Proceedings of the 2007 Winter Simulation Conference
S. G. Henderson, B. Biller, M.-H. Hsieh, J. Shortle, J. D. Tew, and R. R. Barton, eds.

RANKING AND SELECTION TECHNIQUES WITH OVERLAPPING VARIANCE ESTIMATORS

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ABSTRACT

Some ranking and selection (R&S) procedures for steady-state simulation require an estimate of the asymptotic variance parameter of each system to guarantee a certain probability of correct selection. We show that the performance of such R&S procedures depends on the quality of the variance estimates that are used. In this paper, we study the performance of R&S procedures with two new variance estimators — overlapping area and overlapping Cramér-von Mises estimators — which show better long-run performance than other estimators previously used in R&S problems.

1 INTRODUCTION

In ranking and selection (R&S), we are concerned with the selection of the best system out of a number of alternatives. We also require a certain probability of correct selection (PCS) in our procedures. In steady-state simulation, we are usually interested in determining the system that has either the largest or smallest expected value of a specific steady-state performance measure.

Many R&S procedures have been developed assuming that basic observations are independent and identically distributed (i.i.d.) normal random variates. Those R&S procedures can be used for steady-state simulation, if an experimenter is willing to use as basic observations within-replication averages from multiple replications (after deletion of initial data) or batch means from a single replication. However, Goldsman et al. (2002) and Kim and Nelson (2006) found that both approaches could diminish the efficiency of fully sequential procedures and proposed two procedures that take individual observations (such as consecutive wait times) as basic observations from a single replication.

It is often the case that individual observations from steady-state simulations possess an inherent dependence structure, and thus the usual marginal variance is not a good measure for the variability of such dependent data. Instead, most selection procedures require estimates for the so-called variance parameters of the competitors, which are unknown in typical simulation applications; the variance parameter for a particular steady-state process is simply the sum of the covariances at all lags. For instance, the procedures due to Goldsman et al. (2002) and Kim and Nelson (2006) — called R+, KN+, and KN++ — use well-known estimators for the variance parameter that happen to be asymptotically chi-squared distributed.

A number of new variance parameter estimators have recently been developed in the literature. For example, Alexopoulos et al. (2006b) proposed various overlapping standardized time series (STS) estimators. These overlapping STS estimators have smaller asymptotic variance and smaller bias compared to their non-overlapping counterparts. As better variance estimators are introduced, one might become interested in whether these new variance estimators can be incorporated into R&S procedures with beneficial results in terms of the required number of observations and the attained probability of correct selection. In the current paper, we investigate such issues.

This paper is organized as follows: Section 2 defines notation and introduces the variance estimators considered herein. Section 3 gives an overview of three R&S procedures specifically designed for steady-state simulation. In Sections 4 and 5, we discuss our experimental setup and results, followed by conclusions in Section 6.

2 VARIANCE ESTIMATORS

This section describes the notation used throughout the paper and introduces the variance estimators that we will implement in the selection procedures.

2.1 Notation

Let $\mathbf{Y}_i \equiv \{Y_{ij} : j = 1, \ldots, m\}$ be a realization from a single replication of a simulation of system $i = 1, \ldots, k$. For example, Y_{ij} could be the jth individual waiting time in the ith queueing system under consideration. After deleting some

initial data during a carefully chosen warm-up period, this process is believed to be stationary.

Throughout the paper, we assume that $\mathbf{Y}_i$ satisfies a Functional Central Limit Theorem (FCLT):

Assumption 1 *For the process* $\{Y_{ij};\ j = 1,\ldots,m\}$ *there exist constants* μ_i *and* $\sigma_i^2 > 0$ *such that*

$$\frac{\lfloor mt \rfloor\,(\bar{Y}_{i,\lfloor mt \rfloor} - \mu_i)}{\sqrt{m}} \Rightarrow \sigma_i W(t) \text{ for } 0 \le t \le 1,$$

where $\lfloor \cdot \rfloor$ *is the floor function,* $\bar{Y}_{i,\ell} \equiv \sum_{j=1}^{\ell} Y_{ij}/\ell$, $\ell = 1,\ldots,m$, $\Rightarrow$ *denotes weak convergence as* $m \to \infty$, *and* W *stands for a standard Brownian motion process.*

For a stationary process, an FCLT holds using the steady-state mean μ_i and the asymptotic variance parameter $\sigma_i^2 \equiv \lim_{m\to\infty} m\mathrm{Var}[\bar{Y}_{i,m}]$ (see, for example, Glynn and Iglehart 1990). Further, we assume that common random numbers are not employed and thus the following assumption holds:

Assumption 2 *The processes* $\mathbf{Y}_i$ *and* $\mathbf{Y}_\ell$ *for* $i \neq \ell$ *are independent.*

As σ_i^2 is unknown, it needs to be estimated; we consider here three STS variance estimators from the literature: batched area, overlapping area, and overlapping Cramér-von Mises estimators. Healey, Goldsman, and Kim (2007) study the performance of R&S procedures using additional variance estimators, including the modified jackknifed Durbin–Watson estimator (Batur, Goldsman, and Kim 2007) as well as the overlapping version of it.

2.2 Batched Area Estimator

To calculate a batched area estimator, we first split the n data points into b adjacent batches of size m (where $n = bm$). The jth batch mean for system i is defined as

$$\bar{Y}_{i,j,m} \equiv \frac{1}{m}\sum_{\ell=1}^{m} Y_{i,(j-1)m+\ell} \text{ for } j = 1,\ldots,b.$$

Further, the STS for batch j of system i is

$$T_{i,j,m}(t) \equiv \frac{\lfloor mt \rfloor(\bar{Y}_{i,j,m} - \bar{Y}_{i,j,\lfloor mt \rfloor})}{\sigma_i\sqrt{m}}$$

for $t \in [0,1]$ and $j = 1,\ldots,b$.

The area estimator for σ_i^2 from the jth batch from system i is defined by

$$A_{i,j}(f;m) \equiv \left[\frac{1}{m}\sum_{\ell=1}^{m} f(\frac{\ell}{m})\sigma_i T_{i,j,m}(\frac{\ell}{m})\right]^2 \text{ for } j = 1,\ldots,b,$$

where $f(\cdot)$ is a continuous weighting function on the interval $[0,1]$ and normalized so that $\int_0^1\int_0^1 f(s)f(t)(\min(s,t) - st)\,ds\,dt = 1$.

Finally, the batched area (A) estimator for σ_i^2 is defined as

$$A_i(f;b,m) \equiv \frac{1}{b}\sum_{j=1}^{b} A_{i,j}(f;m) \Rightarrow \frac{\sigma_i^2\chi_b^2}{b} \text{ as } m \to \infty$$

where χ_b^2 denotes a chi-squared random variable with b degrees of freedom.

Goldsman, Meketon, and Schruben (1990) show that the A estimator becomes first-order unbiased when $f_2(t) \equiv \sqrt{840}(3t^2 - 3t + 1/2)$ is used as a weighting function. More specifically,

$$\mathrm{E}[A_i(f_2;b,m)] = \sigma_i^2 + o(1/m)$$

and

$$\lim_{m\to\infty} \mathrm{Var}[A_i(f_2;b,m)] = 2\sigma_i^4/b,$$

where the notation $o(1/m)$ indicates a function that goes to zero more quickly than $1/m$ as $m \to \infty$.

2.3 Overlapping Area Estimator

The overlapping area estimator is similar to the batched area estimator, but differs slightly in that we now have $n - m + 1$ overlapping batches of size m. The jth overlapping batch from system i consists of observations $Y_{ij}, Y_{i,j+1},\ldots,Y_{i,j+m-1}$, and the corresponding overlapping batch mean is defined as

$$\bar{Y}^O_{i,j,m} \equiv \frac{1}{m}\sum_{\ell=0}^{m-1} Y_{i,j+\ell}$$

for $j = 1,\ldots,n-m+1$. Then the STS for the jth overlapping batch is given as

$$T^O_{i,j,m}(t) \equiv \frac{\lfloor mt \rfloor(\bar{Y}^O_{i,j,m} - \bar{Y}^O_{i,j,\lfloor mt \rfloor})}{\sigma_i\sqrt{m}}$$

for $t \in [0,1]$ and $j = 1,\ldots,n-m+1$.

Alexopoulos et al. (2006a) define the overlapping area estimator for σ_i^2 from the jth overlapping batch as

$$A_{i,j}^O(f;m) \equiv \left[\frac{1}{m}\sum_{\ell=1}^{m} f(\frac{\ell}{m})\sigma_i T_{i,j,m}^O(\frac{\ell}{m})\right]^2$$

for $j = 1,\ldots,n-m+1$, and the (overall) overlapping area (OA) estimator for σ_i^2 as

$$A_i^O(f;b,m) \equiv \frac{1}{n-m+1}\sum_{j=1}^{n-m+1} A_{i,j}^O(f;m),$$

where $b = n/m$ (but can no longer be interpreted as "the number of batches"). Moreover, Alexopoulos et al. (2006a) show that

$$\mathrm{E}[A_i^O(f_2;b,m)] = \sigma_i^2 + o(1/m),$$

$$\lim_{m\to\infty} \mathrm{Var}[A_i^O(f_2;b,m)] = \frac{3514b-4359}{4290(b-1)^2}\sigma_i^4,$$

and

$$A_i^O(f_2;b,m) \approx \frac{\sigma_i^2\chi_\nu^2}{\nu} \text{ for large } m \text{ and } b,$$

where

$$\nu = \left[\!\!\left[\frac{8580(b-1)^2}{3514b-4359}\right]\!\!\right],$$

and $[\![\cdot]\!]$ rounds to the nearest integer.

2.4 Overlapping Cramér–von Mises Estimator

The Cramér–von Mises (CvM) estimator for σ_i^2, obtained from the jth overlapping batch, is defined as

$$C_{i,j}^O(g;m) \equiv \frac{1}{m}\sum_{\ell=1}^{m} g(\frac{\ell}{m})\left[\sigma_i T_{i,j,m}^O(\frac{\ell}{m})\right]^2$$

for $j = 1,\ldots,n-m+1$, where $g(\cdot)$ is a normalized weighting function on the interval $[0,1]$ such that $\int_0^1 g(t)t(1-t)\,dt = 1$.

Alexopoulos et al. (2006a) define the (overall) overlapping CvM (OCvM) estimator for σ_i^2 as

$$C_i^O(g;b,m) \equiv \frac{1}{n-m+1}\sum_{j=1}^{n-m+1} C_{i,j}^O(g;m).$$

When $g_2^\star(t) \equiv -24+150t-150t^2$ is considered as a weighting function, we have

$$\mathrm{E}[C_i^O(g_2^\star;b,m)] = \sigma_i^2 + o(1/m),$$

$$\lim_{m\to\infty} \mathrm{Var}[C_i^O(g_2^\star;b,m)] = \frac{10768b-13605}{13860(b-1)^2}\sigma_i^4,$$

and

$$C_i^O(g_2^\star;b,m) \approx \frac{\sigma_i^2\chi_{\nu^\star}^2}{\nu^\star} \text{ for large } m \text{ and } b,$$

where

$$\nu^\star = \left[\!\!\left[\frac{27720(b-1)^2}{10768b-13605}\right]\!\!\right].$$

Henceforth, let mV_i^2 denote a generic estimator for σ_i^2 using batch size m.

3 SELECTION PROCEDURES

In this section, we elaborate on the details of three selection procedures, each of which we will implement with the A, OA, and OCvM estimators.

3.1 Extended Rinott Procedure (R+)

The following two-stage "indifference-zone" procedure is an extension of Rinott's (1978) classic procedure, and was studied in Goldsman and Marshall (1999).

1. **Setup:** Select a confidence level $1/k < 1-\alpha < 1$, indifference-zone parameter $\delta > 0$, first-stage sample size $n_0 \geq 2$, and batch size $m_0 < n_0$. The indifference-zone parameter δ is chosen as the smallest difference between systems that the experimenter deems as "worth detecting" and is explained more fully in, for example, Bechhofer, Santner, and Goldsman (1995).

2. **Initialization:** Obtain Rinott's constant $h = h(d,k,1-\alpha)$ from, say, Bechhofer, Santner, and Goldsman (1995), where d is the degrees of freedom for the associated variance estimator. Take n_0 observations $Y_{ij}, j = 1,\ldots,n_0$, from each system $i = 1,\ldots,k$. For $i = 1,\ldots,k$, compute $m_0V_i^2$, the sample asymptotic variance parameter from system i. Let

$$N_i = \max\left\{n_0, \left\lceil\frac{h^2m_0V_i^2}{\delta^2}\right\rceil\right\},$$

for $i = 1,\ldots,k$, where $\lceil\cdot\rceil$ is the ceiling function.

3. **Stopping Rule:** If $n_0 \geq \max_i N_i$ then stop and select the system with the largest first-stage sample mean $\bar{Y}_{i,n_0}$ as the best. Otherwise, take $N_i - n_0$ additional observations $Y_{i,n_0+1}, Y_{i,n_0+2},\ldots,Y_{i,N_i}$ from each system i for which $N_i > n_0$. Select the system with the largest overall sample mean $\bar{Y}_{i,N_i}$ as the best.

3.2 Extended Kim and Nelson Procedure (KN+)

The next procedure, due to Kim and Nelson (2006), is a sequential indifference-zone procedure and is somewhat more efficient with observations than Rinott's method. This savings of observations is gained by screening out clearly inferior systems. Note that we require an estimator for the asymptotic variance of the difference between systems i and ℓ, which is equal to $\sigma_i^2 + \sigma_\ell^2$ under Assumptions 1 and 2. Given the sample size n_0 and batch size m_0, we denote the estimator of this quantity as $m_0 V_{i\ell}^2$, which we calculate using the estimators in Section 2 with the data points of the difference $Z_{i,\ell,j} \equiv Y_{i,j} - Y_{\ell,j}$ for $j = 1, \ldots, n_0$.

1. **Setup:** Select a confidence level $1/k < 1-\alpha < 1$, indifference-zone parameter $\delta > 0$, first-stage sample size $n_0 \geq 2$, and batch size $m_0 < n_0$. Calculate the constant

$$\eta \equiv \frac{1}{2}\left([2(1-(1-\alpha)^{1/(k-1)})]^{-2/d} - 1 \right).$$

2. **Initialization:** Let $I = \{1, \ldots, k\}$ be the set of systems still in contention, and let $h^2 = 2\eta d$, where the degrees of freedom d is determined by which variance estimator is used.

Obtain n_0 observations $Y_{ij}, j = 1, \ldots, n_0$, from each system $i = 1, \ldots, k$. For each system $i = 1, \ldots, k$, compute the first-stage sample mean $\bar{Y}_{i,n_0}$. In addition, for all $i \neq \ell$, use the first n_0 observations to compute the sample asymptotic variance of the difference between systems i and ℓ, $m_0 V_{i\ell}^2$.

Then set the sequential counter $r = n_0$ and go to the **Screening** phase of the procedure.

3. **Screening:** Set $I^{\text{old}} = I$. Let

$$I \equiv \left\{ i : i \in I^{\text{old}} \text{ and } \bar{Y}_{i,r} \geq \bar{Y}_{\ell,r} - W_{i\ell}(r), \forall \ell \in I^{\text{old}}, \ell \neq i \right\},$$

where

$$W_{i\ell}(r) \equiv \max\left\{ 0, \frac{\delta}{2r}\left(\frac{h^2 m_0 V_{i\ell}^2}{\delta^2} - r \right) \right\}.$$

4. **Stopping Rule:** If the cardinality $|I| = 1$, then stop and select the system whose index is in I as the best. Otherwise, take one additional observation $Y_{i,r+1}$ from each system $i \in I$, set $r = r+1$, and go to **Screening**.

3.3 Extended KN Procedure with Updates (KN++)

Goldsman et al. (2002) and Kim and Nelson (2006) present another selection procedure similar to KN+, but one that updates a variance estimator according to a batching sequence (m_r, b_r), where m_r and b_r denote the batch size and number of batches when there are r observations. Both m_r and b_r are non-decreasing functions of r. Goldsman et al. (2002) present three modified batching sequences; we consider the batching sequence leading to more updates than the other two. This batching sequence doubles the number of batches while holding the batch size steady until the first update level at $r = m_0^2$. Once we reach the first update level, the batching stops doubling and takes $m_r = b_r = \lfloor \sqrt{r} \rfloor$.

1. **Setup:** Same setup as KN+.

2. **Initialization:** Let $I = \{1, \ldots, k\}$ be the set of systems still in contention, and let $h^2 = 2\eta d$. Obtain n_0 observations Y_{ij}, $j = 1, \ldots, n_0$, from each system $i = 1, \ldots, k$. Set the observation counter $r = n_0$ and $m_r = m_0$, compute $m_0 V_{i\ell}^2(n_0)$, and proceed to **Screening**.

3. **Update:** If m_r has changed since the last update, then for all $i \neq \ell$, $i, \ell \in I$, calculate the estimator $m_r V_{i\ell}^2(r)$, d, and η.

4. **Screening:** Set $I^{\text{old}} = I$. Let

$$I \equiv \left\{ i : i \in I^{\text{old}} \text{ and } \bar{Y}_{i,r} \geq \bar{Y}_{\ell,r} - W_{i\ell}(r), \forall \ell \in I^{\text{old}}, \ell \neq i \right\}$$

where

$$W_{i\ell}(r) \equiv \max\left\{ 0, \frac{\delta}{2r}\left(\frac{h^2 m_r V_{i\ell}^2(r)}{\delta^2} - r \right) \right\}.$$

5. **Stopping Rule:** If $|I| = 1$, then stop and select the system whose index is in I as the best. Otherwise, take one additional observation $Y_{i,r+1}$ from each system $i \in I$, set $r = r+1$, and go to **Update**.

4 EXPERIMENTAL SETUP

At this point, we are interested in the performance of the R&S procedures when they incorporate the new variance estimators. We follow the same experimental setup that Goldsman et al. (2002) used. In particular, we take system 1 as the best system, i.e., that with the largest mean. For all of the experiments, we set the nominal PCS to 0.95. For purposes of conducting our experiments, we set the indifference-zone parameter $\delta = \sigma_1/\sqrt{n_0}$.

We tested two different configurations for the mean performance measure: the slippage configuration (SC) and the monotone decreasing means (MDM) configuration. For the SC, all inferior systems are separated from the best system by a distance of δ. For example, $\mu_1 = \delta$, while $\mu_2 = \cdots = \mu_k = 0$. For the MDM configuration, we have $\mu_i = \mu_1 - (i-1)\delta$. The MDM configuration exploits a procedure's ability to discard clearly inferior systems quickly, while the SC configuration tends to be a worst-case scenario where

all inferior systems are equal and very close to the best system (and is often used to test the statistical validity of the procedure).

For our analysis, we concentrate on two key measures: actual PCS and the sample average of the total number of raw observations. Experimental results are based on 1000 independent replications.

The experimental study here compares AR(1) processes, but Healey, Goldsman, and Kim (2007) include other examples such as the waiting time process from an M/M/1 queue to illustrate performance under systems with extremely nonnormal observations. Healey, Goldsman, and Kim (2007) show that similar results in terms of savings of observations and preservation of PCS can be expected in these waiting time systems, even with highly nonnormal processes.

AR(1) processes $\mathbf{Y}_i$ for $i = 1,\ldots,k$, are defined as

$$Y_{ij} = \mu_i + \phi(Y_{i,j-1} - \mu_i) + Z_{ij},$$

where Z_{ij}, $j = 1,2,\ldots$, are i.i.d. Norm(0,$1-\phi^2$) random variables with $\phi \in (-1,1)$, and $Y_{i,0}$ are independent and distributed Norm(0,1). In this paper, we chose a fairly high value for the serial correlation coefficient, $\phi = 0.9$.

5 RESULTS

Goldsman et al. (2002) tested the performance of R&S procedures when nonoverlapping batch means, overlapping batch means, and A estimators were considered. Their experimental results show that R&S procedures exhibit the best performance in terms of the expected number of total observations when the A estimator is used. We are interested here in comparing the performance of the R&S procedures incorporating the OA and OCvM estimators with that of the A estimator.

Our experiments show that overlapping variance estimators provide a substantial improvement in observations required, without sacrificing PCS. The savings in observations garnered with the use of the OA or OCvM estimators (compared to the A estimator) depends on the choice of the batch size and selection procedure, but typically ranges from 10% to 50%.

We illustrate our results in pairs of tables, which show the sample average of the total number of raw observations and the estimated PCS for various choices of initial batch size m_0 when n_0 is fixed. The first pair, Tables 1 and 2, display results for $k = 2$ systems under the SC configuration, Tables 3 and 4 show results for $k = 10$ systems under the SC configuration, and Tables 5 and 6 show results for $k = 10$ systems under the MDM configuration.

5.1 Expected Number of Observations

The advantages of implementing the overlapping estimators are most clearly seen in the R+ and KN+ procedures. We notice in Table 1 that, even in the least complicated configuration of an AR(1) process with 2 systems, we record a savings of roughly up to 40% over the A estimator with both of our overlapping variance estimators for R+ and KN+. Tables 3 and 5 show that we get even more savings (up to 66%) when the number of systems increases to 10 for R+ and KN+. These savings are gained because R+ and KN+ calculate variance estimates only once, using the first n_0 observations. Thus, the better variance estimates from the OA and OCvM estimators reduce the number of observations compared to the implementation with the A estimator.

Our savings are much smaller when using OA or OCvM in KN++. As the procedure progresses, the updating procedure of KN++ quickly recovers from a poor variance estimate by allowing us to recalculate variance estimates from much larger collections of data than the initial sample. So the OA, OCvM, and A estimators can all eventually produce good variance estimates. We still can save up to 10% by implementing the OA or OCvM estimator over the A estimator when using KN+.

As we decrease the batch size m_0, fewer observations are needed until a decision is made, but our percentage savings compared to the A estimator tends to decrease for all three R&S procedures. This is because as the batch size decreases for given n_0, the number of batches increases and the χ^2 distributions of the three estimators approach each other, implying that statistical properties (including mean and variance) of the three estimators become similar.

5.2 Probability of Correct Selection

Kim and Nelson (2006) and Goldsman et al. (2002) point out that the actual PCS does not always meet the nominal PCS for the A estimator, and there is some degradation in the actual PCS from the nominal level. However, they showed that such degradation is not significant, a large m_0 helps to satisfy the PCS requirement, and the coverage problem goes away under the MDM configuration. We observed exactly the same tendency when the OA and OCvM estimators are used. Moreover, in most cases, the actual PCS with the OA and OCvM estimators is either equal to or slightly larger than the actual PCS with the A estimator for all three R&S procedures. This implies that the OA and OCvM estimators give us savings in the number of observations without sacrificing the statistical validity of the R&S procedures.

6 CONCLUSION

We have shown that implementing overlapping variance estimators can provide a significant savings in observations needed for certain ranking and selection procedures implemented in steady-state simulation problems, without sacrificing much in the probability of finding the correct system. Implementing either of the two new overlapping variance estimators produces a savings over the A estimator.

ACKNOWLEDGMENTS

This material was supported by the National Science Foundation under Grant Number DMI-0400260.

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Table 1: Sample average of total number of raw observations when AR(1) processes are tested under the SC configuration, $k = 2$, $\phi = 0.9$, $n_0 = 1000$, and $\delta = \sigma_1/n_0$. Entries are shown in units of 10^4.

	R+			KN+			KN++		
m_0	OA	OCvM	A	OA	OCvM	A	OA	OCvM	A
500	2.35	2.35	4.08	1.48	1.47	2.54	0.67	0.67	0.74
250	1.24	1.22	1.74	0.77	0.77	1.10	0.60	0.60	0.64
200	1.14	1.11	1.51	0.72	0.69	0.97	0.57	0.57	0.63
125	0.92	0.91	1.09	0.58	0.58	0.72	0.51	0.51	0.55
100	0.83	0.82	0.90	0.52	0.51	0.59	0.46	0.46	0.48

Table 2: Estimated PCS when AR(1) processes are tested under the SC configuration, $k = 2$, $\phi = 0.9$, $n_0 = 1000$, nominal PCS of 0.95, and $\delta = \sigma_1/n_0$.

	R+			KN+			KN++		
m_0	OA	OCvM	A	OA	OCvM	A	OA	OCvM	A
500	0.970	0.969	0.966	0.953	0.953	0.964	0.934	0.936	0.935
250	0.939	0.945	0.943	0.936	0.936	0.942	0.927	0.931	0.925
200	0.940	0.943	0.951	0.949	0.943	0.949	0.941	0.942	0.942
125	0.934	0.938	0.939	0.924	0.923	0.946	0.931	0.931	0.929
100	0.937	0.932	0.938	0.914	0.912	0.919	0.931	0.932	0.926

Table 3: Sample average of total number of raw observations when AR(1) processes are tested under the SC configuration, $k = 10$, $\phi = 0.9$, $n_0 = 1000$, and $\delta = \sigma_1/n_0$. Entries are shown in units of 10^4.

	R+			KN+			KN++		
m_0	OA	OCvM	A	OA	OCvM	A	OA	OCvM	A
500	51.65	51.54	145.76	32.45	33.06	90.83	6.52	6.56	7.20
250	17.92	17.86	31.03	9.78	9.78	18.51	5.95	5.96	6.34
200	15.83	15.30	24.04	8.48	8.05	13.95	5.79	5.75	6.07
125	12.26	12.08	15.98	6.22	6.12	8.46	5.12	5.09	5.32
100	10.94	10.86	13.34	5.45	5.39	7.02	4.74	4.71	4.85

Table 4: Estimated PCS when AR(1) processes are tested under the SC configuration, $k = 10$, $\phi = 0.9$, $n_0 = 1000$, nominal PCS of 0.95, and $\delta = \sigma_1/n_0$.

	R+			KN+			KN++		
m_0	OA	OCvM	A	OA	OCvM	A	OA	OCvM	A
500	0.988	0.992	0.964	0.987	0.990	0.972	0.943	0.947	0.901
250	0.963	0.968	0.956	0.951	0.958	0.957	0.943	0.947	0.923
200	0.964	0.953	0.963	0.948	0.947	0.945	0.954	0.953	0.929
125	0.964	0.941	0.957	0.933	0.931	0.949	0.922	0.920	0.909
100	0.947	0.923	0.932	0.917	0.917	0.919	0.902	0.900	0.896

Table 5: Sample average of total number of raw observations when AR(1) processes are tested under the MDM configuration, $k = 10$, $\phi = 0.9$, $n_0 = 1000$, and $\delta = \sigma_1/n_0$. Entries are shown in units of 10^4.

	R+			KN+			KN++		
m_0	OA	OCvM	A	OA	OCvM	A	OA	OCvM	A
500	51.33	51.26	143.49	14.25	14.28	40.10	3.47	3.51	4.11
250	18.02	17.96	31.35	4.32	4.31	8.19	2.88	2.88	3.28
200	15.88	15.35	24.00	3.74	3.58	6.02	2.74	2.61	3.08
125	12.28	12.09	15.90	2.77	2.75	3.75	2.44	2.43	2.66
100	10.92	10.86	13.35	2.49	2.46	3.16	2.25	2.24	2.43

Table 6: Estimated PCS when AR(1) processes are tested under the MDM configuration, $k = 10$, $\phi = 0.9$ and $n_0 = 1000$, nominal PCS of 0.95, and $\delta = \sigma_1/n_0$.

	R+			KN+			KN++		
m_0	OA	OCvM	A	OA	OCvM	A	OA	OCvM	A
500	0.995	0.998	0.994	0.999	0.997	0.995	0.992	0.993	0.995
250	0.993	0.994	0.995	0.992	0.993	0.995	0.990	0.990	0.987
200	0.993	0.992	0.995	0.992	0.992	0.996	0.994	0.994	0.990
125	0.985	0.987	0.989	0.987	0.990	0.986	0.985	0.984	0.982
100	0.983	0.985	0.983	0.984	0.984	0.987	0.986	0.986	0.991