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PROGRAM PLANNING UNDER UNCERTAINTY

Kabeh Vaziri
Paul Carr
Linda Nozick

Civil and Envir. Engr.
Hollister Hall, Cornell University
Ithaca, N.Y. 14853, U.S.A.

ABSTRACT

Task durations are subject to uncertainty and they can be influenced by the resources assigned. Commonly there is the opportunity to assign workers to tasks, which require a secondary skill they have. When workers use secondary skills, they are not as efficient as workers for which that skill is their primary skill, however they can contribute to the completion of the task. This paper provides the means for managers to heuristically optimize the allocation of their skilled workers among individual tasks on several competing projects when the task durations are uncertain and workers have multiple skills.

1 INTRODUCTION

Projects are often undertaken in an environment of considerable uncertainty. Uncertainty may be reflected in the duration and outcomes of specific tasks, as well as in the effectiveness of resources applied to them. Project and program managers must decide, in these uncertain conditions, how to allocate and manage scarce resources across many projects that have competing needs. While the model and methodology presented in this paper is applicable to different industries, we have chosen to use construction projects as a means to illustrate these ideas.

Each project is divided into a collection of tasks and there are precedence constraints between the tasks. A crew is a group of skilled workers that are assigned to a task. The skilled workers we focus on are carpenters, laborers, equipment operators and masons. Different categories of tasks require a different balance between the skilled workers; hence there is a unique ideal composition of workers for each task. When workers are assigned to a task in this ideal configuration, the workers are 100% efficient leading to a typical or nominal crew size. Worker assignments to tasks which are multiples of the nominal crew size, and for which the optimal balance of skills across the workers is maintained, also yield workers that are 100% efficient. We refer to a crew with the optimal composition of workers as an ideal crew. We refer to the crew's nominal size as the ideal crew size with the understanding that multiples of this size are equally efficient. By equally efficient we mean that if a crew is a multiple of the nominal crew size the duration is scaled by that multiple.

If workers are added beyond the ideal crew size but these workers do not create a multiple of the ideal crew size, the individual performance of the workers decreases. However, the total output of the crew will increase because of the additional workers. This increase in the output of the crew results in a shorter duration for the corresponding task. One of the key elements of this problem is the development of an understanding of when it is valuable to operate crews that are not the ideal crew size in order to produce a plan of shorter duration.

Each worker has a primary skill at which he is 100% efficient. It may also be possible to assign the worker to a task using some secondary skill that is different than the primary. A worker using a secondary skill will often not be as efficient as a worker for whom that skill is their primary skill. Hence a second key element of this problem is the development of an understanding of how to effectively use the secondary skills of workers.

The focus of this paper is the development of a model formulation and a solution procedure for allocating available resources across projects when there are significant uncertainties associated with the duration and resource needs of individual tasks. This paper makes four significant contributions to the literature in project management for construction activities. First, it creates a mathematical formulation and solution procedure that captures the uncertainty associated with task duration and resource requirements. Second, it includes an explicit mechanism to explore the consequences of cross-functioning personnel. Third, it creates a mechanism to understand the value of re-planning worker assignments as the projects unfold. Fourth, a complex case study is developed illustrating how to use the model in practice.

Section 2, describes the literature that is most relevant to this research. Sections 3 and 4 discuss the model and the solution procedure. Section 5 presents the assumptions for the case study including details on worker skills and wages, and a method to estimate the probability distribution for task duration given the type of task, the number of people assigned and their skills. The case study analysis is presented in section 6. Section 7 describes key conclusions and opportunities for future work.

2 PRIOR RELATED WORK

Elmaghraby (1977) and Moder et al. (1983) describe and formulates several models that explore the time-cost trade-off when that trade-off is continuous. See De et al. (1995) for a review of the discrete time-cost tradeoff problem. Golenko-Ginzburg and Gonik (1998) focus on developing a decision-making process to determine, as a project unfolds, which tasks to start when and what resources to assign. Nozick et al. (2004) uses the notion of resource multipliers which determine the amount of a resource allocated to a task but in a planning context. Changing the resource multipliers affects the probability distribution for the duration of the task. Vaziri et al. (2005) uses the notion of resource multipliers and builds a heuristic that combines simulated annealing with a parallel scheduling scheme to optimize the resource allocation policies. We substantially modify their general model to include cross-functioning which controls for the balance between the workers' skills assigned to a task. Further, this paper also focuses on the value of re-planning as the project unfolds.

Our solution method is related to algorithms for the resource-constrained project scheduling problem (RCPSP). Solving this problem has been a theoretical challenge for researchers. General reviews of efforts to address the RCPSP can be found in Morton and Pentico (1993), Özdamar and Ulusoy (1995), Herroelen et al. (1998), Hartmann (1998), Węglarz (1999), Brucker, et al. (1999), Hartmann and Kolisch (2004) and Kolisch and Hartmann (2006). This problem is also related to a variant on the RCPSP, namely the multi-mode resource constrained project scheduling problem. Relavant papers include Alcaraz, et al. (2003), Jozefowska et al. (2001) and Heilmann (2001)

We focus on the use of priority rule scheduling and meta-heuristics. Kolisch (1996) has explored priority rule scheduling in great detail. Simulated annealing (SA), genetic algorithms (GA), tabu search and greedy search have all been tested by practitioners to solve the RCPSP. Hartmann's (Hartmann, 1998), GA, Bouleimen and Lecocq's (1998) SA approach and Debels et al.'s (2006) hybrid metaheuristic appear to be among the best currently available methods. Valls et al. (2005) shows the benefit of justification as an augmentation to several existing solution procedures. Xu et al. (2007) shows the benefit of incorporating rollout into priority rule scheduling schemes.

3 MODEL FORMULATION

A key element of this formulation is the notion that by adding resources to a task we are able to decrease the mean and variance of the duration of that task. We use two modeling mechanisms to control the allocation of resources to tasks: resource multipliers and duration multipliers. The resource multiplier of a task determines how its resource allocation will change compared to the *default or nominal* allocation. The resource multiplier that corresponds to the *nominal* resource allocation equals one. If the resource multiplier equals one, the task durations follows a *nominal* beta probability distribution. The *nominal* resource allocation and the probability distributions for the task durations are estimates based on expert judgment. If the resource multiplier of a task for some resource is greater than one then the resources allocated to that task are more than the *nominal* allocation and hence the probability distribution of the task duration shifts to the left. On the other hand, a resource multiplier of less than one will cause the probability distribution of the task duration to shift to the right. The duration multiplier determines the amount by which the distribution shifts to the left or right as the resource multiplier varies.

M_{i_l} is the resource multiplier of task i in the l^{th} project and determines the crew size for task i_l while it is active. Once the resource multiplier of a task is known, the size of its crew can be calculated from equation (1) where $n_{i_l 0}$ is the nominal crew size for task i_l and n_{i_l} is the crew size when the resource multiplier is set to M_{i_l}. We assume that there is only one ideal crew configuration for a given task hence no subscript is needed to indicate the crew type.

$$n_{i_l} = n_{i_l 0} M_{i_l} \quad \forall i, l \tag{1}$$

If there are k types of skilled workers, $r_{i_l k}$ identifies the number of workers with skill k assigned to task i_l. Function $g(.)$ is the relationship between crew size and the number of people in the crew with skill k. Therefore, equation (2) shows that the number of workers with each skill as a function of the crew size.

$$r_{i_l k} = g(n_{i_l}) \quad \forall i, k, l \tag{2}$$

Equation (3) illustrates the functional relationship between the resource multiplier and the duration multiplier for task i_l.

$$D_{i_l} = Q(M_{i_l}) \quad \forall i, l \tag{3}$$

If $\mu^o_{i_l 0}$, $\mu^p_{i_l 0}$ and $\mu^m_{i_l 0}$ are the nominal optimistic, pessimistic and most likely durations for task i_l, equations (4) to (6) indicate how these values change when the duration multiplier varies from its nominal value of one.

$$\mu_{i_l}^{o} = \mu_{i_l 0}^{o} D_{i_l} \quad \forall i,l \tag{4}$$

$$\mu_{i_l}^{p} = \mu_{i_l 0}^{p} D_{i_l} \quad \forall i,l \tag{5}$$

$$\mu_{i_l}^{m} = \mu_{i_l 0}^{m} D_{i_l} \quad \forall i,l \tag{6}$$

$S_{i_l t}$ is a binary variable that equals one if task i_l is scheduled to start in period *t* and zero if otherwise. Duration of task i_l is a function of its resource multiplier for the type of crew it will use, the most likely, optimistic and pessimistic durations, i.e., $d_{i_l} = f\,(M_{i_l}, \mu_{i_l 0}^{m}, \mu_{i_l 0}^{a}, \mu_{i_l 0}^{b})$.

Equation (7) simply states that a task can only start when all of its predecessors are finished. $PRED_{j_l}$ is the set of all immediate predecessors of task *j* of project *l*.

$$\sum_{\tau=1}^{T} \tau S_{i_l \tau} + d_{i_l} \le \sum_{\tau=1}^{T} \tau S_{j_l \tau} \quad \forall i_l \in PRED_{j_l} \tag{7}$$

Assume N_l is the completion task for project l. This task is a dummy that has duration of zero with no resource requirements. If the planning horizon for all projects has the maximum possible length of T, then equation (8) represents the completion constraint for each project l.

$$\sum_{\tau=1}^{T} S_{N_l \tau} = 1 \quad \forall l \tag{8}$$

Equation (9) represents the resource requirement constraints. In these constraints, $r_{i_l k}$ is the number of skilled workers from category *k* allocated to task i_l. Let R_k represent the number of skilled workers from category *k* available in each period during the planning horizon. Also, let $f_{i_l}(t)$ be the probability density function for the duration of task i_l, then $F_{i_l}(t-\tau)$ shows the probability that this task would complete at t had it started at time τ. The inner summation in equation (9) represent the probability that task i_l is active in period *t*. Equation (9) states that the expected use of skilled workers in period *t* cannot be more than what is available.

$$\sum_{i,l} r_{i_l k} \sum_{\tau=1}^{t} S_{i_l \tau}[1 - F_{i_l}(t-\tau)] \le R_k \quad \forall k,t \tag{9}$$

Every worker has a primary skill for which they are 100% efficient. Based on the primary skill, the worker may have secondary skills for which they would be at most 100% efficient. Since workers may have multiple skills, it is useful to generalize equation (9) to represent this cross-functioning. This can be done by defining a new variable $X_{i_l k_1 k_2 t}$ to represent the number of workers whose primary skill set is k_1 but are using their secondary skill set k_2 for task i_l during period *t*. $e_{i_l k_1 k_2}$ is the efficiency of worker k_1 who is doing job k_2 ($0 \le e_{i_l k_1 k_2} \le 1$). If cross-functioning cannot exist, the efficiency equals zero. Equations (10) and (11) represent a more generalized version of equation (9) that includes the possibility of using the secondary skills of workers.

$$\sum_{i,l} r_{i_l k_2} \sum_{\tau=1}^{t} S_{i_l \tau}[1 - F_{i_l}(t-\tau)] \le \sum_{i,l,k_1} X_{i_l k_1 k_2 t} e_{i_l k_1 k_2} \quad \forall k_2, t \tag{10}$$

The total number of workers used when cross-functioning is allowed cannot exceed what is available. This notion is captured in equation (11).

$$\sum_{i,l,k_2} X_{i_l k k_2 t} \le R_k \quad \forall k,t \tag{11}$$

Equation (12) creates the bounds on the resource multiplier for every task i_l to prevent the assignment of too few or too many workers.

$$M_{i_l}^{min} \le M_{i_l} \le M_{i_l}^{max} \quad \forall i,l \tag{12}$$

For the purpose of this analysis we focus on minimizing the expected makespan of the collection of the projects subject to precedence constraints and resource limitations. If *N* represents the dummy node that marks the completion of all the projects, then the objective function is given in (13) and is accomplished subject to equations (1) through (12).

$$Min \sum_{t=1}^{T} tE[S_{Nt}] \tag{13}$$

This analysis is easily generalized to other objectives including minimizing a weighted average of the tardiness of each project and a weighted average of each project's makespan.

4 SOLUTION PROCEDURE

The formulation that is presented in the previous section cannot be solved directly because the task durations are uncertain. The solution procedure is based on iteratively updating the resource multipliers using simulated annealing and evaluating the implications of the new policy on the duration of the projects using Monte Carlo simulation based on a parallel scheduling scheme.

The Simulated Annealing (SA) algorithm starts off with an initial set of randomly chosen resource multipliers. After evaluation of the current solution, a neighboring set of resource multipliers is generated from the current solution. If the neighboring solution improves the performance measure, which is the expected makespan of the projects, the new set of resource multipliers replaces the current solution. However, if the new solution does not improve the performance measure, there is still some probability of accepting it over the current solution. The algorithm is composed of four key elements that are discussed in the following sections.

4.1 Initial Solution Generation

The initial solution for each run is chosen by random sampling. Each set of resource multipliers is uniformly and independently chosen from the solution space such that it falls between the stated minimum and maximum bounds indicated by equation (12). Once the initial resource multipliers have been identified, they can be evaluated.

4.2 Evaluation of a Set of Resource Multipliers

Once the resource multipliers are known, we use simulation to evaluate the expected makespan of the collection of the projects resulting from that resource allocation policy. Each replication of the simulation creates a feasible schedule using sampled task durations. The information collected from the simulation is used to update the resource multipliers and generate a neighboring solution to the current one.

Assume that the resource multipliers are known. The duration multiplier for each task can be calculated from equation (3). The optimistic, pessimistic and most likely durations for each task can be calculated from equations (4), (5) and (6). The scheduling is done using the Parallel Scheduling Scheme (PSS) where tasks are randomly selected from the decision with a bias based on the total number of successors.

4.3 Selection Policy

Once the initial set of resource multipliers is evaluated, meaning that the expected makespan that corresponds to that set of resource multipliers is known, a new set of resource multipliers is generated. This new set is produced using the neighborhood definition discussed in the next subsection. The new solution is sampled only enough times, that the Wilcoxon rank-sum test can determine which of the solutions is better. If the new solution is better, it will be accepted. However, if the solutions are of the same quality, meaning that they produce the same expected makespan value, or if the new solution is worse than current, there is still some probability of accepting the new solution. This procedure is repeated each time a new set of resource multipliers is generated. In the last iteration, double justification introduced in Valls et al. (2005) is used to improve our estimate of the project duration.

4.4 Neighborhood Definition

The neighboring solution to the current one is identified by changing the resource allocation policy. The key point is to increase the resources assigned for the tasks that are on the critical path most of the time by releasing resources from those tasks that are not on the critical path. Let CP_{i_l} be the percent of the time that task i_l is on the critical path. In this discussion we consider the critical path for the collection of projects not each of them individually. If task i_l has been on the critical path for at least one replication, then we increase the resource multiplier of that task randomly between zero and $CP_{i_l}(1-u)(1-n_{i_l}/N)$ where u is the average utilization of people in the current solution and n_{i_l}/N is the size of crew c of task i_l for people when N is the total number of skilled workers available. If task i_l has never been part of the critical path in any replications, the resource multiplier of that task for all of the resources will be decreased by a random number between zero and $(E(LFT_{i_l} - EST_{i_l})/E(TD))u(n_{i_l}/N)$ where TD stands for total duration of the project and EST_{i_l} and LFT_{i_l} represent the earliest start time and latest finish time of task i_l. All changes in the resource multipliers occur subject to the availability of the people during the planning horizon.

5 Case Study

To illustrate these ideas, we focus on a small contractor with about five million dollars annually in construction projects that was currently executing three projects. The first project (Victim's Assistance Center) is made up of two identical subprojects of 21 tasks each. The second project (Cerebral Palsy Center) consists of 73 tasks and the third one (Methodist Church) has 77 tasks. Including dummies that mark the single beginning and single end of the projects, the total number of tasks is 193. We assume that the tasks are numbered based on their precedence, i.e., if task i_l is an immediate predecessor to task j_l, then $i_l < j_l$.

5.1 Resources

There are four general types of people available throughout the planning horizon: carpenters, laborers, masons and equipment operators. The contractor has a pool of 18 laborers, 17 carpenters, 6 masons and 6 equipment operators that have to be allocated to projects efficiently. We also assume that the resource requirements of the tasks are constant for the time they are active. Scarcity of material and equipment is not considered in this analysis because labor is the driving concern.

There are three categories of tasks. The milestones are assumed to have no resource requirements and no duration. The tasks that are sub-contracted require no resources from the contractor pools but do require a fixed period of time to complete. The remaining crew types are defined by the number of laborers, carpenters, equipment operators and masons, respectively. Each task is mapped to only one crew type. The crew types are assigned to the tasks based on expert judgment. Table 1 shows the combination of workers in each of these 11 crew types. For example if a task is of type 3, the nominal crew composition is two laborers, one carpenter and two equipment operators and the nominal crew size is 5. If the nominal crew is assigned to the task while it is active, its task duration is described by its nominal distribution.

The hourly wages of laborers, carpenters, operators and masons are assumed to be \$28.83, \$33.67, \$39.70, \$34.94, respectively. These wages include workers compensation at 18%, social security at 7.65%, and other taxes at 5.5%. As is typical in construction, a worker is assumed to be paid the higher rate between his primary skill and the skill used. For example if a carpenter is asked to work as a laborer, he will be compensated as a carpenter because the wage of a carpenter is more than a laborer but if a carpenter is asked to work as an equipment operator, he will be paid as an equipment operator. Further, if a worker is not called to work for some period of time, he is not paid.

Table 1: Crew Types

Crew Type	Laborer	Carpenter	Operator	Mason
1	1	3	1	
2	3	1	1	
3	2	1	2	
4	2	3		
5	2		3	
6	2			3
7	3	2		
8	1	4		
9	4	1		
10	5			
11		5		

5.2 Cross-Functioning

The workers in this case study are categorized based on their primary skill set as carpenters, laborers, operators or masons. However, it is possible to make use of their secondary specialties. Based on expert input, we have assigned efficiency factors for the use of secondary skills given the primary skill. These efficiency factors are as follows. Laborers can do carpenter tasks with 70% efficiency if it involves framing and forming. Also since all of the masonry in this case study involves block-work, the laborers can do that work with 70% of the efficiency of a mason. On the other hand, masons and carpenters can do laborer tasks with 90% efficiency. The equipment operators cannot be cross-functioned to do any of the other jobs, however, since these projects are fairly small and they do not need complicated equipment, carpenters, laborers and masons can do operator tasks with 100% of efficiency (the same efficiency of an operator).

5.3 Impact of Resource Assignments on Task Duration

As more skilled workers are added to the crew, or eliminated from the crew, the average worker efficiency as well as the duration multiplier for that task changes. Figure 1 shows a potential relationship of duration multiplier of a task of crew type 8 and the crew size for that task. The crew size is determined once the resource multiplier is known. If the crew size is increased, the duration multiplier of the task decreases, and vice versa. There is a similar relationship between the crew size and duration multiplier for other tasks.

It is important to notice that in this application, the increments and decrements to the crew composition can only happen in specific ways. These rules must be specified for each crew type and are thus important input data for the model. Table 2 shows how changes in the crew size of a task that uses crew type of 8 will change the crew composition.

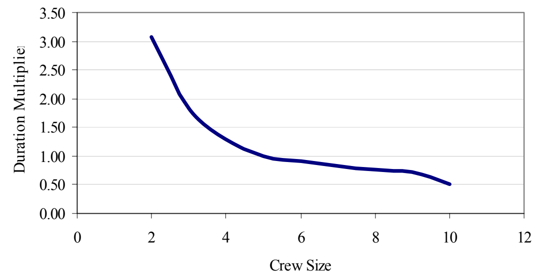

Figure 1: Relationship of the Duration Multiplier and the Resouce Multiplier (Function Q(.)).

Table 2: Calculation of Function *g(.)* for a Crew of Type 8

Size	Laborer	Carpenter
1		
2	1	1
3	1	2
4	1	3
5	**1**	**4**
6	1	5
7	1	6
8	1	7
9	1	8
10	2	8

6 RESULTS

6.1 Program Planning

In this section three cases are discussed and they are called *Nominal*, *Optimized* and *CrossF Optimized*. *Nominal* is when the default resource allocation to the tasks is used. In this case, the resource multiplier for each task is equal to one. This yields a probability distribution of total duration for all projects with a mean of 193 days with standard deviation of 4 days. The standard deviation of this distribution is quite small in comparison to the mean indicating that there is relatively little schedule risk. The mean labor cost of the projects is about $765,000 with a standard deviation of about $12,000.

Optimized is when the resource multipliers are optimized but cross-functioning is not allowed and therefore each worker performs tasks based on his primary skill only. In this case, the mean and standard deviation of labor cost are about $765,000 and $12,000 and the mean and standard deviation of duration are about 163 and 4 days. With no change in the average direct labor cost, it is possible to achieve more than a 15% reduction in the average duration. Again the standard deviation of this distribution is quite small in comparison to the mean hence the schedule risk has not increased even though the duration has shrunk.

CrossF Optimized is when the resource allocation is optimized and cross-functioning is allowed. The mean and standard deviation of the total duration are about 151 and 5 days respectively. The average cost increases to about $801,000 with an increase in the standard deviation to about $16,000. With a 4% increase in average total labor cost, the mean duration drops by 6% compared to *Optimized* and more than 21% compared to *Nominal*. In this case study the total value of the projects including labor, materials, subcontractors and profit is about $2.6 million and the performance period is about 193 days. As mentioned previously, about 11.3% of total construction project cost is overhead hence the daily overhead rate for these projects based on the Eichely formula (Levin, 1998) is about $1,400. In this example, speeding up the projects using cross-functioning is not economically effective if only overhead benefits are considered. The overhead savings from *Optimized* solution is about $17,000 but the additional labor costs are about $36,000.

In this case study the schedule risk as measured by the standard deviation of the duration distribution to the mean of that distribution is quite small indicating relatively little schedule risk. The *Optimized* solution provides substantial reduction in the distribution for duration at effectively no additional cost. The *CrossF Optimized* solution provides for a further reduction in duration but at a higher cost. If just the overhead savings resulting from the schedule acceleration is considered against the higher labor costs it does not appear to be financially attractive to accelerate the schedule beyond that indicated by the *Optimized* solution.

Figure 2 shows a part of the network of Cerebral Palsy Center to illustrate how *Optimized* and *CrossF Optimized* solutions affect the schedule. Each box represents a task. For example task ID 71 is Frost Wall. Frost wall has to be completed before vertical insulation and form and prep walls can begin. The notation of *(X,O)=(2,1)* in the box indicates that in *CrossF Optimized (X)*, the resource multiplier for this task is set to 2 and in *Optimized (O)* the resource multiplier is set to 1.

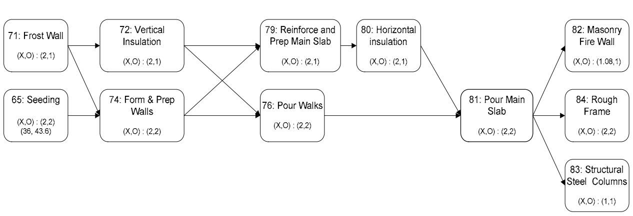

Figure 2: Portion of the Project Network for the Cerebral Palsey Center

In the Nominal solution all the resource multipliers are set to one. In *Optimized* and *CrossF Optimized* solutions, the resource multipliers of many of the tasks on the critical paths increase. However, in *CrossF Optimized* solution more of these multipliers increase further because the resources are now available to make this feasible. For example, tasks 72, 74, 76, 79, 80, 81 and 84 are illustrated in Fig. 2. 74, 76, 81 and 84 are more frequently found on the critical hence their multipliers are set to 2. However, tasks 72, 79 and 80 are sometimes found on the critical path but there are insufficient resources available when cross-functioning is not allowed to increase these resource multipliers to the point where they have a meaningful positive impact on the durations of the projects. Much of the schedule benefit achieved in the cross-functioned solution stems from the ability of laborers to perform many of the tasks of carpenters. Also, as a further result of cross-functioning, some of the tasks associated with the Methodist church can also be done about 10 days earlier and concurrent with the tasks in Figure 2. These tasks include excavate shallow footer, install rebar and forms for shallow footer, place concrete for shallow footer, install rebar and forms at shallow wall, etc.

Figure 3 and Figure 4 show the impacts of *Optimized* and *CrossF Optimized* solutions on the distribution of makespan and labor costs for the projects. Both shift the probability distribution of the total duration to the left. Notice that the cost distribution for *Optimized* solution is the same as for *Nominal*, however; the duration distribution is shifted substantially to the left and is therefore stochastically smaller. *CrossF Optimized* solution has a stochastically larger distribution for total labor cost but a stochastically smaller distribution for total duration.

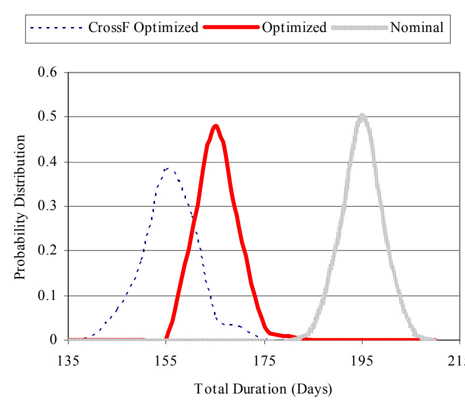

Figure 3: Effect of Change in Resource Multipliers on the Duration Probability Distribution

As mentioned previously, the pool of skilled workers of this small contractor consists of 17 carpenters, 18 laborers, 6 masons and 6 equipment operators. Figure 5 to Figure 8 show the expected utilization of the workers during the makespan of the projects.

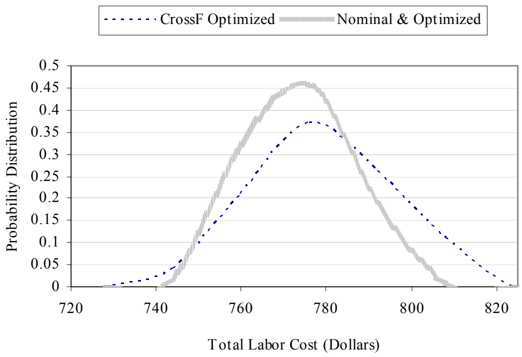

Figure 4: Effect of Change in Resource Multipliers on the Labor Cost Probability Distribution

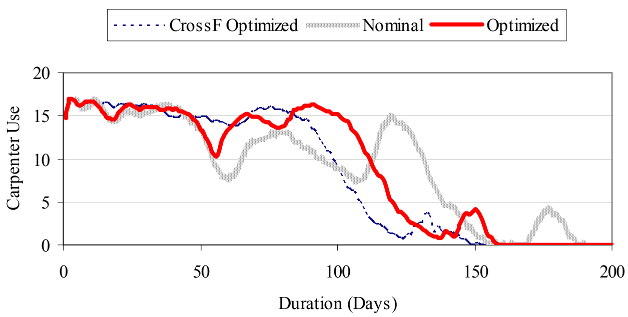

Figure 5: Average Carpenter Use over Time

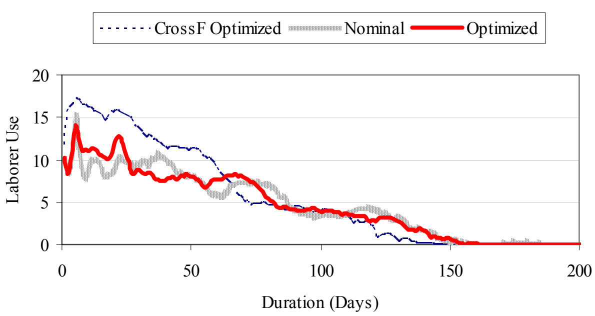

Figure 6: Average Laborer Use over Time

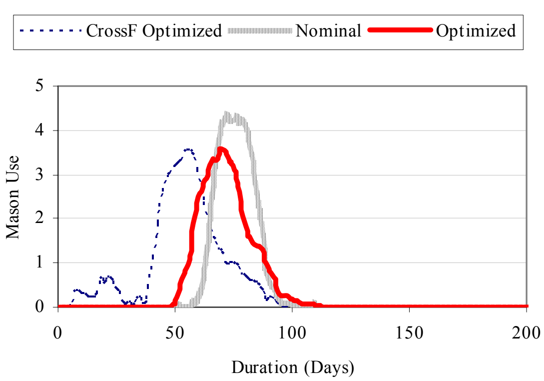

Figure 7: Average Mason Use over Time

As Figure 5 shows, the period of prolonged high utilization for carpenter is longer with both *Optimized* and *CrossF Optimized* solutions compared to *Nominal*. It is desirable that in the *CrossF Optimized* solution the use profile for carpenters is high and then becomes very low quickly because it indicated that the carpenters can essentially move on to other work.

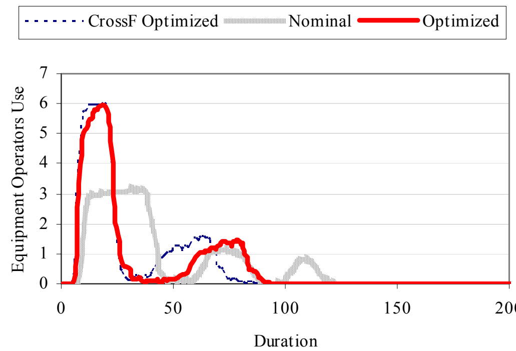

Figure 8: Average Equipment Operator Use over Time

Figure 6 illustrates that the laborer use increases when cross-functioning is allowed because laborers are now performing some carpenter tasks. The drop in labor use is quite gradual over time in contrast to that for carpenters.

In these projects, less than 5% of the tasks require masons. In Figure 7, the distribution of use of the masons over time is shown for all three solutions. Clearly there is an issue with masons in the contractor's labor pool during this time period. The contractor really does not have any need for actual masons across these projects because all of the masonry work can be done by laborers. If the contractor believes this condition is likely to exist into the future it is more realistic to eliminate masons from the resource pool and simply employ laborers to do these tasks.

At the beginning of the planning horizon, there are some tasks that require equipment operators, e.g., rough grade or select fill at the basement slab. Since the carpenters and laborers are already busy with their own tasks, and that these tasks are time critical, in the *CrossF Optimized* solution the operator tasks are done by operators. Since the tasks are time critical, resource multipliers of two are assigned to these tasks so they use all 6 equipment operators for a short period of time. Aside from this period, note that in Figure 8 at least two-thirds of the equipment operators are idle as the projects unfold. This is due to the fact that the equipment operators do not have a secondary skill which can be utilized.

6.2 Program Control

The method discussed in the previous sections and applied in the case study focuses on the assignment of workers to tasks (optimization of the resource multipliers) from a planning point of view. That is, it focuses on the creation of an up-front plan and estimates the impact of that plan if it is followed without modification as the projects unfold. However, it may be possible to achieve better outcomes if the plan is modified as the projects unfold. From an operational perspective, this means modifying the resource multipliers as the projects progress. The key question is then, how much of a benefit in performance is possible if the contractor goes through the effort to re-plan on a regular basis. To understand what the answer to this question might be, we can apply the algorithm describes above on a rolling horizon basis by sequentially drawing an observation from each of the task duration distribution as those tasks are executed and as each task completes, re-optimizing the resource multipliers for all the tasks which have yet to be started.

Figure 9 illustrates the distributions for total duration and cost with and without regular re-planning of the resource assignments when cross-functioning is allowed. The key insight from this figure is that by re-planning the contractor can reduce his schedule risk. Effectively the upper tail is removed from the distribution of duration. In these projects there is relatively little schedule risk hence the benefit is somewhat modest however it does illustrate that benefits are possible.

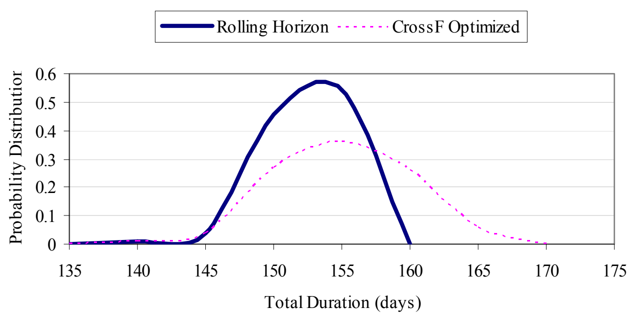

Figure 9: Impact of Re-Optimizing the Resource Multipliers as the Project Unfolds

Overall, updating the resource assignments throughout the project provides a better outcome because it uses information as it becomes available throughout the course of the project. In general, the resource multipliers are not modified substantially over the course of the projects. Most changes in the resource multipliers are associated with tasks that require a lot of carpentry, i.e. in tasks that map to crew types 1, 4, 8 and 11. Since carpenters are highly utilized, when a task takes longer that requires substantial support from them, there is a risk that other tasks that also require substantial support from carpenters will be delayed. By carefully managing these tasks and hence workers with skills in high demand, the overall duration can be better managed.

7 CONCLUSIONS

In this paper, we develop a model and solution procedure that allocates available workers among competing projects for a single contractor to decrease the expected duration of the projects. This formulation includes an explicit mechanism to reflect the impact of changes in the resources assigned to a task on the probability distribution for task duration and cost. Further this formulation also includes opportunities to capitalize on the secondary skills of workers through cross-functioning. These opportunities provide for the acceleration of work albeit at an increase in cost. This model also includes an explicit mechanism based on expert opinion to reflect the impact of crew sizes and compositions on crew productivity and therefore on task duration and cost.

This model and solution procedure has been applied to a realistic case study based on a small contractor. The case study has illustrated the following. First, careful assignment of resources to tasks can substantially decrease project duration. Second, cross-functioning can create opportunities to finish projects faster, however since cross-functioned workers are not as efficient and because the workers are paid based on what they do not their primary specialty, cross-functioning increases the labor costs. Third, cross-functioning may provide an opportunity to substantially reduce the size of labor pools that are not in high demand based on the nature of the work the contractor engages in and for which there is another type of worker that can effectively do much of that work. This allows the contractor to potentially offer more work to employees that are felt to be "more critical" hence making it easier to retain those workers. Fourth, reconsidering the assignment of workers to future tasks as projects unfold can improve schedule performance. Much of this benefit stems from carefully managing the work assignments of individuals with skills in high demand.

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AUTHOR BIOGRAPHIES

KABEH VAZIRI is a strategic research consultant at the Corporate Executive Board Company where she is focused on case-based best practice research in project management. Her research interests include logistics and program risk management. She has a Ph.D, from Cornell University in transportation systems engineering. Her email address is kv37@cornell.edu.

PAUL CARR is an adjunct associate professor in the School of Civil and Environmental Engineering. Previously he was Chief Executive Officer of the Bernier Carr and Associates, P.C., a multi-disciplined engineering, architecture and construction management firm. He is currently a consultant focused on dispute resolution in the building industry. He has a Ph.D. in Construction Engineering from Virginia Tech. His web page can be found at <http://www.cee.cornell.edu/fbxk/fcbo.cfm?pid=202>. His email address is pgc3@cornell.edu.

LINDA NOZICK is a professor in the School of Civil and Environmental Engineering at Cornell University. Her research interests are in the development of mathematical models for use in the management of complex systems. She has a particular interest in systems that can be represented mathematically as networks, including transportation and logistics systems, civil infrastructure networks and project networks. Her web page can be found at <http://www.cee.cornell.edu/fbxk/fcbo.cfm?pid=194>. Her email address is lkn3@cornell.edu.