MODELLING PATIENT ARRIVALS WHEN SIMULATING AN ACCIDENT AND EMERGENCY UNIT

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ABSTRACT

This paper presents a case study of a discrete event simulation model of an Accident and Emergency Unit in a hospital in the UK. The objective of the study is to create a simulation study of the A&E Unit, to evaluate alternative scenarios and hence reducing patient waiting time. The case study uses a novel approach to predict the arrival time of patients and hence results in a more realistic platform on which to base the subsequent scenario analysis. The scenario analysis illustrates that significant reductions in the waiting time of patients can be obtained by relatively minor changes in operations.

1 INTRODUCTION

For more than twenty years, simulation had been used to solve healthcare problems in the US and UK. For example Pitt (1997) simulated patient flows in a hospital based on State Transition Networks. Spry and Lawley (2005) developed a model to evaluate pharmacy staffing and work scheduling and Swisher and Jacobson (2002) implemented multidimensional performance measurement of a Family Practice Healthcare Clinic using simulation. Jun et al. (1999) present a survey of 117 applications of simulation applications in health care clinics.

Most simulation studies within healthcare have concentrated on relatively tightly constrained environments of operational care (eg. the organisation of Accident and Emergency Departments (Miller et al. 2004) or have needed to vastly simplify the modelled domain in order to produce usable results. In general, such studies have been directed at specific issues of concern within identified institutions of care (Pitt 1997).

One of the common areas of concern in any health-care institution is the reduction of queue time or length of stay of patients. This also forms part of the quality in the management of the healthcare organisation. Today, many healthcare organisations have adopted different quality management techniques, such as Business Process Reengineering, Total Quality Management and Continuous Improvement, to improve their processes. As simulation

can model complex and highly variable environments, it a useful tool in these studies.

A significant percentage of a hospital's admission come through the accident and emergency (A&E) unit and it also attends to the most urgent cases of the day, it is crucial that the service in the department is efficient at all times. A number of case studies have been performed on A&E units. Garcia et al. (1995) proposed using simulation to look into the possibility of reducing time in an emergency room via a fast track. In another simulation study, McGuire (1998) suggests employing an extra patient care co-ordinator during peak hours, having an alternative room for admitted patients and a fast track to improve the length of stay in the emergency department. Takakuwa and Shiozaki (2004) simulated patient flows using ARENA and discovered that patients in an A&E Unit spent the majority of their time waiting for treatment and that waiting for emergency beds, doctors, drips and stretchers accounted for most of the waiting time. Miller et al (2004) illustrated the use of simulation for the continuous improvement of an A&E Unit and in particularly used EXTEND to implement Experimental Design techniaues.

One of the problems of simulating clinics such as an A&E Unit is accurately representing the arrival rates of patients. Random "walk-ins" (and emergencies) are superimposed upon scheduled appointments (for review etc) and so it is very difficult to predict and therefore manage patient arrivals at any point in time. In previous studies patients are usually grouped into appointment cases and random walk-ins (for example Kaukainen (1986)). The patients who have appointments will arrive at scheduled hours while the arrivals of the walk-in patients will be randomly generated. Senriech and Marmor (2004) develop a probabilistic model for predicting potential patient arrivals to an emergency department. However these approaches do not take into account the possibility of peaks and troughs in so called daily "random" arrivals. For example peak times in patient arrivals often occur in late morning and early afternoon.

This paper presents a simulation case study (using the MedModel simulation package) which was performed in conjunction with a re-engineering study of an A&E department in the UK. A novel approach to modelling the arrival pattern of patients is utilised which represents the random and deterministic nature of the arrivals. The potential of using simulation in a dynamic and complex environment is illustrated and the results of the model, based on the proposed model for arrival times, are used to establish a set of alternative scenarios.

2 THE A&E UNIT

The case study is based on a public hospital in a conurbation of around 200,000 people in the UK. The hospital A&E Unit sees an average patient load of 160-200 daily. Though the hospital is a non-profitable organisation, it has managed competitively by the standards set by the UK National Health Service (NHS).

Patients are categorised into five groups depending on their condition which is assessed during triage: red, orange, yellow, green and blue (the highest priority being red). Patients with high priority are attended within a time specified by NHS and patients with lower priority wait longer.

For comparisons with NHS standards, the performance measures presented in this study are: (a) average waiting time for triage and (b) average waiting time for patients in the red category. Table 1 shows the NHS standards.

Table 1 NHS Waiting time Standards

Category	Standards set up NHS (mins)
Red	15 minutes
Orange	30 minutes
Yellow	About an 1 hour
Green	- no specified standards

Current Operations: Figure 1 shows the layout of the accident and emergency unit. All patients arrive at the reception counter, where their details are recorded and the patients then wait for triage. During triage, the nurse will record vital signs which are determined through direct and external contact in the triage room. Patients without appointment are categorised into one of the five groups: red, orange, yellow, green and blue while, the nurse pages the doctors (also known as Senior House Officers (SHO)) to see patients who have appointments. For patients who arrive by ambulance, the registration clerk and the triage nurse are called to the trolley area. Here, the nurse will carry out a urine test or ECG, for patients were required. The doctor will pick up a folder, based on the acuity of patient, and call the patient to either a treatment room or the minors department. The doctors will then decide if

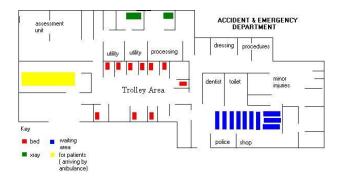


Figure 1: Floor plan of the A & E Unit

the patient requires a blood test and/or x-rays. Patients who require an x-ray are sent to the x-ray department. After the x-ray procedure the patient returns to the treatment room where the doctor will decide on the appropriate course of treatment. It is noted that all blood test results will take at least one hour to return (as a porter delivers the test results once an hour) and can take as long as 2 hours. Due to medical insurance liability, the results cannot be sent via email.

Patients in the red category have to be attended to immediately while those in the orange category have to be attended to within 10 minutes. All patients in these 2 categories will also require a bed. 65% of "yellow" patients and 30% of "green" patients will require a bed. All patients who do not require a bed will be seen by a nurse practitioner in the minors department during the day. A SHO is on duty in the minors department.

Patients who require a bed will be sent to the trolley area once a bed is made available (using a FIFO priority queue). Patients will be attended by a doctor (SHO). It will then be decided whether the patient needs a blood test and/or x-ray.

If it is decided that the patient needs to be sent to a specialist, he/she will then wait for the specialist in the trolley area or waiting area (depending on acuity). Meanwhile, the specialist will be paged to see the patient in the trolley area. For the patient who is waiting for a specialist in the trolley area, a nurse needs to check on the patient every half an hour and to inform him/her or his/her relatives of his/her progress.

The Minors Department, which deals with patients with minor injuries, opens from 9am to midnight. Most of these patients are attended by nurse practitioners. In the evenings (especially after midnight), any patient who needs to be admitted to the wards will be kept in the trolley area. This is to prevent disturbing the rest of the patients in the ward who are sleeping. The patients in the trolley area, are attended by a nurse every 20 - 30 minutes depending on the their condition. A SHO is stationed in the trolley area at all times.

The specialist, or the houseman assisting the specialist, will then attend to the patient in the trolley area and decide whether to admit him or to give him an appointment to return on another day. Patients who attend the fracture clinic will proceed to the clinic itself, and those attending the review clinic wait to be serviced by the A & E consultant.

3 MODEL BUILDING

A simulation model of the A&E unit was built using MedModel, a simulation-based software tool for evaluating, planning or re-designing healthcare systems in the healthcare industry. There are 6 basic resources represented in the simulation model: consultants, doctors, nurses, auxiliary nurses, nurse practitioner and technicians

3.1 Assumptions

It is assumed that the number of patients who attend the review clinic is equal to the number of patients who are scheduled for the review clinic. Although in reality most (but not all) patients arriving by ambulance have high priority, the model assumes that all patients arriving by ambulance will have high priority. In reality, the percentage of patients who arrive in the evening are more seriously ill than those who attend the hospital in the day, however it is assumed that the distribution of acuity of the patients in the day and night are equivalent.

3.2 Data Collection

Data was obtained from past records from the department and collected and analyzed for a period of one month. Time studies were performed on treatment times and statistical distributions were then fitted to the data. Historical data for patient arrivals was segregated by day of the week, then by hour of the day.

It is noted that in McGuire (1998), the number of patients arriving each day was represented by Poisson distribution while the patient arrivals was further broken down into a percentage of patients for each two-hour segment of the day. Exponential inter-arrival times were then set for the 12 two-hour segment of each day. McGuire (1998) uses Weibull distribution for service times. The majority of service times in Garcia et al. (1995) followed a uniform distribution.

3.3 Modelling the Arrival Rate

One of the important input distributions to be included in the model is the arrival rate of patients. This section describes the problem the arrival rate of this department presents and how it can be overcome with the help of a mathematical model. Analysis of the data from the current study showed that the arrivals peaked at particular times of the day consistently every day of the week (Monday to Friday). The structure of the data resembled periodic component with a random component superimposed. This is consistent with a scheduled arrival pattern with random walk-ins. It was therefore decided to model the arrival pattern with a combination of a Fourier Series to model the periodic component and a non-Gaussian Autoregressive (AR) process to model the random arrivals. This novel technique was originally proposed to model the arrival pattern in a healthcare environment by the authors (Meng and Spedding 2008).

3.4 Determination of the arrival distribution using a mathematical model.

The following methodoloogy was used to model the arrivals to the A&E unit.

- 1) Firstly, the collected arrival times was divided into days of the week and hours of the day.
- 2) ANOVA and t-test were carried out on the data to determine if there was any significant difference between the days of the week (Monday to Friday). It was found that they are similar.
- Next, the arrivals per day was linked together to obtain a time series which represented the arrivals for the entire week...
- 4) An autocorrelation function of this time series was obtained. It was observed that the series is periodic.
- 5) A Fourier series is fitted onto the time series to model the periodic components.
- 6) These Fourier terms are then subtracted from the original time series.
- 7) The autocorrelation function of the new series shows no signs of cyclic function and that the remaining series is totally random.
- 8) The distribution of the remaining random component is identified as a log-logistic distribution.

3.5 Time Series

A time series is a record taken through time. It will express any observation in the series as a mathematical function of some combination of time, other variables, known values of interest, and one or more random components. A suitable model for the data may be

$$X(t) = T(t) + S(t) + R(t)$$

Where X(t) = time series under investigation,

T(t) = trend

S(t) = seasonal term, and

R(t) = random component.

As the data seems to exhibit both cyclic and random components, it is necessary to decompose the series (Wold 1958). After separating these components, a low-order Gaussian AR process is used to model the remaining random part. This approach may be extended by substituting a non Gaussian ARMA process to model the random component and thus leading to a more accurate representation (Watson and Spedding 1982).

ARIMA (Autoregressive Integrated Moving Average) methods combine three processes: autoregression (AR); differentiation to remove the integration (I), and moving average (MA) (Box et al. 1994). The general ARIMA model, neglecting seasonality, is traditionally and mathematically written as ARIMA(p,d,q) where p is the order of autoregression, d is the degree of differentiation and q is the order of the moving average involved. Each of the three processes is described by a small integer. The three processes are closely related to each other and can be examined separately.

ARIMA models can only describe stationary series or a series that have been made stationary by differentiation. The general model can be expressed by the following equation (Box et al. 1994). The present observation Yt, is a combination of an autoregressive and moving average model. This means Yt, is a linear function of past stationary observations plus present and past forecasting errors:

$$Y_{t} = \delta + \phi_{1}Z_{t-1} + \phi_{2}Z_{t-2} + \dots + \phi_{p}Z_{t-p} + \varepsilon_{t} - \theta_{1}\varepsilon_{t-1} - \theta_{2}\varepsilon_{t-2} - \dots - \theta_{a}\varepsilon_{t-a}$$

Where

Y_t = the stationary observation;

 Y_{t-1} , Y_{t-2} ,..., Y_{t-p} = past stationary observations (usually not more than two)

 δ =the constant of the model;

 $\phi_1, \phi_2, \dots, \phi_p$ = the autoregressive coefficients.

 ε_t = the random error of the present time period (expected value is equal to 0)

 $\epsilon_{t\text{-}1}, \, \epsilon_{t\text{-}2}, \, , \epsilon_{t\text{-}q} = \text{past forecasting error (usually not more than 2 are used)}$

 $\theta_1, \theta_2, \theta_q$ =the moving average coefficients.

In most instances, including the present case study the autoregressive process alone provides a good approximation of the stochastic component. Fourier analysis approximates a function, i.e., the given time series, by a sum of sine and cosine terms called the Fourier series.

$$f(t) = \frac{a_0}{2} + \sum_{r=1}^{k} a_r \cos(2\pi f t) + b_r \sin(2\pi f t)$$

Where f(t) = time series under investigation a_0 = constant

 a_r = coefficient or random amplitude of the cosine functions

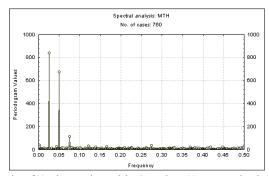
 b_r = coefficient or random amplitude of the sine functions

f = frequency of the harmonic analysis

The most significant frequency is identified from the periodogram. Coefficients, a_r and b_r , are approximated by numerical integration. By combining several periodic functions of the most significant periodicities, the time series can be represented by the Fourier series. The number of periodic cycles was chosen so that there is an equitable balance between residual error and the number of terms in the model. The periodogram of patient arrivals is shown in Figure 2. Therefore, the time series to represent the arrival time series of the month is given by

$$X(t) = S(t) + R(t)$$

where S(t) is represented by the approximate Fourier



Series f(t) shown in Table 2 and R(t) = Log-logistic(-7, 6.9, 6.84).

Figure 2 Periodogram of Patient Arrrivals

f b r a 0 6.9 1 0.0208-1.6928-1.5259 2 0.0417 0.4484 -0.4558 3 0.0448 0.3491 0.1630 4 0.0604 0.2883 0.1826 5 0.0625 0.2925 0.5214 -0.2707 6 0.0770 0.1890 7 -0.30950.01670.0337 8 0.0115 0.2663 -0.11709 0.1667 0.1854 0.2027 -0.2882 10 0.4072 0.0899

Table 2 Fourier Coefficients

4 RESULTS AND DISCUSSION

he results were verified using the "walk-through" technique. Meetings were carried out with personnel of the

hospital to ensure that the process flow and the results of the simulation of the actual system were correct. The models used to simulate the patient "random" arrivals were superimposed on scheduled patient arrivals and were determined to effectively model the arrival pattern experienced by the A&E unit. A scenario analysis was then carried out. The scenarios are described in Tables 3 and 4 and the results shown in Table 5.

The simulation results show that Scenario 1, the removal of the review and dressing clinic can reduce the average waiting time of patients by approximately 7 minutes. The overall time spent in the department is also reduced by approximately 14 to 18 minutes. However, the number of patients treated remains about the same. This is because the number of patients using the review clinic is about 5%. Patients take up about 20 minutes of either the nurse or the doctors' time, hence, the removal of this section will only bring about improvement in the range of 14 to 18 minutes.

Table 3 Scenarios and their Description

Scenario Description and changes					
	1 8				
Base	Simulation reflective of actual situation.				
Model	Review clinic exists. Blood tests return in 2				
	hrs, Consultant arrives 2 hrs after called, x-				
	ray dept opens 9am-5pm, after patients sent				
	to x-ray department outside hospital, &				
	returns to dept after obtaining results.				
1	Removes review clinic, others unchanged				
2	15 mins wait for consultant				
3	15 mins wait for blood test results				
4	15 trolley beds				
5	24-hour x-ray department				
6	15 minutes wait for consultant + have 15				
	trolley beds				
7	15 minutes wait for consultant and 24-hour				
	x-ray department				
8	15 minutes wait for a consultant and no				
	review clinic.				

Table 4 Scenario Settings

	Changes				
Sce- nario	Rev Clinic	Blood tests (hrs)	Cons- ultant (hrs)	Trol beds	X-ray dept open
Base	Yes	2	2	10	9-5
1	No	2	2	10	9-5
2	Yes	2	0.25	10	9-5
3	Yes	0.25	2	10	9-5
4	Yes	2	2	15	9-5
5	Yes	2	2	10	24 hrs
6	Yes	2	0.25	15	9-5
7	Yes	2	0.25	10	24 hr
8	No	2	0.25	10	9-5

Scenario 2 shows that if the waiting time for the consultant is reduced from 2hours to 15 minutes, there is a drastic change in the average waiting time for the patient. When a patient is waiting for the consultant, some of

them may be occupying trolley bed space, at the same time, nurses and doctors have to attend to them at intervals. Reducing this time will "free up" the time of nurses and doctors so that they can attend to more patients. It is observed that both the average waiting time for a consultant and the overall time spent in the clinic have been greatly reduced.

Table 5 Results of Scenario Analysis

	base	1	2	3	4
A	7.2	7.0	6.6	15.6	5.9
В	80.0	72.6	11.3	87.0	30.0
С		7.4	68.7	-7.0	50.0
D	95.3	88.0	23.5	106	45.8
E		7.3	71.8	-10.7	49.4
F	205	191	104	210	161
G		14.1	101	-5.6	43.7
Н	237	218	122	218	244
I		18.7	114	18.5	-7.0
J	13	14	4.7	10.9	12.8
K		-0.7	8.6	2.5	0.5

	5	6	7	8
A	6.8	14.0	10.0	9.4
В	72.1	7.7	11.3	17.1
C	8.0	72.3	68.8	62.9
D	85.0	23.2	19.7	25.7
E	10.2	72.0	75.6	69.5
F	193	108	97	109
G	12.3	96.8	108	96.6
Н	223	126	121	128
I	14.2	111	115	108
J	12.0	4.0	3.6	4.6
K	1.4	9.4	9.8	8.8

A = Red triage time

B = Average waiting time (yellow)

C = Imrovement 1 *

D = Average waiting time (green)

E = Improvement 2 *

F = Average time spent in total (green)

 $G = Improvement_3^{1} *$

H = Average time spent in total (yellow)

I = Improvement 4 *

J = No of people at the end of the day

K= Improvement 5 *

* All comparison made against Base Model

All Times in minutes

A comparison of Scenario 3 and the base model shows that bringing down the waiting time for a blood test result will generally reduce the waiting time of the patients by about 20 minutes. The average overall time

spent in the department is reduced by approximately 30 minutes. This is probably because some of these patients may not be waiting in the trolley area while they are waiting for blood tests. So, the effect of reducing the waiting time for a blood test may not give as significant a result as reducing the waiting time for the consultant.

When Scenario 4 is compared to the base model, it shows that having a larger trolley area cuts down the waiting time of patients by 45 to 55 minutes, as some trolleys are taken up by patients waiting for consultants. However, this alternative will not bring about any benefit to the yellow category patients as it increases the yellow patient category using the trolley area and they will then wait longer for the consultant or nurse to serve them.

Comparison of Scenario 5 to the base model shows that having a 24-hour x-ray clinic does not have a significant effect on the waiting time of patients. Although x-ray patients now get attended to in a shorter time span, the average waiting time, which measures the average time the doctor first sees the patient, is reduced by approximately 7 to 10 minutes as the patients continue to be served at the same rate. It is also noted that the overall time spent in the clinic is reduced and the number of patients served is slightly increased.

From Table 5, it is observed that among the single changes from Scenario 1 to 5, Scenarios 2 and 4 give the largest reduction to waiting time. Scenario 2 provides the largest reduction in the average overall time spent in the clinic by patients in both categories. Hence, different combinations of Scenario 1,2,4 and 5 are analysed giving rise to Scenarios 6 to 8. The results of this further analysis are also given in Table 3.

Scenario 6 is a combination of Scenario 2 and Scenario 4, the waiting time for a consultant is placed at 15 minutes instead of 2 hours and the number of trolley beds are increased by 5 to 15. It is found that average waiting time of both the green and the yellow categories are reduced drastically by approximately 70 minutes. number of people served also increased by 6. The overall time spent in the clinic is reduced by about 90 minutes for the green category and 100 minutes for the yellow category. Compared to Scenario 4, the average waiting time improves by approximately 20 minutes, and the average overall time spent in the department improves by almost an hour for the patients in the green category. When compared with Scenario 2, the average waiting time in this scenario improves marginally, and there is a slight improvement of about 3 minutes in the average overall time spent in the clinic for patients in the yellow category. The number of patients who are treated are about the same.

Scenario 7 combines Scenario 2 and 5. The waiting time for a consultant is 15 minutes and a 24-hour x-ray department has been implemented. Compared to the base model, the average waiting time of both the yellow and

green category is reduced by approximately 70 minutes. The overall time spent in the department the green and yellow category patients is reduced by an approximately 110 and 115 minutes respectively. The number of patients served increased by approximately 9. Compared to Scenario 2, there is little improvement in average waiting time of both categories, however, more patients are treated and the average overall time spent in the department is reduced. This scenario shows improvement in all aspects when compared to Scenario 5.

In Scenario 8, the review clinic is removed and the waiting time for the consultant is reduced from 2 hours to 15 minutes. This is a combination of Scenario 1 and 2. It is noted that this brings about a decrease of average waiting time of both categories by approximately 60 to 70 minutes. The average overall time spent in the clinic is reduced by approximately 95 to 112 minutes for the green category and however no improvement is made for the yellow category.

From the above analysis, it is found that the reduction of waiting time for the consultant will bring about the greatest benefit to the department. Scenario 6, which depicts a reduction of waiting time for the consultant as well as an increase in the number of trolley beds provides the greatest reduction in waiting time. Alternatively, Scenario 7, which depicts the setting up of the 24 hour x-ray clinic and the reduction of waiting time for consultant, provides the greatest reduction in overall time spent in the clinic. It is important that patients are attended as soon as possible and so it is recommended that the waiting time for the consultant be reduced and the number of trolley beds in the department to be increased.

5 CONCLUSIONS

This paper has presented a simulation case study of an Accident and Emergency Department. Arrivals to the A&E unit are modelled using a combination of Fourier Series and an Autoregressive Process. Alternative scenarios were analysed using the average waiting time of patients as the key performance indicator. The procedure for representing the combination of scheduled and stochastic arrival of patients resulted in a more realistic simulation model on which the scenario analysis was based. It is found that the reduction in waiting time for the consultant be reduced from 2 hours to 15 minutes provided the most benefit for the department. On the basis that it is important that the patients be attended to as soon as possible, it is recommended that the waiting time for the consultant to be reduced and the number of trolley beds be increased.

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