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AN EFFICIENT RANKING AND SELECTION PROCEDURE FOR A LINEAR TRANSIENT MEAN PERFORMANCE MEASURE

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ABSTRACT

We develop a Ranking and Selection procedure for selecting the best configuration based on a transient mean performance measure. The procedure extends the OCBA approach to systems whose means are a function of some other variable such as time. In particular, we characterize this as a prediction problem and imbed a regression model in the OCBA procedure. In this paper, we analyze the linear case and discuss a number of extensions. Additionally, we provide some motivating examples for this approach.

1 INTRODUCTION

In this paper, we develop a ranking and selection procedure to select the best out of multiple configurations based on mean performance. The procedure is complicated by the fact that the true underlying mean of each configuration is not constant but is a function of some other variable such as time. This problem was motivated by a simulation comparison of multiple configurations to select the configuration whose mean performance measure has achieved the best level of performance after a certain amount of simulated time. An example is found in new product development where multiple different prototypes are being simulated simultaneously and the one that is able to achieve the best specifications (e.g., based on performance, quality, safety, etc.) after a certain amount of development time is selected. Another example is simulation comparison of multiple configurations whose mean performance measures are in transient phases as a result of the initial start up transient (Law and Kelton 2000, Section 9.5.1) or as a result of parameter changes in a transient sensitivity analysis experiment such as the one described in Morrice and Schruben (2001). In either case, the system that is able to achieve the best level of performance within a certain amount of time is selected.

To illustrate, Figure 1 depicts a single configuration with a performance measure that is a function of time t. For traditional approaches to ranking and selection, the unknown mean performance measure for each configuration must be same as the mean of each simulation sample. To meet this requirement, simulation has to be conducted for the entire length of the run. Namely, if the performance measure of interest is some metric at M, then simulation has to be performed until M as shown in Figure 1. The existing ranking and selection literature focuses on determination of the number of simulation sample paths/runs for each configuration. The computational requirements for this type of analysis might render it impractical.

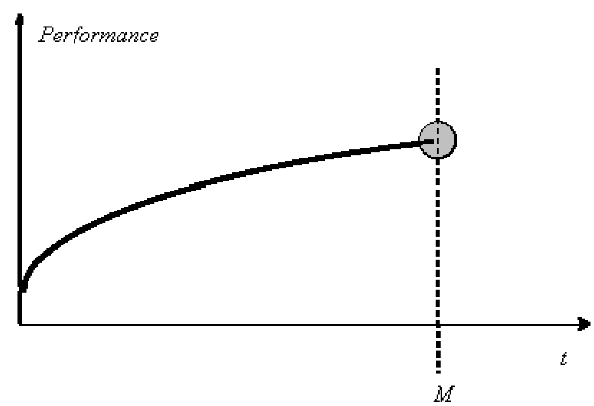

Figure 1: The mean performance of a single configuration up to time M

Suppose we are still interested in the performance measure for a simulation with length M but we simulate up to time m (<M) in order to reduce the computational burden. In our research, the objective is to predict the mean performance measure at M using the simulation results conducted only until m as shown in Figure 2. Further, if there are multiple configurations as depicted in Figure 3, at time *m* we want to be able to correctly *predict* which configuration is best at *M*. In other words, our research combines prediction with ranking and selection.

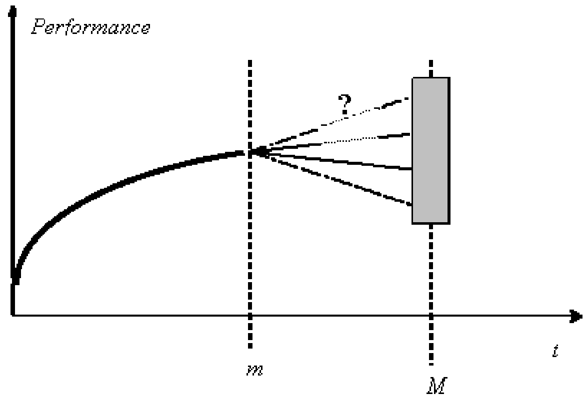

Figure 2: The mean performance of a single configuration up to time *m* with prediction of mean performance at time *M*

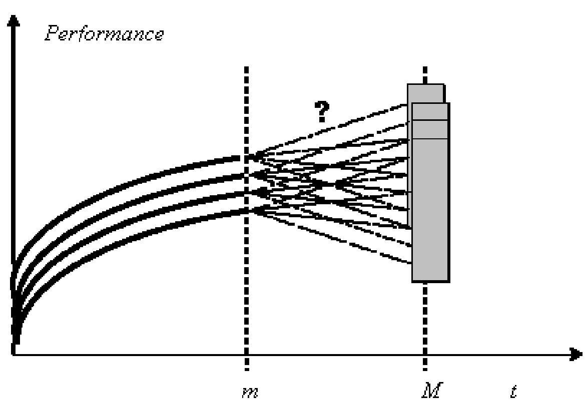

Figure 3: The mean performance of multiple configurations up to time *m* with prediction of mean performance at time *M* for ranking and selection

In the motivating examples given thus far, the mean performance is a function of time. However, it could be function of some other quantity (or quantities). For example, if one is interested in evaluating the effectiveness of different advertising approaches by forecasting sales revenues as a function of advertising dollars spent, the mean performance (sales revenue) for each advertising approach (configuration) is a function of advertising dollars spent.

We develop a regression-based approach and embed it within the OCBA procedure (Chen et al. 2000 and 2008) to address the prediction problem within the ranking and selection context. While we make a number of simplifying assumptions, the approach is general enough to accommodate an arbitrary independent variable (i.e., more than just time) and lends itself to a number of extensions.

The remainder of the paper is organized in the following manner. Section 2 provides a problem statement including the modeling assumptions. We describe the regression framework in Section 3. In Section 4, we extend OCBA by imbedding the regression approach in it to do prediction. Section 5 contains discussion, conclusions, and future research.

2 PROBLEM STATEMENT

This paper explores a problem with the principle goal of selecting the best of $\boldsymbol{k}$ alternative configurations. Without loss of generality, we consider the minimization problem shown below where the "best" configuration is the one with smallest expected performance measure.

$$\min_{j} \boldsymbol{y}_j(\boldsymbol{x}_M);\ \boldsymbol{j} = 1,\ldots \boldsymbol{k}\ .$$

We consider a common case where $\boldsymbol{y}_j(\boldsymbol{x}_i)$ must be estimated via simulation with noise and that the simulation output $\boldsymbol{f}_j(\boldsymbol{x}_i)$ is independent from replication to replication such that

$$\boldsymbol{f}_j(\boldsymbol{x}_i) = \boldsymbol{y}_j(\boldsymbol{x}_i) + \boldsymbol{\varepsilon};\quad \boldsymbol{i} = 1,\ldots,\boldsymbol{k},$$

where $\boldsymbol{\varepsilon} \sim N(0,\boldsymbol{\sigma}_j^2)$. (1)

We consider a discrete domain $\boldsymbol{x}_i \in [\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_m, \cdots, \boldsymbol{x}_M]$ and assume that we have a maximum simulation budget of $\boldsymbol{T}$ simulation runs on the simulation domain $\boldsymbol{x}_i \in [\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_m]$.

In this paper, we consider that the expectation of the unknown underlying function for each configuration is linear on the prescribed domain, i.e.,

$$\boldsymbol{y}_j(\boldsymbol{x}_i) = \boldsymbol{\beta}_{0j} + \boldsymbol{\beta}_{1j}\boldsymbol{x}_i\ . \tag{2}$$

The parameters $\boldsymbol{\beta}$ are unknown so $\boldsymbol{y}(\boldsymbol{x}_i)$ are also unknown. However, we can find an estimated expected performance measure at $\boldsymbol{x}_i$, that we define as $\hat{\boldsymbol{y}}_j(\boldsymbol{x}_i)$, by using a least squares estimate of the form shown in (3) below where $\hat{\boldsymbol{\beta}}_{0j}$ and $\hat{\boldsymbol{\beta}}_{1j}$ are the least squares parameter estimates for the corresponding parameters associated with the constant and linear terms in (1).

$$\hat{\boldsymbol{y}}_j(\boldsymbol{x}_i) = \hat{\boldsymbol{\beta}}_{0j} + \hat{\boldsymbol{\beta}}_{1j}\boldsymbol{x}_i \tag{3}$$

For ease of notation, we will define $\hat{\boldsymbol{\beta}}_j = [\hat{\boldsymbol{\beta}}_{0j} \;\; \hat{\boldsymbol{\beta}}_{1j}]$.

In order to obtain the least squares parameter estimates for configuration $\boldsymbol{j}$, we take a total of $\boldsymbol{n}_j$ samples on any choice of $\boldsymbol{x}_i$ (on at least two in order to avoid a singular solution). Given the $\boldsymbol{n}_j$ samples, we define $\boldsymbol{F}_j$ as the $\boldsymbol{n}_j$ dimensional vector containing the replication output measures $\boldsymbol{f}_j(\boldsymbol{x}_i)$ and $\boldsymbol{X}_j$ as the $\boldsymbol{n}_j \times 2$ matrix composed of rows consisting of $[1 \;\; \boldsymbol{x}_i]$ with each row corresponding to its respective entry of $\boldsymbol{f}_j(\boldsymbol{x}_i)$ in $\boldsymbol{F}_j$. Using the matrix notation, we determine the least squares estimate for the parameters $\boldsymbol{\beta}_j$ which minimize the sum of the squares of the error terms $(\boldsymbol{F}_j - \boldsymbol{X}_j\boldsymbol{\beta}_j)^T(\boldsymbol{F}_j - \boldsymbol{X}_j\boldsymbol{\beta}_j)$. As shown in most regression texts, we obtain the least squares estimate for the parameters as shown below:

$$\hat{\boldsymbol{\beta}}_j = (\boldsymbol{X}_j^T \boldsymbol{X}_j)^{-1} \boldsymbol{X}_j^T \boldsymbol{F}_j .$$

Our problem is to select the configuration associated with the smallest mean performance measure from among the k configurations within the constraint of a computing budget with only $\boldsymbol{T}$ simulation replications. Given the least squares estimates for the parameters, we can use (3) to estimate the expected performance measure at each configuration. We designate the configuration with the smallest estimated mean performance measure at $\boldsymbol{x}_M$ as $\hat{\boldsymbol{y}}_b(\boldsymbol{x}_M)$ so that $\boldsymbol{b} = \arg\min_j \hat{\boldsymbol{y}}_j(\boldsymbol{x}_M)$. Given the uncertainty of the estimate of the underlying function, $\boldsymbol{b}$ is a random variable and we define Correct Selection as the event where $\boldsymbol{b}$ is indeed the best configuration.

We define $\boldsymbol{N}_{ij}$ as the number of simulation replications conducted for configuration $\boldsymbol{j}$ at $\boldsymbol{x}_i$. Since the simulation is expensive and the computing budget is restricted, we seek to develop an allocation rule for each $\boldsymbol{N}_{ij}$ in order to provide as much information as possible for the identification of the best configuration. Our goal then is to determine the optimal allocations to the configurations that maximize the probability that we correctly select the best configuration (PCS). This Optimal Computing Budget Allocation (OCBA) problem is reflected in (4) below.

$$\begin{aligned} \max_{N_{ij}} \quad & \boldsymbol{PCS} = \boldsymbol{P}\{\boldsymbol{y}_b(\boldsymbol{x}_M) \le \boldsymbol{y}_j(\boldsymbol{x}_M) \,\forall\, \boldsymbol{j}\} \\ \boldsymbol{s.t.} \quad & \sum_{\boldsymbol{i}=1}^{\boldsymbol{m}} \sum_{\boldsymbol{j}=1}^{\boldsymbol{k}} \boldsymbol{N}_{ij} = \boldsymbol{T} \end{aligned} \tag{4}$$

The constraint $\sum_{\boldsymbol{i}=1}^{\boldsymbol{m}} \sum_{\boldsymbol{j}=1}^{\boldsymbol{k}} \boldsymbol{N}_{ij} = \boldsymbol{T}$ denotes the total computational cost and implicitly assumes that the simulation execution times for one sample are constant across the domain.

3 REGRESSION FRAMEWORK

In order to solve the problem in (4), we must obtain estimates for the parameters $\boldsymbol{\beta}_j$ which are assumed to be unknown and are treated as random variables. We will proceed with a Bayesian linear regression framework due to the ease of the derivation (Brantley et al. 2008 and DeGroot, 1970). We aim to find the posterior distributions of $\boldsymbol{\beta}_j$ as the simulation replications are conducted and use these distributions to update the posterior distribution of the performance measures for each configuration. We can then perform the comparisons with the performance measure for $\boldsymbol{y}_b(\boldsymbol{x}_M)$ as expressed in (4).

Given a set of initial $\boldsymbol{n}_j$ simulation runs with the output contained in vector $\boldsymbol{F}_j$, DeGroot (1970) shows that we can obtain the following results

$$\boldsymbol{E}(\boldsymbol{\beta}_j \mid \boldsymbol{F}_j) = (\Sigma^{-1} + \boldsymbol{X}_j^T \boldsymbol{X}_j)^{-1}(\Sigma^{-1}\mathrm{B} + \boldsymbol{X}_j^T \boldsymbol{F}_j)$$

$$\mathrm{cov}(\boldsymbol{\beta}_j \mid \boldsymbol{F}_j) = \boldsymbol{\sigma}_j^2 \left(\Sigma^{-1} + \boldsymbol{X}_j^T \boldsymbol{X}_j\right)^{-1} .$$

Note that for ease of notation, we will use $\tilde{\boldsymbol{\beta}}_j$ and $\tilde{\boldsymbol{y}}_j(\boldsymbol{x}_i)$ to denote the random variables whose probability distributions are the posterior distributions of $\boldsymbol{\beta}_j$ and $\boldsymbol{y}_j(\boldsymbol{x}_i)$ conditional on $\boldsymbol{F}_j$ given samples respectively. We can then use the posterior distribution of $\boldsymbol{\beta}_j$ to update the posterior distribution of the performance measures for each configuration such that

$$\tilde{\boldsymbol{y}}_j(\boldsymbol{x}_i) = \tilde{\boldsymbol{\beta}}_{j0} + \tilde{\boldsymbol{\beta}}_{j1}\boldsymbol{x}_i .$$

Therefore, given a set of initial $\boldsymbol{n}_j$ simulation runs for each configuration conducted at the locations described in $\boldsymbol{X}_j$ with the output contained in vector $\boldsymbol{F}_j$ and the configuration $\boldsymbol{y}_b(\boldsymbol{x}_M)$ obtained from the least squares results derived in the previous section, the probability of correct selection from (4) can be expressed as

$$\begin{aligned} \max \quad & \boldsymbol{PCS} = \boldsymbol{P}\{\tilde{\boldsymbol{y}}_b(\boldsymbol{x}_M) \le \tilde{\boldsymbol{y}}_j(\boldsymbol{x}_M) \,\forall\, \boldsymbol{j}\} \\ \boldsymbol{s.t.} \quad & \sum_{i=1}^{m} \sum_{j=1}^{k} \boldsymbol{N}_{ij} = \boldsymbol{T}. \end{aligned} \tag{5}$$

Using a non-informative prior distribution, the posterior distribution of $\boldsymbol{\beta}_j$ is then given by (6) below. The non-informative prior distribution represents the fact that we have no information about the unknown parameters and places emphasis upon the information we collect with our simulation runs.

$$\tilde{\boldsymbol{\beta}}_j \sim N[(\boldsymbol{X}_j^T \boldsymbol{X}_j)^{-1} \boldsymbol{X}_j^T \boldsymbol{F}_j \,, \boldsymbol{\sigma}_j^2 (\boldsymbol{X}_j^T \boldsymbol{X}_j)^{-1}] \,. \tag{6}$$

Since $\tilde{\boldsymbol{y}}_j(\boldsymbol{x}_M)$ is a linear combination of the $\tilde{\boldsymbol{\beta}}$ elements, this means that $\tilde{\boldsymbol{y}}_j(\boldsymbol{x}_M)$ has a Gaussian distribution of the form

$$\begin{aligned} & \tilde{y}_j(x_M) \sim \\ & N[X_M (X_j^T X_j)^{-1} X_j^T F_j \,, \sigma_j^2 X_M^T (X_j^T X_j)^{-1} X_M], \end{aligned} \tag{7}$$

where $\boldsymbol{X}_M^T = [1 \;\; \boldsymbol{x}_M]$.

We can estimate $\boldsymbol{\sigma}_j^2$ from our least squares results and can calculate $\tilde{\boldsymbol{y}}_j(\boldsymbol{x}_M)$ using (7). We can then use Monte Carlo simulation with (5) in order to estimate the PCS. However, estimating the PCS via Monte Carlo simulation can be time consuming. The next section presents a way to approximate the PCS without running Monte Carlo Simulations.

4 OPTIMAL ALLOCATIONS FOR PREDICTED PERFORMANCE

The previous section demonstrated how we can use the linear structure of the unknown underlying function for each configuration to provide estimates of the performance measure at $\boldsymbol{x}_M$. In this section, we will develop a method to optimally allocate simulation runs $\boldsymbol{N}_{i,j}$ in order to maximize our PCS.

Define

$$\boldsymbol{\delta}_{b,j} \equiv \tilde{y}_j(x_M) - \tilde{y}_b(x_M) = (\tilde{\beta}_{1j} x_M + \tilde{\beta}_{0j}) - (\tilde{\beta}_{1b} x_M + \tilde{\beta}_{0b}) \,.$$

Since $\boldsymbol{\delta}_{b,j}$ is a linear combination of the $\tilde{\boldsymbol{y}}_j(\boldsymbol{x}_M)$ and $\tilde{\boldsymbol{y}}_b(\boldsymbol{x}_M)$ elements, the $\boldsymbol{\delta}_{b,j}$ terms are also asymptotically normally distributed.

Denote

$$\boldsymbol{\sigma}_{b,j}^2 = \mathrm{var}\{\boldsymbol{\delta}_{b,j}\} = \mathrm{var}\{\tilde{\boldsymbol{y}}_j(\boldsymbol{x}_M) - \tilde{\boldsymbol{y}}_b(\boldsymbol{x}_M)\} \,.$$

Since the simulation runs for each configuration are independent,

$$\boldsymbol{\sigma}_{b,j}^2 = \mathrm{var}\{\tilde{\boldsymbol{y}}_j(\boldsymbol{x}_M)\} + \mathrm{var}\{\tilde{\boldsymbol{y}}_b(\boldsymbol{x}_M)\} \,.$$

Using the results of our linear regression equation for each configuration,

$$\boldsymbol{\sigma}_{b,j}^2 = \boldsymbol{\sigma}_j^2 \boldsymbol{X}_M^T (\boldsymbol{X}_j^T \boldsymbol{X}_j)^{-1} \boldsymbol{X}_M + \boldsymbol{\sigma}_b^2 \boldsymbol{X}_M^T (\boldsymbol{X}_b^T \boldsymbol{X}_b)^{-1} \boldsymbol{X}_M \tag{8}$$

Our aim is to efficiently allocate the computing budget between the configurations and to the time steps associated within each configuration. Therefore, we will rewrite this variance term so that it is expressed in terms of the $\boldsymbol{N}_{ij}$. Since we have a discrete domain, we can write the $\boldsymbol{X}_j^T \boldsymbol{X}_j$ matrix as

$$\boldsymbol{X}_j^T \boldsymbol{X}_j = \begin{bmatrix} \sum_{i=1}^{m} \boldsymbol{N}_{ij} & \sum_{i=1}^{m} \boldsymbol{N}_{ij} \boldsymbol{x}_i \\ \sum_{i=1}^{m} \boldsymbol{N}_{ij} \boldsymbol{x}_i & \sum_{i=1}^{m} \boldsymbol{N}_{ij} \boldsymbol{x}_i^2 \end{bmatrix}.$$

It is critical to note for this paper, the constraint $\sum_{i=1}^{m} \sum_{j=1}^{k} \boldsymbol{N}_{ij} = \boldsymbol{T}$ imposed in (4) does not require us to allocate simulation runs at every $\boldsymbol{x}_i$ for each configuration. For notation sake, we will refer to the $\boldsymbol{x}_i$ receiving simulation run allocations as support points and present the following theorem.

Theorem 1 *Given that we assume the expectation of our underlying function is linear, we require only two support points at the extreme locations on the domain (* $\boldsymbol{x}_1$ *and* $\boldsymbol{x}_m$*) to obtain all of the information in the* $\boldsymbol{X}_j^T \boldsymbol{X}_j$ *matrix.*

Proof: De la Garza (1954) establishes that for a polynomial of degree $\boldsymbol{m}$ and a discrete domain with more than $\boldsymbol{m}+1$ support points, the information obtained in the $\boldsymbol{X}^T \boldsymbol{X}$ matrix by a spacing at more than $\boldsymbol{m}+1$ support points can always be attained by spacing the same information at only $\boldsymbol{m}+1$ of the support point locations. Kiefer (1959) extends this result by proving that, regardless of the design of experiments optimality criteria, two of the support points will be located at the extremes.

Given these results and the symmetric nature of the $X^T X$ matrix, for each configuration it can be rewritten as

$$X_j^T X_j = \begin{pmatrix} 1 & 1 \\ x_1 & x_m \end{pmatrix} \begin{pmatrix} N_{1j} & 0 \\ 0 & N_{mj} \end{pmatrix} \begin{pmatrix} 1 & x_1 \\ 1 & x_m \end{pmatrix}.$$

Using a basic theorem for matrices where $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$, we obtain

$$\left(X_j^T X_j\right)^{-1} = \begin{pmatrix} 1 & 1 \\ x_1 & x_m \end{pmatrix}^{-1} \begin{pmatrix} N_{1j} & 0 \\ 0 & N_{mj} \end{pmatrix}^{-1} \begin{pmatrix} 1 & x_1 \\ 1 & x_m \end{pmatrix}^{-1}.$$

Therefore, we may write (8) in the simplified form expressed in (9) below:

$$\sigma_{b,j}^2 = \sigma_j^2 \left[\frac{D_m}{N_{1j}} + \frac{D_1}{N_{mj}} \right] + \sigma_b^2 \left[\frac{D_m}{N_{1b}} + \frac{D_1}{N_{mb}} \right],$$

where $D_1 = \dfrac{(x_M - x_1)^2}{(x_1 - x_m)^2}$ and $D_m = \dfrac{(x_M - x_m)^2}{(x_1 - x_m)^2}$. (9)

Note that this variance term is proportional to distances from x_M to the support points and we obtain

$$\sigma_{b,j}^2 = \frac{\sigma_j^2}{N_{mj}} + \frac{\sigma_b^2}{N_{mb}} \text{ when } x_M = x_m .$$

Using a Lagrangian relaxation of the budget constraint and the Bonferroni inequality, the objective function as shown in (4) can be approximated as:

$$F = 1 - \sum_{j=1, j \neq b}^{k} \int_{\frac{-\delta_{b,j}}{\sigma_{b,j}}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-t^2}{2}} dt + \lambda \Big(T - \sum_{j=1}^{k} N_{1j} + N_{mj}\Big) . \quad (10)$$

Investigating $\dfrac{\partial F}{\partial N_{1j}}$ and $\dfrac{\partial F}{\partial N_{mj}}$ to determine the allocations, we can use the chain rule to establish that:

$$\frac{\partial F}{\partial N_{1j}} = -\frac{\partial F}{\partial \sigma_{b,j}} \frac{\partial \sigma_{b,j}}{\partial N_{1j}} - \lambda$$

From (10) we obtain

$$\frac{\partial F}{\partial \sigma_{b,j}} = \frac{1}{2\sqrt{2\pi}} \exp\left[\frac{-\delta_{b,j}^2}{2\sigma_{b,j}^2}\right] \frac{\delta_{b,j}}{\sigma_{b,j}^2}.$$

From (9) we obtain

$$\frac{\partial \sigma_{b,j}}{\partial N_{1j}} = -\sigma_j^2 \frac{D_m}{N_{1j}^2} \frac{1}{\left(\sigma_{b,i}^2\right)^{1/2}}.$$

Substituting these results we determine that for every $j \neq b$

$$\frac{\partial F}{\partial N_{1j}} = \frac{-1}{2\sqrt{2\pi}} \exp\left[\frac{-\delta_{b,j}^2}{2\sigma_{b,j}^2}\right] \frac{\delta_{b,j}\sigma_j^2}{\left(\sigma_{b,j}^2\right)^{3/2}} \frac{D_m}{N_{1j}^2} - \lambda = 0. \quad (11)$$

A similar method provides that for every $j \neq b$,

$$\frac{\partial F}{\partial N_{mj}} = \frac{-1}{2\sqrt{2\pi}} \exp\left[\frac{-\delta_{b,j}^2}{2\sigma_{b,j}^2}\right] \frac{\delta_{b,j}\sigma_j^2}{\left(\sigma_{b,j}^2\right)^{3/2}} \frac{D_1}{N_{mj}^2} - \lambda = 0. \quad (12)$$

Setting (11) and (12) equal to each other, we can establish the within configuration allocation ratios as

$$\frac{N_{mj}^2}{(x_M - x_1)^2} = \frac{N_{1j}^2}{(x_M - x_m)^2}$$

Since $x_1 < x_m < x_M$, this simplifies to

$$\frac{N_{mj}}{N_{1j}} = \frac{(x_M - x_1)}{(x_M - x_m)}. \quad (13)$$

Note that the closer x_m is to x_M, the larger the proportion of the within configuration allocation that we provide to N_{mj}.

Taking a similar approach,

$$\frac{\partial F}{\partial N_{1b}} = \frac{-1}{2\sqrt{2\pi}} \sum_{i \neq b} \exp\left[\frac{-\delta_{b,i}^2}{2\sigma_{b,i}^2}\right] \frac{\delta_{b,i}\sigma_b^2}{\left(\sigma_{b,i}^2\right)^{3/2}} \frac{D_m}{N_{1b}^2} - \lambda = 0, \quad (14)$$

and

$$\frac{\partial F}{\partial N_{mb}} = \frac{-1}{2\sqrt{2\pi}} \sum_{i \neq b} \exp\left[\frac{-\delta_{b,i}^2}{2\sigma_{b,i}^2}\right] \frac{\delta_{b,i}\sigma_b^2}{\left(\sigma_{b,i}^2\right)^{3/2}} \frac{D_1}{N_{mb}^2} - \lambda = 0. \quad (15)$$

Setting (14) and (15) equal to each other provides the same within configuration allocation ratio for configuration $\boldsymbol{b}$ as shown in (13).

In order to determine the between configuration allocation ratios, we will use the terms associated with the allocations at $\boldsymbol{x_m}$. We note from (12) that for every $\boldsymbol{j} \neq \boldsymbol{b}$

$$\frac{-1}{2\sqrt{2\pi}} \exp\left[\frac{-\delta_{b,j}^2}{2\sigma_{b,j}^2}\right] \frac{\delta_{b,j}}{\left(\sigma_{b,j}^2\right)^{3/2}} \boldsymbol{D}_1 = \lambda \frac{N_{mj}^2}{\sigma_j^2} .$$

Substituting these results into (15) yields

$$N_{mb} = \sigma_b \sqrt{\sum_{j=1, j\neq b}^{k} \frac{N_{mj}^2}{\sigma_j^2}} . \tag{16}$$

If we assume that $\left(N_{1b} + N_{mb}\right) >> \left(N_{1j} + N_{mj}\right)$, then $\text{var}\{\widetilde{y}_j(x_M)\} >> \text{var}\{\widetilde{y}_b(x_M)\}$ and $\sigma_{b,j}^2 \approx \text{var}\{\widetilde{y}_j(x_M)\}$. Applying an asymptotic rule similar to Chen et al (2000), from (12) we can also establish that

$$\frac{-\delta_{b,i}^2}{\sigma_i^2\left[\dfrac{D_m}{N_{1i}} + \dfrac{D_1}{N_{mi}}\right]} = \frac{-\delta_{b,j}^2}{\sigma_j^2\left[\dfrac{D_m}{N_{1j}} + \dfrac{D_1}{N_{mj}}\right]} .$$

Using the results in (13) and simplifying, we obtain

$$\frac{N_{mi}}{N_{mj}} = \frac{\sigma_i^2 \delta_{b,j}^2}{\sigma_j^2 \delta_{b,i}^2} . \tag{17}$$

We can determine the between configuration allocation ratios for the terms associated with the allocations at $\boldsymbol{x}_1$ in the same manner.

The results obtained in this section can be summarized by the following theorem.

Theorem 2 *An approximation for the PCS expressed in (4) can be asymptotically maximized with allocations that satisfy:*

Within Configurations:

$$N_{ij} = 0 \ \forall\ i \neq 1, m \ ; \ \frac{N_{mj}}{N_{1j}} = \frac{(x_M - x_1)}{(x_M - x_m)}$$

Between Configurations:

$$\frac{N_{1i}}{N_{1j}} = \frac{N_{mi}}{N_{mj}} = \frac{\sigma_i^2 \delta_{b,j}^2}{\sigma_j^2 \delta_{b,i}^2} \ ; \ N_{sb} = \sigma_b \sqrt{\sum_{j=1, j\neq b}^{k} \frac{N_{sj}^2}{\sigma_j^2}}, \ s = 1, m.$$

These allocations provide a solution that is easy to compute. While the between configuration equations are very similar to results for the OCBA procedure (Chen et al. 2000), the σ_i^2 and $\delta_{b,i}^2$ terms are calculated using the regression equation and not by conducting simulation runs at x_M.

5 DISCUSSIONS

We have presented a Ranking and Selection procedure for selecting the best configuration based on a transient mean performance measure. The procedure extends the OCBA approach to systems whose means are changing over time when we can assume that the underlying function is linear. There are certainly ways to improve upon these results. We believe that the method can be extended to consider nonlinear transient mean performance measures as well as cases with non-equal sampling costs. We anticipate that the former will primarily affect the within configuration allocation results while the later can affect the within and between allocation results depending upon the sampling cost variation scenario. Finally, we believe that the method can be extended by imposing sequential sampling constraints. Examples of this include limiting the number of simulation runs at each portion of the simulation domain or imposing time based constraints such that we cannot sample at events that have already occurred.

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