as a max-out), the controller would be forced to end the green regardless of the number of vehicles in DZ. In this paper, we use the Markov process (MP) as a means to predict the number of vehicles in dilemma zone in the near future, and evaluate the impact of ending the green at any given times before max-out. The Markov process has been proved capable of simulating a wide range of systems (Ross 2006). However, in the field of the traffic signal control, few MP-based signal control applications were reported. In this paper, we considered the number of vehicles in DZ during the green as a discrete Markov variant. In light of this idea, the number of vehicles in DZ is predicted with the current number of vehicles in DZ and the state transition matrix. Specifically, using the transition matrix, the new algorithm first predicts the numbers of vehicles in DZ per hour for all the time steps before the maximum green. Then the algorithm seeks the best time to end the green to obtain the fewest hourly vehicles caught in DZ. If the best time is now, the algorithm will end the green immediately, if the best time is in future, the algorithm will extend the green one more step. This process is repeated until either the algorithm considers now is the best time to end the green or the green phase reaches the maximum.

In order to elaborate the new algorithm, we structured this paper into three parts: the first part includes the literature review on the MP's applications to the transportation field, the explanation how the new MP-based algorithm works and the description how a VISSIM-based simulation environment was developed in order to test and evaluate the new algorithm; the second part is to evaluate the new algorithm using data from a high-speed signalized intersection in Christiansburg, VA. The geometry of that intersection and its dynamic traffic patterns were modeled into the simulation environment exactly to obtain a close-to-reality traffic network; the third part will analyze the safety performance of the new algorithm.

2 LITERATURE REVIEW

The Markov process has been proved capable of simulating highly stochastic, non-linear traffic systems. A typical Markov control model is composed of four items: state space, control actions, states transition probabilities and reward matrix.
The state space X: it is a *Borel*[2] *Space* and each element in the space is called *state*. In the context of the traffic system, the state space is defined to reflect traffic dynamics and it can be, for example, queue length, number of vehicles in DZ or control

delay. Previous studies on stochastic traffic dynamics primarily focused on how to improve the mobility and therefore they usually used the number of vehicles (e.g., the queue length or the platoon length) to define the state spaces. Taken as examples, Botma used the number of vehicles in a queue to model the stochastic traffic(Botma 1999); Botma's model was later used in Hoogendoorn's study on the robust control of stochastic traffic networks(Hoogendoorn *et al.* 2008); Alfa and Neuts used the number of vehicles in a platoon to define the state space for the random traffic arrival profile(Alfa and Neuts 1995); Viti and Zuylen used the number of vehicles in queues as the state space to re-cast the queuing models at signalized intersections(Viti and Van Zuylen 2004); Geroliminis and Skabardonis used the queue length as the state space to model the stochastic traffic dynamics along signalized arterials(Geroliminis and Skabardonis 2005); Cascetta modeled traffic assignment evolution as a stochastic process using the Markov process(Cascetta 1989). Obviously, the more detailed a state space is modeled, the closer to the reality it is. Excessive details, however, may also dramatically increase a problem's dimension and result in excessively long computing time. For this reason, in their Markov-process-based adaptive signal control framework, Yu and Recker used a binary state space, congested vs. uncongested, rather than the queue length on each approach to ensure the optimizing algorithm can work fast. When the number of vehicles in a link is greater than a threshold value, that link is marked as "congested", otherwise "uncongested"(Yu and Recker 2006). Similarly, Kim *et al.* used "congested" and "uncongested" to mark the states for each node of the network in their vehicle routing studies(Kim *et al.* 2005).

The possible control actions A: it is a *Borel Space* and defined as the set of all possible controls (or alternatives). Each element x in the state space X is associated with, or results from, a subset of A. From the perspective of the traffic signal control, it is usually translated into control strategies. Taken as examples, some well-known control strategies include: SCOOT(Hunt *et al.* 1981), SCATS(Sims and Dobinson 1980), OPAC(Gartner 1983), RHODES(Head *et al.* 1992) and TRPS(Abbas and Sharma 2006). However, although most signal control strategies implicitly incorporate the traffic's stochastic features, they are in general based on deterministic models rather than stochastic models. As a result, those traditional strategies sometimes may not function well under highly dynamic traffic patterns. Another issue about the control strategies is how to efficiently optimize the strategies on-line or off-line. Off-line optimization works well when a traffic pattern is repeatable. Some well-known studies include: the *Hill Climbing* optimizing technique in the package of TRANSYT-7(Robertson 1969); neural networks in self-organizing traffic control strategies(Nakatsuji and Kaku 1991); *Genetic Algorithm* to optimize the signal control strategies in TRANSYT-7F (version 8.1)(Park *et al.* 1999) and *Multiple objective Genetic Algorithm* to optimize traffic responsive signal control framework(Abbas *et al.* 2005).

On-line adaptive optimal signal control strategies have a higher requirement for computing efficiency. There are two optimizing approaches for adaptive control strategies:

1. *Binary choice logic*, in which time is divided into successive small steps. Between the minimum green and the maximum green, a binary decision (i.e., end vs. extend current green) is made at each time step. Examples of the binary control logics include vehicle actuated signal control strategies, and the stepwise adjustment of timing plans(Lin and Vijayakumar 1988; Lin 1990). The binary choice logic considers a short-term future (10sec~30sec) and can typically be optimized fast.
2. Sequential approach, in which the decision-making window is longer (50sec~100sec) than the binary logic. *Dynamic programming* (DP) is commonly used here. The original dynamic programming may need excessively long time with the increase of the problem dimensions and therefore it is often revised in practice. For example, the well-known adaptive signal control strategies, OPAC(Gartner 1983) and RHODES(Head *et al.* 1992) use the *rolling-horizon* and iterative dynamic programming technique to increase the optimizing speed. Yu et al. used a rapid-calculate version of dynamic programming to obtain the optimal control strategies(Yu and Recker 2006).

The probability measure space: it is typically described in a matrix P and each element p^k_{ij} in P stands for a transition probability from state i to state j under control measure k. A stochastic process is called *Markov Process* if its future probabilities are only determined by its most recent states. In the field of the traffic signal control, most related studies before considered the traffic dynamics (e.g., traffic arrival profile, queue length) as the Markov process. Viti and Zuylen designed a mesoscopic Markov model for queues at controlled intersections(Zuylen *et al.* 2006); Yu and Recker used the *Poisson Process* to analyze the queue length changing probabilities in their MP-based adaptive signal control(Yu and Recker 2006); Geroliminis and Skabardonis used MP to model arrival profiles and queue lengths along signalized arterials(Geroliminis and Skabardonis 2005); Alfa and Neuts used a discrete time Markovian arrival process to model traffic platoons(Alfa and Neuts 1995); Adam et al. used Markov transition matrix in the dilemma zone protection policy studies(Adam *et al.* 2009; Adam *et al.* 2009); Kim et al. used travel time estimation to calculate the transition probabilities between states(Kim *et al.* 2005); Sun et al. used Gaussian Mixture Model (GMM) and Maximum Likelihood Estimation (MLE) to estimate the transition matrix(Sun *et al.* 2004); Sherlaw-Johnson et al. used the *Maximum Likelihood Estimation* (MLE) to estimate the transition matrix(Sherlaw-Johnson *et al.* 1995); Hazelton and Walting used linear exponential learning filter(Hazelton and Watling

2004); Gaussian process to derive the transition matrix their traffic equilibrium distribution studies(Hazelton and Watling 2004).

One-step reward R is the immediate result from the control action. Reward is important for on-line optimization in adaptive signal control strategies. Reward is defined according to the problems. Taken as examples, Yu and Recker assigned a high reward for a control policy if the traffic state transit from the congested(Yu and Recker 2006); Adam et al used the number of vehicles in the dilemma zone(Adam *et al.* 2009; Adam *et al.* 2009); Kim *et al.* estimated the cost proportional to the travel times(Kim *et al.* 2005).

3 PROBLEM STATEMENT

Although previous studies have proved that the Markov process could be applied to many traffic problems, its applications to the dilemma zone issues have been limited. Adam et al. designed a dilemma zone protection policy using reinforcement learning. The algorithm compares the current number of vehicles in DZ and the predicted numbers of vehicles in DZ to arrive at the optimum policy. This paper uses the same state space framework, but with a MP transition matrix that is updated in real time to respond to the changing traffic conditions.

This paper evaluates the new MP-based dilemma zone protection algorithm under dynamic traffic. The new algorithm was directly deployed and evaluated in VISSIM® via a middleware, namely *VTDatex*. In general, if a new signal control algorithm involves complex computing, it will have difficulty in embedding into most traffic simulation packages in the market. As a result, many studies evaluated their signal-control algorithms in simplified simulation environments, which is likely to cause bias since many driving behaviors are ignored.

4 MODEL DESCRIPTION

Markov-chain-based dilemma zone protection algorithm

Let N_{t0} denote the current number of vehicles in DZ at time t_0 and $P(t_0)=\left[p_{ij}(t_0)\right]$ denote the state transition matrix. Then the state transition matrix at time t_n after n time steps is:

$$P(t_n)=\left[p_{ij}(t_0)\right]^n=\left[p_{ij}(t_n)\right] \tag{1}$$

And the probabilities in which the state transits from N_{t0} to other states after n time steps are:

$$\Pi^{t_n}=(p_{N_{t_0},0}(t_n),p_{N_{t_0},1}(t_n),p_{N_{t_0},2}(t_n),.\ \ .\ (\sum_j p_{N_{t_0},j}(t_n)=1) \tag{2}$$

Where $p_{N_{t_0},j}(t_n)$ is the corresponding element in the state transition matrix $P(t_n)$.

With Eq. (1) and (2), the predicted number of vehicles at time t_n is:

$$N_t=0\times\pi_0^{t_n}+1\times\pi_1^{t_n}+2\times\pi_2^{t_n}+.... \tag{3}$$

$$\text{Where } \pi_j^{t_n}=p_{N_{t_0},j}(t_n)$$

At each time step (either current or future), the hourly number of vehicles in DZ can be calculated with the predicted number with Eq. (3) and the corresponding cycle length.

$$N_t^{Hourly}=N_t*\frac{3600}{G_t+l+\sum_i(g_i+l)} \tag{4}$$

Where:

- N_t^{Hourly}: Calculated number of vehicles caught in DZ over an hour;
- N_t: Predicted number of vehicle caught in DZ at time step t.
- G_t: Current green duration at time step t;
- l: Time loss between greens (i.e., yellow+all-red);
- g_i: Average green durations on conflict phases;

At any particular point in time, if the calculated number of vehicles in DZ per hour (which is caused by ending the green now) is fewer than any predicted vehicles in DZ per hour (which is caused by ending the green in future) the algorithm will end the green immediately. Otherwise, the algorithm extends the green one time step. In case that the observed number and certain predicted number are equally minimal, the green will be extended. The rationale is to lengthen the cycle length and reduce the hourly number of vehicles in DZ.

4.1 Updating the State Transition Matrix

The state transition matrix $\left[p_{ij}(t)\right]$ is critical for the prediction. In order to respond to the latest traffic, the matrix needs to be updated periodically according to the new incoming observations. We applied the *rolling horizon* concept to the matrix updating process. The rolling horizon concept is used by operations research analysts in production-inventory control and was introduced into the signal control field by Gartner in his adaptive signal control algorithm, OPAC (Gartner 1983). OPAC The new algorithm collects state transitions during the "head" time of each stage, updates the matrix according to the new data then applies the new matrix during the "tail" time (Figure 1).

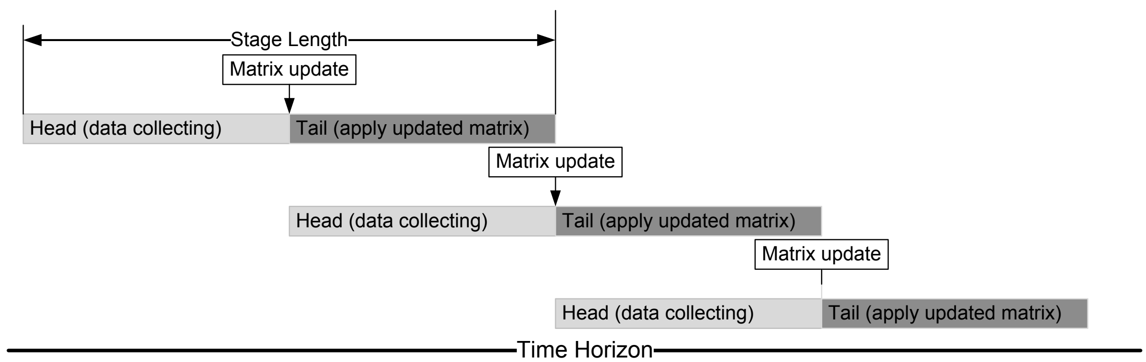

Figure 1: Updating Markov matrix using rolling horizon technique

Let Ω denote the finite state space and $X_1, X_{2,}...$ be the transitions between states. Then X is a Markov chain on Ω with the transition matrix $\left[p_{ij}(t)\right]$. The empirical state transition matrix can be derived using the new observations with entries:

$$\hat{p}_n(i,j) = \frac{\sum_{k=1}^{n} 1(X_k = i, X_{k+1} = j)}{\sum_{k=1}^{n} 1(X_k = i)} (i, j \in \Omega) \tag{5}$$

We reasonably assume that the denominator in Eq. (5) is positive for all the possible states after long observation (i.e., the head time is long enough).

While deriving the transition matrix with the observations, one issue is that the observed transitions within one head time may be biased due to the system's inherent randomness. To mitigate this bias, the algorithm does not derive the transition matrix merely with the new incoming observations. Rather, it estimates the number of observations using both the historical observations and the new observations. The underlying rationale is that although the traffic is highly dynamic, the traffic pattern is repeated day by day. As a result, the traffic patterns at the same time of different days are mostly similar.

As in Figure 2, the historical observations were stored in a series of time-dependent matrixes and each cell of those matrixes stands for the transitions between states during certain time. Whenever new observations come in, the algorithm estimates the transitions for each cell using the new incoming transitions, the historical transitions at the same time of a day, the historical transitions head-time units before that time of a day, the historical transitions two head-time units before that time of a day, the historical transitions one head-time units after that time of a day and the historical transitions two head-time units after that time of a day.

Least-square estimation is used and its mathematical expression is as follows:

Let $N_{i,j}(t)$ (variable) denote the estimation of the transitions between *i* and *j* at time t; $N_{i,j}^{old}(t)$ denote the historical transitions between *i* and *j* at time t; $N_{i,j}^{new}(t)$ denote the new incoming transitions between *i* and *j*. Then the estimation using the least-square estimation can be formulated as:

$$\begin{aligned} Min\ & \left(N_{i,j}(t)-N_{i,j}^{new}(t)\right)^2+\left(N_{i,j}(t)-N_{i,j}^{old}(t-1)\right)^2+\left(N_{i,j}(t)-N_{i,j}^{old}(t-2)\right)^2 \\ & +\left(N_{i,j}(t)-N_{i,j}^{old}(t+1)\right)^2+\left(N_{i,j}(t)-N_{i,j}^{old}(t+2)\right)^2 \end{aligned} \tag{6}$$

After all $N_{i,j}(t)$ s are estimated, they are first used to derive the new transition matrix, and then they replace the data in the historical data matrixes for the corresponding time period. The approximating function for each cell is updated as well. This method will not only mitigate the possible bias generated during the matrix updating but also prevent the historical data from getting obsolete.

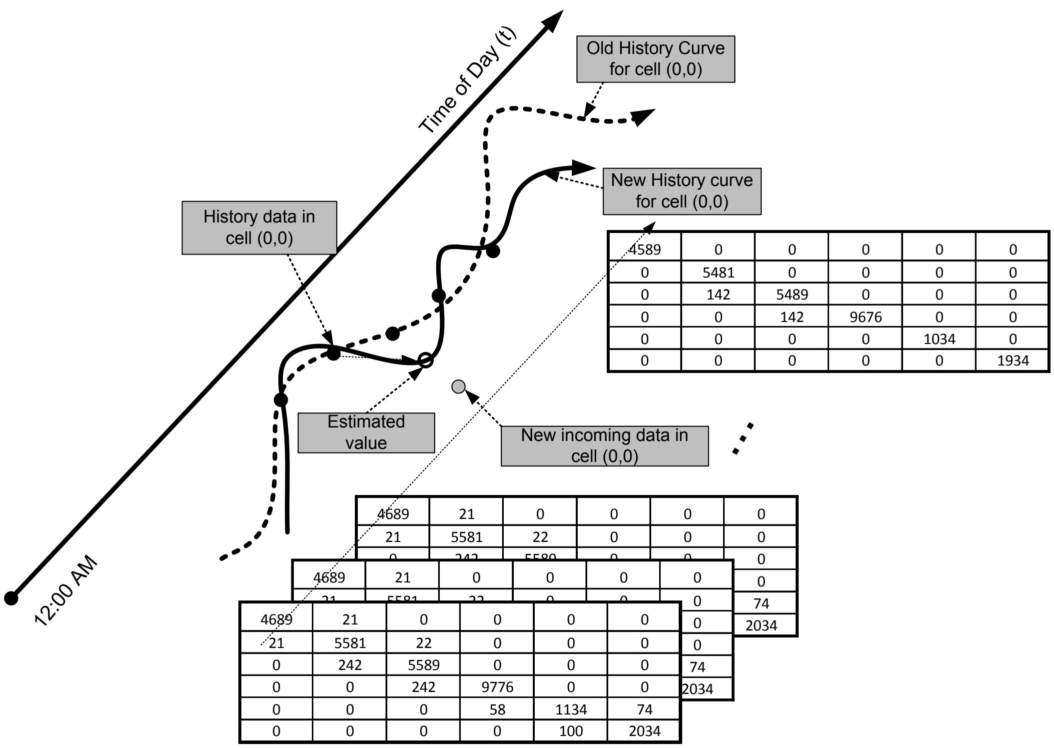

Figure 2: Illustrations how to update the transition matrix using both historical data and new incoming data

In summary, the MP-based dilemma zone protection algorithm can be described as in Figure 3.

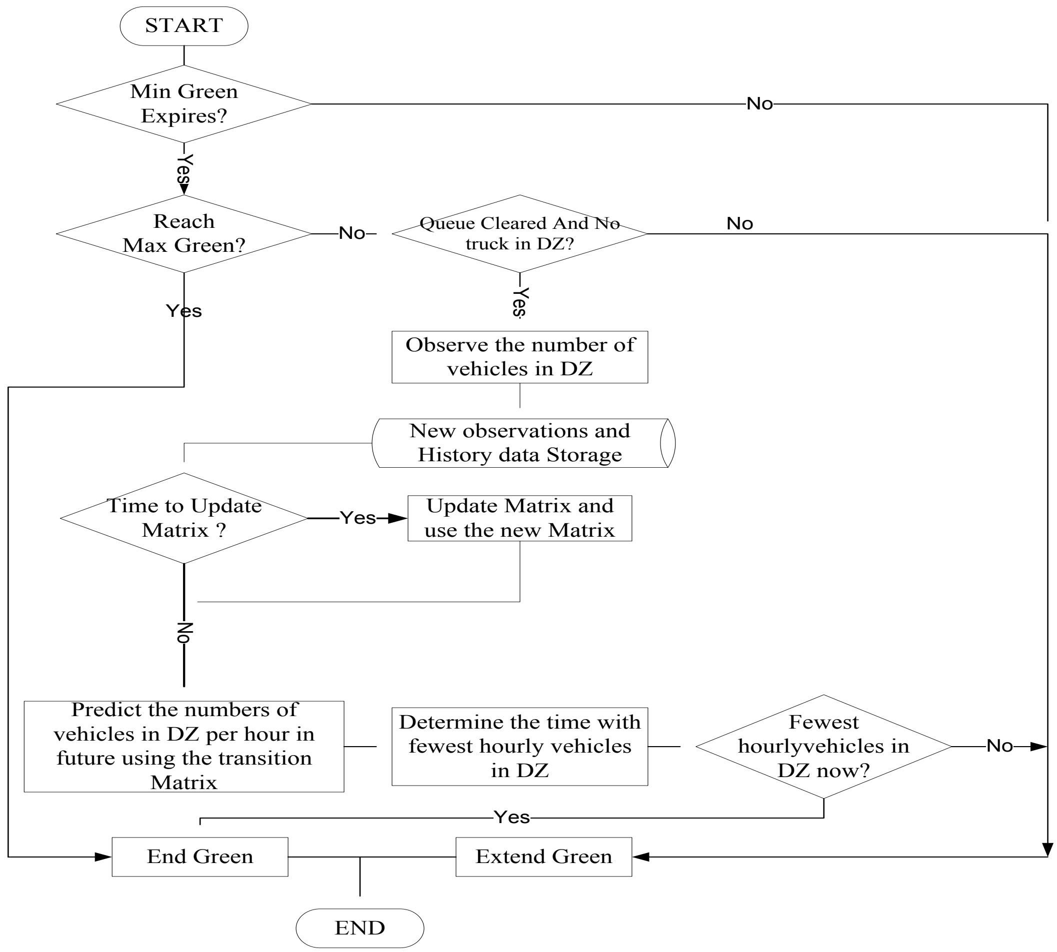

Figure 3: Flow chart of the new MP-based dilemma zone protection algorithm

4.2 Algorithm deployment in VISSIM

The new algorithm was deployed and evaluated in a commercial microscopic traffic simulation environment, VISSIM (PTV-AG Inc. 2008). The advantages of VISSIM over other simulation packages include:

1. VISSIM provides the largest flexibility for users to calibrate the driving behaviors and traffic conditions;
2. VISSIM was developed under .NET framework, which brings flexibility for add-on program development;
3. VISSIM provides the best tools for the development of the signal control strategies, such as the NEMA controller emulator, Vehicle Actuated Programming (VAP) language, signal control Application Programming Interfaces (SCAPI), etc.

VISSIM SCAPIs method was used to develop the MP-based signal control emulator in this research and was written in C++ language.

The algorithm also needs real-time vehicle trajectory data to calculate the number of vehicles in DZ. this information was acquired via VISSIM's Common Object Module (VISSIM COM) and sent to the external control emulator. The controller then runs the algorithm and makes decisions according to the incoming data and returns the new desired phase states to the VISSIM network. The concept of this simulation environment is illustrated as in Figure 4(Li and Abbas 2009).

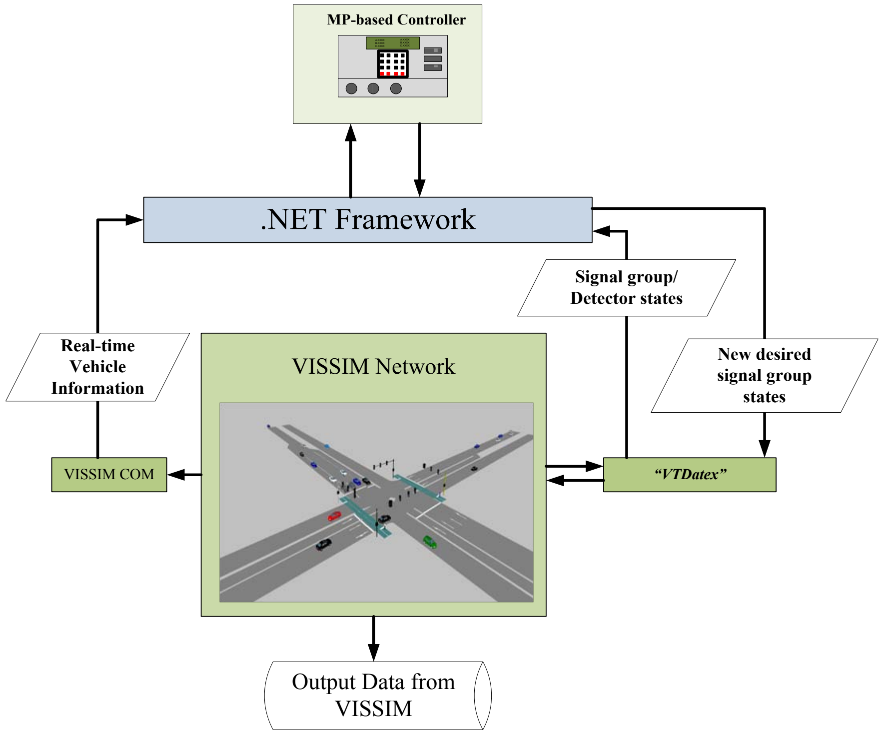

Figure 4: Illustration of VISSIM Simulation Environment

4.3 Study Intersection

The study intersection is located on the Peppers Ferry Road and the North Franklin Street in Christiansburg, VA. It has high-speed (45 MPH) lanes dedicated to the through traffic on the north-bound and south-bound approaches. Those lanes are our study lanes.

We collected the traffic volume on the study lanes every 15 minutes with a data acquisition system on the high-speed approaches and the whole counting lasted 9 hours. The through-traffic volumes were plotted as in Figure 5. These volumes were modeled into the VISSIM network to obtain a close-to-reality traffic volume pattern. The volumes on other approaches are listed in Table 1.

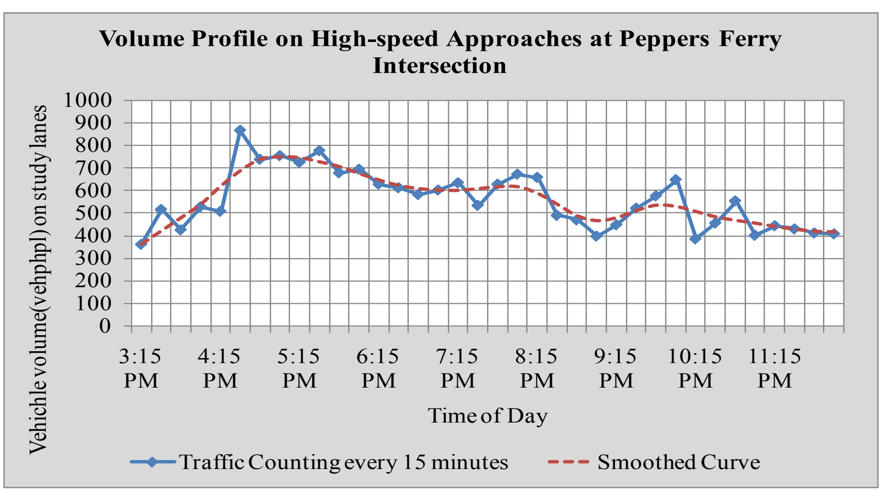

Figure 5: Traffic volume profiles on the study lanes

4.4 Experiment Results

The MP based DZ protection system was applied to the two major approaches' through phases. Meanwhile, the traditional green extension system was also integrated into the same VISSIM network to compare how much additional benefit the MP-based DZ protection system could bring. The mechanism of the green extension system is to detect vehicles via advance detector(s) and protect the vehicles by extending the green until they leave DZ (Figure 6).

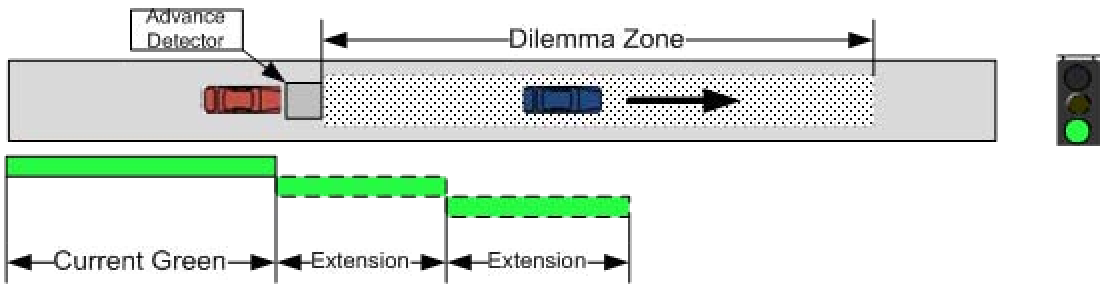

Figure 6: Illustration of single-detector green extension system

The traffic safety can be evaluated using two measures: 1) the probabilities of max-outs in an hour (since all protection is lost when a max-out occurs), and 2) the average number of vehicles caught in the dilemma zone. Figure 7 shows the hourly max-out ratios and it is clear that the max-out probability was kept at a very low level (less than 8% even though traffic volume was high) whereas the green extension system failed to protect vehicles in DZ due to its high max-out ratios(as much as 53%) under heavy traffic volume.

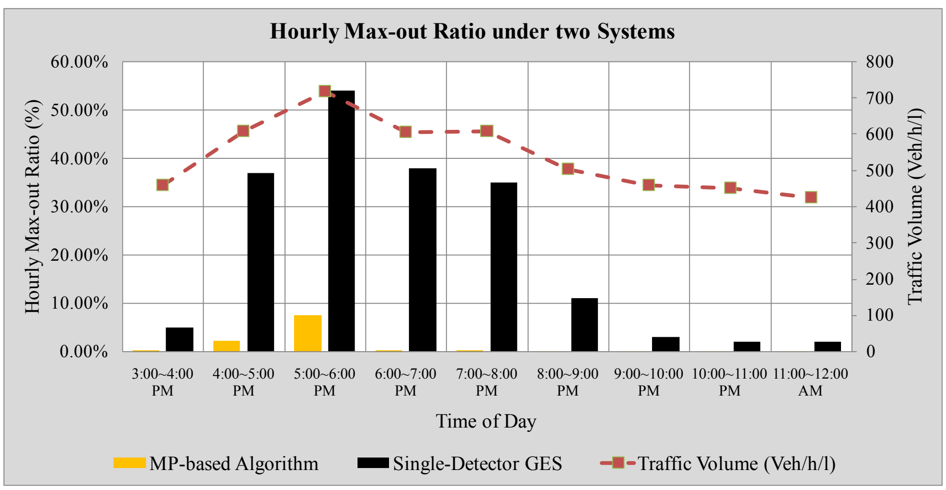

Figure 7: Hourly max-out ratios under two DZ protection systems

The number of vehicles caught in dilemma zone was also calculated as the total number of vehicles in DZ in one hour divided by the total number of approaching vehicles. Figure 8 shows that the new algorithm caught fewer vehicles in DZ than the green extension system under both high and moderate traffic conditions.

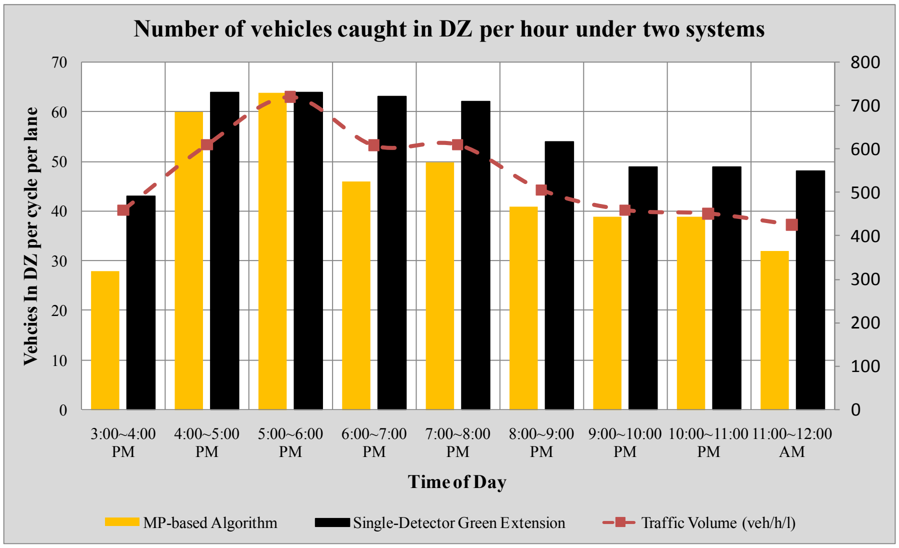

Figure 8: Vehicles in DZ per cycle under the new algorithm and the green extension system

5 CONCLUSIONS

In this paper, we address the dilemma zone issue by applying the Markov process concept to the designing of the dilemma zone protection algorithms. The new algorithm predicts the numbers of vehicles in DZ in a short-term future and compares them with current observation then it determines the best time to end the green. The Markov state transition matrix is critical in the predicting process. In order to respond to the latest traffic, the state transition matrix is periodically updated using a rolling-horizon approach.

The new algorithm was emulated in a close-to-reality VISSIM environment with multiple replications. The emulator was coupled with VISSIM's simulation engine via shared memory and the simulation was driven by VISSIM common object Module. The new algorithm was compared with another traditional dilemma zone protection system (the green extension system). The results showed that the new MP-based algorithm can maintain a lower max-out ratio and catch less approaching vehicles in DZ than the traditional system.

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AUTHOR BIOGRAPHIES

PENGFEI LI is a Ph.D. Candidate at Virginia Tech. He is expected to complete his Ph.D. in Civil Engineering in August 2009. His e-mail is <pli@vt.edu>.

MONTASIR ABBAS is an Assistant Professor at Virginia Tech. He received his Ph.D. in Civil Engineering from Purdue University in 2001. He has previously worked as an Assistant Research Engineer at Texas Transportation Institute and as a Visiting Assistant Professor at Texas A&M. His e-mail is <abbas@vt.edu>.