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REAL OPTIONS VALUATION

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ABSTRACT

Managerial flexibility has value. The ability of their managers to make smart decisions in the face of volatile market and technological conditions is essential for firms in any competitive industry. This advanced tutorial describes the use of Monte Carlo simulation and stochastic optimization for the valuation of real options that arise from the abilities of managers to influence the cash flows of the projects under their control.

Option pricing theory supplements discounted cash flow methods of valuation by considering managerial flexibility. Managers' options to take actions that affect real investment projects are comparable to options on the sale or purchase of financial assets. Just as a financial option derives much of its value from the potential price movements of the underlying financial asset, a real option derives much of its value from the potential fluctuations of the cash flows generating the value of the investment project.

1 INTRODUCTION

Discounted cash flow (DCF) techniques are standard methods used for evaluation of capital budgeting projects. Under DCF the expected cash inflows and outflows from a project are stated in present value terms by using a discount rate selected to account for the project's risk and the time value of money. The discounted cash flows are summed, and investment costs are subtracted to obtain the net present value (NPV) of the project. Theory holds that if the NPV is positive, the project should be undertaken to increase shareholder value.

The assumption that all investments are irreversible is a fundamental weakness of most DCF methods. Managers often have the ability to influence the results of a project and have recourse to abandon a project if results are poor, while maintaining the opportunity to expand projects if results are better than expected. This managerial flexibility is not valued with the traditional NPV method.

Option pricing theory offers a supplement to the NPV method that considers managerial flexibility in making decisions regarding the real assets of the firm. Managers' options on real investment projects are comparable to investors' options on financial assets, such as stocks. A financial option is the right, without the obligation, to purchase or sell an underlying asset within a given time for a stated price. A financial option is itself an asset that derives its value from (1) the underlying asset's value, which can fluctuate dramatically prior to the date when the opportunity expires to purchase or sell the underlying asset, and (2) the decisions made by the investor to exercise or hold the option. Financial option pricing methods have been developed to estimate option values from parameters characterizing the underlying asset's value and investor behavior.

Myers (1984) was among the first to publish in the academic literature the notion that financial option pricing methods could be applied to strategic issues concerning real assets rather than just financial assets. In the practitioner literature, Kester (1984) suggested that the traditional NPV methods in use at that time ignored the value of important flexibilities inherent in many investment projects and that methods of valuing this flexibility were needed. Real options valuation is most effective when competing projects have similar values obtained with the traditional NPV method.

This advanced tutorial describes the use of Monte Carlo simulation and stochastic optimization for the valuation of real options that arise from the abilities of managers to influence the cash flows of the projects under their control. In the next section, we discuss methods of financial option valuation that have appeared in the literature. In §3, we describe how these methods have been applied to real options valuation. Some general applications of real options valuation are presented in §4, and applications in regulated industries such as telecommunications are presented in §5. Section 6 concludes. See Charnes (2007) and the references therein for more about using Monte Carlo simulation for financial risk analysis and real option valuation.

2 FINANCIAL OPTION VALUATION

The Nobel-prize winning breakthrough of Black and Scholes (1973) provided an analytical solution to the value of an option on a financial asset with a single exercise date, which is known as a European option. Merton (1973) adapted the model to include options on dividend paying stocks. The Black-Scholes-Merton (BSM) price for a European call option trading at time t is:

$$C_t(S_t, T-t) = S_t\Phi(d_1) - Xe^{-r(T-t)}\Phi(d_2), \qquad (1)$$

where

$$d_1 = \frac{\log(S_t/X) + \left(r - \delta + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}, \qquad (2)$$

$$d_2 = d_1 - \sigma\sqrt{T-t}, \qquad (3)$$

S_t is the price of the underlying stock at time t, $\Phi(d_i)$ is the cumulative distribution value for a standard normal random variable with value d_i, X is the strike (exercise) price, r is the risk-free rate of interest, δ is the dividend rate, and T is the time of expiration.

The BSM price for a European put option trading at time t is:

$$P_t(S_t, T-t) = -S_t\Phi(-d_1) + Xe^{-r(T-t)}\Phi(-d_2), \qquad (4)$$

where d_1 and d_2 are given by expressions (2) and (3) above.

An American option grants its holder the right, but not the obligation, to buy or sell a share of common stock for a specified exercise price, X, at or anytime until the option's expiration at time T. Valuation of American-style options is more difficult than pricing European options because American options can be exercised on multiple dates. The BSM formula yields an approximation for the value of an American option, but in practice numerical techniques are used to obtain closer approximations of options that can be exercised at times other than the expiration time. The fair value of an American option is the discounted expected value of its future cash flows. The cash flows arise because the option can be exercised at the next instant, or at the following instant if not previously exercised, ..., *ad infinitum*. An American option has a higher value than an otherwise similar European option because of its greater optionality. Thus, the price of a European option provides an lower bound for the price of an American option with similar parameters.

In practice, American options are usually approximated by securities that can be exercised at only a finite number, k, of opportunities before expiration. These types of financial instruments are called Bermudan options. By choosing k large enough, the computed value of a Bermudan option will be practically equal to the value of an American option. The early exercise feature of Bermudan options makes their valuation more difficult because the optimal exercise policy must be estimated as part of the valuation. An exercise policy is defined by a set of stock prices and dates that the option holder uses to decide whether to exercise or hold the option. The optimal exercise policy is one that maximizes the discounted expected value of the future cash flows.

Publication of the BSM valuation formula for European options led to the development of numerous other financial option valuation methods. Cox, Ross and Rubinstein (1979) developed a binomial lattice method for valuing American or European financial options. When used to price a European option, this model provides a discrete-time approximation to the continuous-time Black-Scholes-Merton result. By working backwards from the expiration date of an American option, the lattice method allows valuation of American-type options with multiple exercise dates.

Several Monte Carlo simulation methods focus on valuing American options by pricing similar Bermudan options. These methods are highlighted in the remainder of this section.

2.1 Monte Carlo Simulation

Boyle (1977) uses Monte Carlo simulation as an alternative to numerical integration (Parkinson 1977) and finite difference (Schwartz 1977) methods for valuing options on financial assets. This method is particularly advantageous when the underlying asset follows a process characterized by difference equations that are difficult or impossible to solve analytically. Under this method, the distribution of terminal stock values is determined by the process generating future stock price movements; this process in turn determines the distribution of future terminal option values. To obtain an estimate of the option value at time t, a number of sample values are generated at random from the distribution describing the terminal (time T) values of the option. In turn, these terminal values are discounted and averaged over the number of trials. Charnes (2000) uses Boyle's technique to price various exotic options and also demonstrates variance reduction techniques to increase the precision of estimates of option values obtained by Monte Carlo simulation.

Barraquand and Martineau (1995) developed a numerical method for valuing American options with multiple underlying sources of uncertainty which uses Monte Carlo simulation. Their technique relies on partitioning the state space of possible exercise opportunities into a tractable number of cells, then computing an optimal cash flow management strategy that is constant over each cell. The option value is based on the strategy with the maximum value.

Grant et al. (1997) consider how to incorporate optimal early exercise in the Monte Carlo method by linking forward-moving simulation and backward-moving dynamic programming through an iterative search process. They simplify the problem by optimizing the option value with respect to a piece-wise linear early exercise hurdle, albeit at the expense of biasing the option value downward. After the exercise boundary is established at each potential exercise point, the price is estimated in a forward simulation based on the obtained boundaries.

American-style securities can be priced using simulation (Broadie and Glasserman 1997a) by developing a “high” and “low” estimator, then using the average to estimate the value of the option. While both estimators are biased, both are also consistent, so as the number of trials in the simulation is increased, the error bounds on the estimate narrow.

Longstaff and Schwartz (2001) present another method for valuing American options with simulation that utilizes least squares regression. First, a number of paths of the underlying asset are randomly generated and the cash flows from a corresponding European option in the last period are generated for each path. In the next to last period, the paths that are ”in the money” are selected and the cash flows are discounted to the current period. To estimate the expected cash flows from continuing the option’s life conditional on the stock price in the next-to-last period, the discounted option payoffs are regressed on basis functions of the stock price. With this conditional expectation function, the value of immediate exercise in the next to last period and the value from continuing the option can be compared. Using the optimal decision, the cash flow matrix for the next-to-last period is generated and the process is repeated. Given the sample paths, a stopping rule is created for each sample path. These cash flows are then discounted to the current period and averaged over all paths to estimate the option value.

Additional implementations of Monte Carlo simulation for pricing American-style options are described by Bossaerts (1989), Fu (1995), Fu and Hu (1995), Carriere (1996), Raymar and Zwecher (1997), and Ibanez and Zapatero (2004). Glasserman (2004) also provides a comprehensive textbook covering financial option valuation methods using Monte Carlo simulation. Stochastic dynamic programming and stochastic differential equation methods have also been presented as alternatives to pricing American-type options. These methods are similar to the aforementioned simulation methods in that they begin by evaluating cash flows in the last period of the option, then work recursively to determine the option value. The next section presents another method.

2.2 Simulation and Optimization

Fu et al. (2001) introduces a simulation-based approach that parameterizes the early exercise curve and casts the valuation problem as an optimization problem of maximizing the option value with respect to the associated parameters. This approach simultaneously optimizes the option value with respect to a parameter vector by iterative updates via a stochastic approximation algorithm. This approach is compared with two dynamic programming techniques (Tilley 1993, Grant et al. 1997) and the stochastic mesh and simulated tree methods of Broadie and Glasserman (1997a, 1997b, 1998) on a test bed of several American-style options. Wu and Fu (2003) gives further details of the application of this technique to American-Asian options.

Charnes (2000) shows how to use stochastic optimization to value a Bermudan put option by determining an optional exercise rule based on the stock price at each exercise date. This method uses commercially available software (Glover, et al., 1996). Cobb and Charnes (2004b) later extend this method to fit piece-wise linear and cubic Bézier curve exercise thresholds for American options to reduce the number of parameter estimates required.

3 REAL OPTIONS VALUATION

As methods for valuing financial options were developed, researchers began to recognize the potential for applying these techniques to valuation of real asset investment options.

Copeland and Keenan (1998a, 1998b) state that NPV ignores the flexibility to defer, abandon, expand, or undergo sequential investment. Real options valuation (ROV) is cited as being particularly useful when decisions must be made regarding later stages of projects and when learning options exist. A learning option is one that, upon exercise, yields a cash flow and another option. One difficulty in applying ROV is that real asset investments are affected by multiple sources of uncertainty. Such options are typically called rainbow options. Combinations of rainbow and learning options often exist in practice. ROV typically has the most effectiveness when competing projects have similar values when evaluated with traditional NPV methods.

One reason for using ROV, according to Brabazon (1999), is that thinking about investment projects in option terms encourages managers to decompose an investment into its component options and risks, which can lead to valuable insights about sources of uncertainty and how uncertainty will be resolved over time. This mindset also encourages managers to consider how best to enhance the value of their investments by building in more flexibility. Bowman and Moskowitz (2001) find that ROV is useful because it challenges the type of investment proposals that are submitted and encourages managers to think proactively and creatively.

ROV has the potential to allow companies to examine programs of capital expenditures as multi-year investments, rather than as individual projects (Copeland 2001). These programs of investments are typically strategic and highly dependent on market outcomes, a decision climate under which Miller and Park (2002) find ROV to be most useful.

ROV and DCF may still be complementary techniques, with DCF being suitable for basic replacement decisions.

Dixit and Pindyck (1994) and Trigeorgis (1996) provide texts that summarize much of the early work done in applying financial options valuation methodology to real options problems. Leslie and Michaels (1997) examine the parameters in the Black-Scholes-Merton model and their analogs in the context of the real options framework. These relationships are summarized in Table 1.

Early work on real options valuation demonstrates that if the analogous real options parameters can be estimated, any method used to value financial options can potentially be used to value real options. Often, many of the assumptions must be relaxed to make the connection. Amram and Kutilitilaka (1999), Copeland and Antikarov (2001) and Mun (2002a, 2002b) suggest feasible methods of applying ROV to problems in practice, providing step-by-step instructions for creating models that can be used to value projects.

4 APPLICATIONS

This section describes specific instances where ROV has been applied using various modeling techniques.

4.1 Black-Scholes model

Taudes et al. (2000) suggest using option pricing theory to value software platform decisions. Using managerial intuition and industry standards, they estimate parameters for the Black-Scholes model and compare results to an NPV analysis for a case study problem. Campbell (2002) uses the Black-Scholes model to evaluate the costs of waiting to invest, thus determining the optimal start-up date for an IS investment project. By calculating foregone cash flows and interest over the time the project is delayed, then discounting these to review times over the investment horizon, a decision rule is established. More frequent review periods were found to increase deferral time.

4.2 Lattice models

Trigeorgis (1993) uses the binomial lattice method to demonstrate the non-additivity in value of certain combinations of multiple options on the same project. Factors that affect the joint probability of exercising multiple options include type of option, whether in or out of the money, and the order of exercise of the options. Interactions between options can be positive or negative, depending on these factors.

Herath and Park (2002) use a compound binomial lattice model to value a multi-stage investment. This method assumes the value pertaining to each downstream investment and the volatility per unit time can be estimated directly from the cash flow estimates by developing the distribution for the rate of return; their simulation method (which is also suggested by Copeland and Antikarov (2001)) of estimating volatility uses only uncorrelated variables. Cobb and Charnes (2004a) show that correlation between inputs significantly affects volatility estimates created using simulation. Brandão et al. (2005) also use the simulation method for estimating project volatility, then apply this estimate to model real options using a decision tree based on a binomial lattice. Smith (2005) suggests an adaptation of the simulation method for estimating volatility, and applies this along with the Longstaff-Schwartz least squares method in a binomial lattice scheme to value an oil production project with buyout and divestiture options.

Kellogg and Charnes (2000) compare decision tree and binomial lattice methods for calculating the value of a biotechnology firm, including a growth option in the latter model. A volatility parameter is selected for the binomial lattice based on assuming a maximum value for the project after 12 years, then solving for a volatility level that allows cash flows to grow to this level. Including the growth option improves the estimates of the firm's value.

4.3 Dynamic programming

Smith and McCardle (1998) use a dynamic programming model with mean reversion in the underlying stochastic process for project value. Mean reversion greatly decreases the value of waiting to develop, particularly when facing long lead times. Moreover, because it implies narrower long-run confidence bands with a decreased probability of sustaining high or low future prices, mean reversion implies there is significantly less risk and value associated with long-term projects.

Dangl (1999) uses a dynamic programming approach to determine optimal timing for investment based on a threshold of the demand shift parameter. After the timing decision is resolved, the decision on installed capacity is made to maximize profit. Results indicate that increasing uncertainty causes an increase in project size but also increasing installation delays.

A model can be designed to account for technological progress in a duopoly where firms compete in the adoption of new technologies (Huisman and Kort 2000). In this model, increased uncertainty delays adoption of the current technology, which increases the probability that new technology is invented before the investment in new technology commences. This leads to the conclusion that increased revenue uncertainty induces a higher probability of moving to a new technology; uncertainty raises technological level.

Huchzermeier and Loch (2001) model an R&D project using dynamic programming. In their model, market payoffs are determined by a one-dimensional parameter and performance uncertainty manifests itself in the variability of a probability distribution; the value of managerial flexibility is enhanced by an increase in market payoff. When

Table 1: Analogous parameters in financial and real option models. The left column lists the six parameters that serve as inputs to the Black-Scholes-Merton (BSM) financial option pricing models given by expressions (1) and (4). The center column lists the real option valuation (ROV) parameters corresponding to the financial option parameters in the left column. The right column lists examples of the sources of uncertainty for the corresponding real option valuation model parameters.

BSM Parameter	Analogous ROV Parameters	Example Sources of Uncertainty
Stock price, S	Present value of expected cash flows from investment	Market demand for products and services, labor supply and cost, materials supply and cost
Exercise price, X	Present value of required investment costs in real asset	Availability, timing and price of real assets to be purchased
Stock price volatility, σ	Volatility of underlying cash flows	Volatility in market demand, labor cost, materials cost, correlation of model assumptions
Time to expiration, T	Period for which investment opportunity is available	Product life cycle, competitive advantage
Dividend rate, δ	Cash flows lost to competitors	Product life cycle, competitive advantage, convenience yield
Risk-free interest rate, r	Risk-free interest rate	Inflation, money market behavior

uncertainty is resolved after all decisions are made, more variability smears out contingencies and thus reduces the value of flexibility.

The types of investments a firm chooses will depend in part on what it expects those investments to reveal about its competencies, according to Bernardo and Chowdry (2002). They develop a model that captures the value of information a company can expect to learn after an investment and employ a variation of the implicit finite difference method to find an approximate solution. Their solution shows that higher volatility does not necessarily increase option value when the volatility results from noise that inhibits relevant signals about firm performance.

4.4 Monte Carlo Simulation

Gamba (2002) extends the Longstaff-Schwartz least-squares regression simulation method to determine the optimal stopping time and value of three combinations of real options. A portfolio of independent options on the same project is examined, compound real options are valued, and mutually exclusive, but not independent options are modelled. Analysis takes place within a Cox-Ross-Ingersoll economy with stochastic state variables. Schwartz (2002) also extends the Longstaff-Schwartz approach to patent valuation by modeling both time to completion and duration of cash flows protected by the patent as random variables. The dynamics of the cost to completion are described by a controlled diffusion process which includes control and technical uncertainty terms. The dynamics of net cash flow rate are described by a geometric Brownian motion process and may be correlated with the uncertainty in the expected cost to completion of the project. Interestingly, the methodology finds corner solutions: invest at the maximum rate or do not invest.

Implementation time is an irrelevant factor for financial option pricing but one that must be considered in real options valuation (Nembhard et al. 2002). A simulation model can be developed to incorporate a lag time between option exercise and the beginning of project cash flows. Nembhard et al. (2000) present a simulation model where project values are valued by dividing the life of the variable into four time intervals, similar to a lattice approach.

Tseng and Barz (2002) value a power plant using a multistage stochastic real option model and propose a solution procedure that integrates forward-moving Monte Carlo simulation with backward-moving dynamic programming. Prices for electricity and fuel are characterized by stochastic uncertainties and a commitment decision must be made before unit prices are revealed. The model represents physical constraints of the plant and the optimal decision strategy is determined by using simulation within each period.

Cortazar (2002) compares the backward induction procedures of dynamic programming, binomial and multinomial lattices, and finite difference procedures. Each of these procedures starts from a boundary condition and solves simultaneously for the asset value and optimal exercise policy. He finds that each of these procedures properly value American options, but that forward looking simulation cannot handle the valuation problem because the optimal strategy is not known in advance. He states that more research is needed to develop methods of simulation that combine forward and backward procedures for valuing American options.

A straightforward solution to the valuation of real options is obtained using commercially available software in a manner similar to that used by Charnes (2000) to value a Bermudan put option. This method is especially well suited to complicated situations involving many decisions. The stochastic optimization algorithms built into the software use efficient search techniques to pare down the solution space quickly and obtain near-optimal solutions easily, overcoming the difficulties identified by Cortazar (2002) and others regarding using forward simulation of project cash flows to obtain real option values. Cobb and Charnes (2003) utilize this simulation optimization method to price a Bermudan abandonment option on a project that has project cash flows that follow a mean-reverting, first-order autoregressive stochastic process. Cobb (2005) combines this simulation-optimization method with the financial options valuations techniques presented by Cobb and Charnes (2004b) to parameterize a piece-wise linear exercise threshold for a Bermudan abandonment option.

4.5 Decision analysis

Decision analysis methods have been used for capital budgeting, and several researchers have proposed an integration of decision analysis tools and ROV. The combination of these concepts may allow models that produce a solution for the value of a project and an optimal investment decision rule more intuitively and efficiently.

Lander (1997) and Lander and Shenoy (1999) compare influence diagrams, decision trees, and binomial lattice methods of valuing real options. The solved decision tree and binomial tree yield different valuations due to the different probabilities and discount rates used; however, the optimal strategies suggested are usually the same. Depending on the conditional probability distributions specified, an influence diagram model can emulate either a decision tree or binomial model.

Real options can be valued using the opportunity loss concept when a risk-free arbitrage method is not feasible (Herath and Park 2001, Park and Herath 2001). Using this approach, the expected value of perfect information is the maximum quasi-real option value. Since perfect information is rarely available, an approach to determining the appropriate expected value of sample information is explored.

De Reyck et al. (2002) achieve the same results by using the replicating portfolio approach and constructing a decision tree with the appropriate discount rate prevailing at each chance node. The decision tree method is regarded as more practical and intuitive than other ROV approaches because the structure of the tree models managerial flexibility.

Demirer et al. (2007) recognize that the effectiveness of ROV is affected by how well uncertainty is represented in the model. Influence diagrams can represent uncertainty compactly, so they are suggested as an alternative tool for ROV. The representation of different fundamental sources of uncertainty in influence diagrams improves the predictive capability of the model and leads to better estimates of the value of a biotechnology firm, when compared to decision tree and binomial lattice methods.

Use of simulation for multiple-stage decision problems within a decision analysis framework is possible, as shown by Charnes and Shenoy (2004). They propose a forward Monte Carlo method that generates observations at each stage of the problem from a small set of variables in an influence diagram. In each stage, a decision function is determined for a selected decision variable. Such a method may hold promise for efficient evaluation of complex real options problems.

5 ROV IN REGULATED INDUSTRIES

Teisberg (1994) characterizes regulation in a real options model by using three parameters: 1) the current market value of a completed project, 2) the expected rate of foregone earnings due to regulation, and 3) the fraction of cost recovery allowed upon abandonment. Regulation that restricts profits and losses effectively reduces the sensitivity of the project value to the firm's construction strategy (i.e. the value of flexibility is reduced). Option effects are lessened for a regulated firm because profit restrictions reduce the expected value of the project and increase the incentive to invest sooner since delay becomes more costly as profit restrictions take hold.

Using a stochastic differential equation model, Falco and Campo (2001) value a regulated project by adding a mean reverting term which describes the expected rate of change over time as influenced by regulatory and market conditions. Regulation in this model is deemed to reduce a project's value when it reflects high profits and limit losses when the value of the project is low.

ROV has the potential to assist telecommunications firms with investment decisions because the industry is characterized by high future uncertainty regarding the market for products and services (Bhagat 1999, Trigeorgis 1999). Economides (1999) encourages use of ROV in telecommunications to estimate the costs of providing services, as required for pricing regulated services. Emmerson (1999) and Sharkey (1999) agree and state that prices set equal to long-run marginal costs are not sufficient to achieve efficiency if only the costs of physical and financial resources are included in the calculations. Additional price increases to cover increased value from risky resource commitments are necessary and could be estimated using ROV.

Jamison (1999) applies ROV to estimate costs used as the basis for regulatory pricing decisions. Using a model that assumes the carrier produces multiple products and makes investment decisions subject to uncertain demand and regulatory controls, ROV helps to improve incentives to invest efficiently. Tardiff (1999) suggests the use of ROV for establishing a cost basis because they can better consider forward-looking, as opposed to historical, costs.

Strauss (1999) suggests that telecommunications can learn from applications of ROV to power plant valuation because the telecommunications and electric power industries have similarities in their cost structures. Both industries can benefit from a model that starts with a characterization of market price volatility and values the ability of the power plant (or telecommunications network) to respond to fluctuating markets.

Use of ROV in specific telecommunications applications is not yet widespread. One application examines the underlying drivers of the bandwidth market and applies a real options framework to the problem of optimal timing of investment into new capacity (d'Halluin et al. 2002). A solution is determined using differential equations, the results of which suggest that increases in volatility tend to delay the investment decision; additionally, as the growth rate in demand increases, the upgrade point occurs at a lower percentage of current network usage.

6 CONCLUSION

Applications of financial option pricing methods to real asset investments have become more widespread in the past decade. In the abstract, if the parameters required in a financial option pricing method can be related to similar parameters in a real asset investment problem, any method used to value financial options can be used to value real options. Estimating these parameters, particularly the volatility parameter, is often difficult though, because the underlying asset is not traded. In practice, managers making decisions on whether to exercise real options typically have no method of observing the precise value of the underlying asset at the time the option must be exercised.

One implicit assumption of financial options pricing methods is that the value of the underlying asset is known at the time the exercise decision is made, which makes the decision rule obvious. For example, a European put option on a traded stock will always be exercised if the market price is less than the exercise price on the expiration date. As a result, financial option pricing methods are most concerned with providing a value for the option so an investor can determine whether to invest in the option. However, real options valuation must be concerned with determining both a value and an optimal exercise decision rule. The decision rule is often based on an observation of market factors or project performance in the periods leading up to the exercise date. In some instances, the decision rule might be based on an updated forecast of expected future performance. Neither market observations or updated estimates, however, provide perfect information, so the value placed on a real option by a method that assumes underlying asset value is observable is an upper bound.

Given the difficulty in observing relevant information prior to the exercise date in real options valuation, future research in this area is needed on methods of determining an optimal decision rule, along with a value for the real option if the optimal decision rule is followed. Since multiple uncertainties are often not constant over the life of real options, methods that do not require combination of uncertainties into one volatility parameter show most promise. Additionally, the risk-free arbitrage assumptions implicit in financial options must be relaxed, as there is no opportunity for a risk-free hedge with most real options.

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